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Connected Sum of Graphs in 2-Dimension with Algorithm

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Abstract:

In this paper, we introduce algorithm on connected sum of the graphs 2-dimension (tape graph and complete tape graph) when the graphs are weighted, the connected sum in case of removing the vertices and removing the edges of the graphs. Some theorems related to these results are obtained. Also, some applications are obtained.

***Keywords*:** Algorithm, Connected sum, Tape graph, complete tape graph.

***Mathematics Subject Classification*:** 5C5, 68 R 10. 2010

1. *Introduction and background* :

In mathematics a graph is intuitively a finite set of points in space, called the vertices of the graph, some pairs of vertices being joined by arcs, called the edges of the graph [4,6].

The tape graph G is a diagram consisting of a finite non empty set of the elements with "line or curve" shape called "vertices" denoted by V(G) together with elements, with "tape" shape called "edges" denoted by E(G) [2].

The complete tape graph is a simple tape graph in which each pair of distinct vertices are adjacent is called a complete tape graph. We denote the complete tape graph of n vertices k; has n(n-1)/2 edges [5].

In mathematics and computer science, an algorithm is an effective method expressed as a finite list of well-defined instruction for calculating a function. In simple words an algorithm is a step-by-step procedure for calculations [8].

Graph algorithms are one of the oldest classes of algorithms and they have been studied for almost 300 years (in 1736) which solve problems related to graph theory. There are some of important algorithms for solving these problems [8].

Weighted graph is a graph for which each edge has an associated real number weight [4,7]. In Kruskal's algorithm, the edges of weighted graph are examined one by one in order. What will of increasing weight. At each stage the edge being examined is added to become the minimum spanning tree, provided that this addition doesn't create a circuit.

After n -1 edges have been added (where n is the number of vertices of the graph), these edges, together with the vertices of the graph form a minimum spanning tree for the graph [4,7].

The weighted Kruskal's algorithm

How it works:
Input: G (weighted connected undirected graph with n vertices).
Algorithm body:

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Build a subgraph T of G which consists of all the vertices of G with edges added at each stage.
1. Initialized T (empty graph) to have all vertices of G.
2. Let E be the set of all edges of G.
3. Find an edge e in E of least weight.
4. Delete e from E.
5. If addition of e to edge set of T doesn't produce a circuit.
   Then add e to the edge set of T.
   T is a minimum spanning tree of G [4,7].

Minimal spanning tree for a weighted graph is a spanning tree that has at least possible total weight compared to all other spanning trees for the graphs.

It is minimum spanning tree in a connected weighted graph with n ≥1 vertex carry out the following procedure:

Step (1) *Find an edge of least weight and call this* e^1. *Set k=1*

step (2) *While k<n, if there exists an edge e such that* $\{e\} \cup \{e^1,...,e^k\}$ *does not contain a circuit, let* e^{k+1} *be such an edge of least weight replace k by k+1, else output* $e^1,e^2,...,e^k$ *and stop.*
End while [4].

The connected sum defines by, let S_1^2 and S_2^2 are two compact surfaces, then the connect sum of S_1^2 and S_2^2, denoted by $S_1^2 \# S_2^2$, is constructed by the following steps:
1. Remove a small open disk from each of the sets S_1^2 and S_2^2, leaving the boundary circles on each of the surface.
2. Glue together the boundary circles to form the connected sum. It is clear in Fig.(1.1) [1,3].

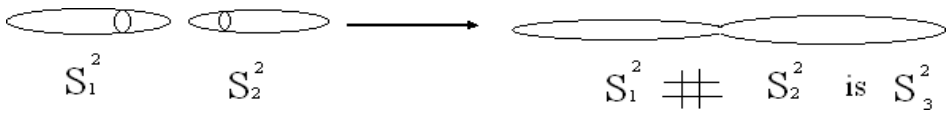

Fig.(1.1)

2. Main results:

El-Ghoul, M. [2] submitted the work of a connected sum of some graphs. He also works on algorithms of some types of graphs. In this paper we will introduce connected sum on algorithm of the graphs of 2-dim. When the graphs are weighted.

2.1. Connected sum of tape graph with algorithm:

We will compute the connected sum on algorithm of the tape graph when the tape graph is weighted by Kruscal's algorithm in case of removing the vertices and the corresponding edges of the tape graph.

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Case (1): Removing the vertices:

Let G_1 be a tape graph with two vertices v_0,v_1 and one edge e_0 and let G_2 be a tape graph with two vertices v_2,v3 and one edge e_1 and(dim 2), we can compute the connected sum on algorithm as follows:

Input: G_1(tape graph with vertices v_0,v_1),G_2(tape graph with vertices v_2,v_3)

Algorithm body:

Build a graph G_3 to G_1,G_2 by connected sum.

1. Visit v_0.
2. Attach the edge e_0,e_1 and go to the edge e_{01} in G_3.
3. Removing the vertices v_1,v_2
4. Go to step 1 for the other vertex v_3.
   End while.
5. Output G_3. End
   algorithm.

When we measure the weight of the tape graphs such as: G_1 ($v_0v_1 = e_0 = \varepsilon_0$),$G_2$ ($v_2v_3 = e_1 = \varepsilon_1$) and G_3 ($v_0v_3 = \varepsilon_0 + \varepsilon_1$). Where $\varepsilon_1 \geq \varepsilon_0$, it is shown in Fig.(2.1).

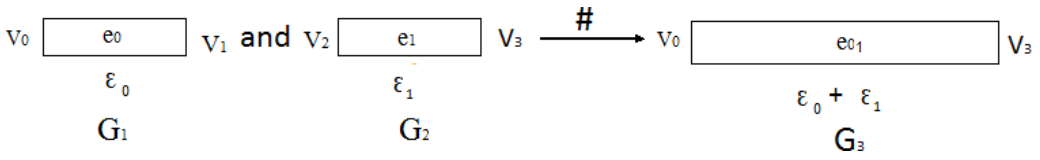

Fig.(2.1)

By using Kruscal's algorithm we find the minimum spanning tree as follows in Table (2.1).

Iteration no.	Edge considered	Weight	Action taken
1	v_0- v_3	$\varepsilon_0 + \varepsilon_1$	Added

Table (2.1)

The minimum spanning tree is a tape graph, shown in Fig.(2.2).

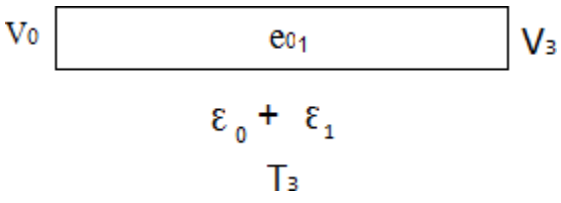

Fig.(2.2)

Case (2): Removing the corresponding edges:

<u>First:</u>

Input: G_1 (tape graph with vertices v_0,v_1),G_2 (tape graph with vertices v_2,v_3).

Algorithm body:

Build a graph G_3 from G_1,G_2 by connected sum.

1. Attach the vertices v_0 ,v_2 and go to the vertex v_{02} in G_3.
2. Attach the vertices v_1 ,v_3 and go to the vertex v_{13} in G_3.
3. Removing the edges e_0,e_1 from G_1 and G_2. End while.
4. Output G_3. End algorithm.

When we measure the weight of the tape graphs such as: G_1 (v_0v_1= e_0= ε_0), G_2 (v_2v_3= e_1= ε_1) and the weight of G_3 (v_{02}= ε_2, v_{13}= ε_3), where $\varepsilon_1 \geq \varepsilon_0$, ε_2, $\varepsilon_3 < \varepsilon_0,\varepsilon_1$, as shown in Fig.(2.3).

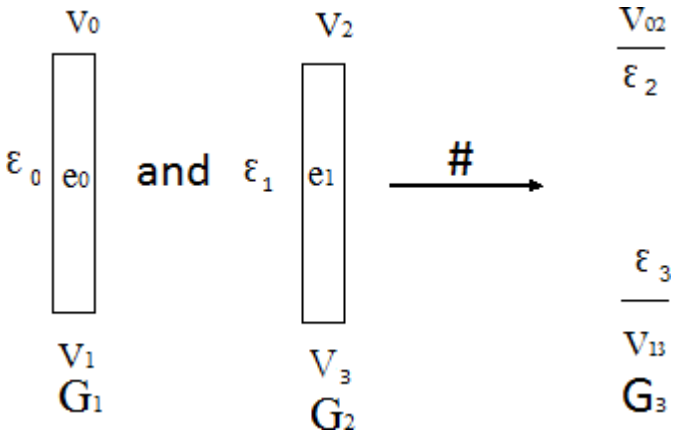

Fig.(2.3)

By using Kruscal's algorithm we find the minimum spanning tree as follows in Table (2.2).

Iteration no.	Edge considered	Weight	Action taken
1	v_{02}	ε_2	Added
2	v_{13}	ε_3	Added

Table (2.2)

The minimum spanning tree is a null graph, shown in Fig.(2.4).

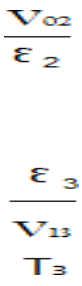

Fig.(2.4)

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Second:

Input: G_1 (tape graph with vertices v_0,v_1),G_2 (tape graph with vertices v_2,v_3). Algorithm body: Build a graph G_3 from G_1,G_2 by connected sum.

1. Visit v_0.
2. Attach the edge e_2, e_3 in G_3, where e_2, $e_3 \subset e_0$,e_1.
3. Go to step 1 for the other vertices v_1,v_2,v_3. End while.
4. Output G_3. End algorithm.

When we measure the weight of the tape graphs such as: G_1 ($v_0v_1 = e_0 = \varepsilon_0$),$G_2$ ($v_2v_3 = e_1 = \varepsilon_1$) and the weight of G_3 ($v_0v_2 = e_2 = \varepsilon_2$, $v_1v_3 = e_3 = 3$),where $\varepsilon_1 \geq \varepsilon_0$, ε_2, $\varepsilon_3 < \varepsilon_0 + \varepsilon_1$ it is shown in Fig.(2.5).

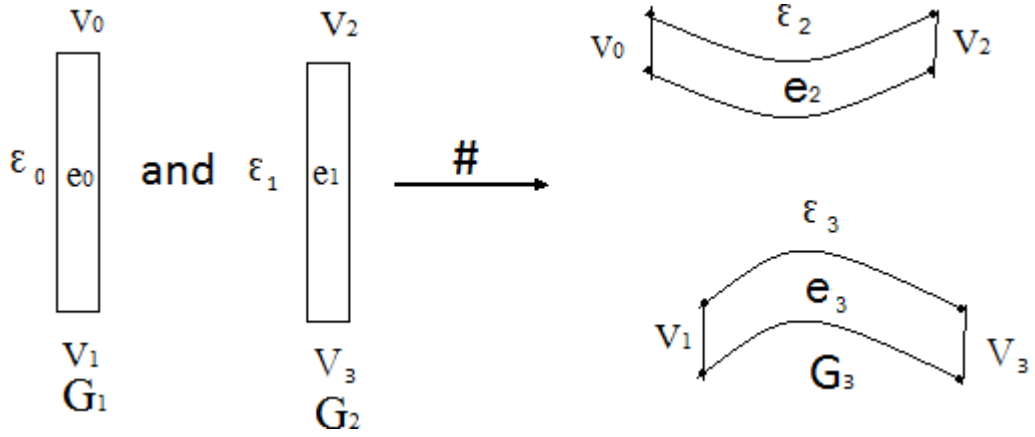

Fig.(2.5)

By using Kruscal's algorithm we find the minimum spanning tree as follows in Table (2.3).

Iteration no.	Edge considered	Weight	Action taken
1	v_0-v_2	ε_2	Added
2	v_1-v_3	ε_3	Added

Table (2.3)

The minimum spanning tree a graph, shown in Fig.(2.6).

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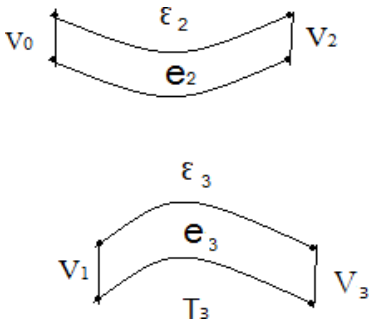

Fig.(2.6)

Theorem (1)
In case of removing all the corresponding edges, the connected sum on algorithm of the weighted tape graphs is weighted null graph.

Proof
The proof is clear from the above discussion, see Fig.(2.3) and Fig.(2.4).

2.2 Connected sum of complete tape graph with algorithm:
We will compute the connected sum on algorithm of the complete tape graph when the complete tape graph weighted by Kruscal's algorithm in case of removing the vertices and the corresponding edges of the tape graph.

Case(1): Removing the vertices:
Let G_1,G_2 be two complete tape graphs, and G_3 is a graph, let G_1 be a complete tape graph with three vertices v_0,v_1,v_2 and three edges e_0,e_1,e_2, let G_2 be a complete tape graph with three vertices v_3,v_4,v_5 and three edges e_3,e_4,e_5 (with dim 2), we can compute there algorithm as follows:
Input: G_1 (complete tape graph with vertices v_0,v_1,v_2),G_2 (complete tape graph with vertices v_3,v_4,v_5).
Algorithm body:
Build a graph G_3 from G_1,G_2 by connected sum.

1. Visit v_0.
2. Attach the edge e_0 to G_3.
3. Removing the vertices v_2,v_3.
4. Attach the edges e_2 ,e_5 and go to the edge e_{25} and the edges e_1,e_3 go to the edge e_{13} in G_3.
5. Go to step 1 for the other vertices v_1,v_4,v_5. End while.
6. Output G_3. End algorithm.

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When we measure the weight of the complete tape graphs such as: $G_1(v_0v_1= e_0= \varepsilon_0, v_1v_2 = e_1= \varepsilon_1, v_2v_0= e_2= \varepsilon_2)$, G_2 $(v_3v_4= e_3= \varepsilon_3, v_4v_5= e_4= \varepsilon_4, v_5v_3= e_5= \varepsilon_5)$ and G_3 $(v_0v_1= e_0= \varepsilon_0, v_0v_5= e_{25} = \varepsilon_7, v_5v_4= e_4= \varepsilon_4, e_{13}= \varepsilon_6)$,where $\varepsilon_1> \varepsilon_0, \varepsilon_2> \varepsilon_1, \varepsilon_3> \varepsilon_2, \varepsilon_4 > \varepsilon_3, \varepsilon_5> \varepsilon_4, \varepsilon_6< \varepsilon_1+ \varepsilon_3, \varepsilon_7 < \varepsilon_2+ \varepsilon_5$.

It is shown in Fig.(2.7).

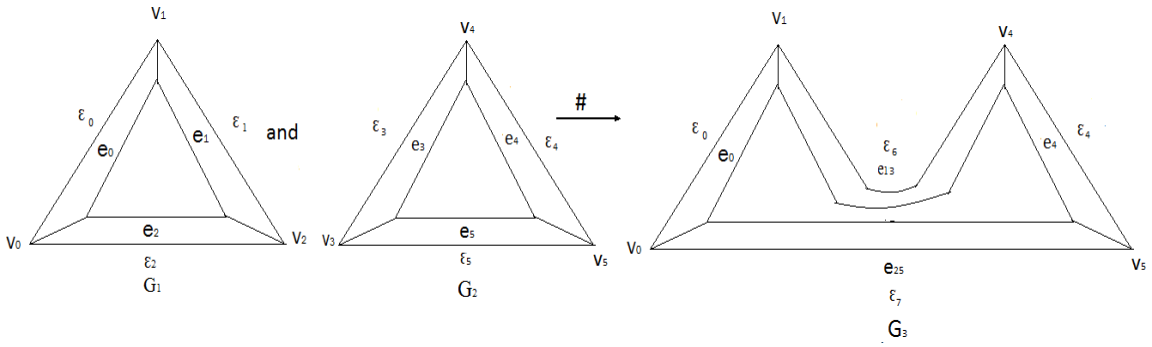

Fig.(2.7)

By using Kruscal's algorithm we find the minimum spanning tree as follows in table (2.4).

Iteration no.	Edge considered	Weight	Action taken
1	v_0-v_1	ε_0	Added
2	v_5-v_4	ε_4	Added
3	e_{13}	ε_6	Added
4	v_5-v_0	ε_7	Not added

Table (2.4)

The minimum spanning tree is a graph, shown in Fig.(2.8).

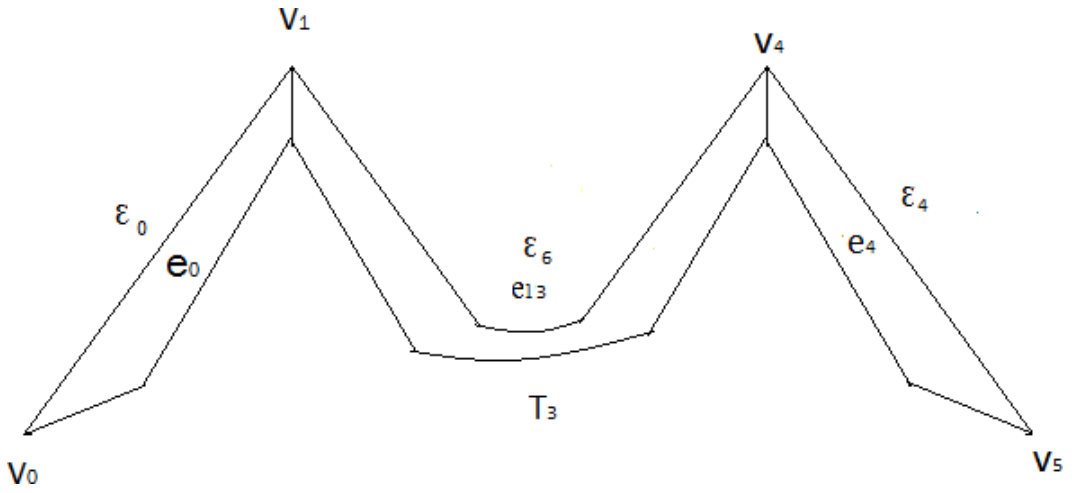

Fig.(2.8)

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Case (2): Removing the corresponding edges:

First:

Input: G_1 (complete tape graph with vertices v_0,v_1,v_2),G_2 (complete tape graph with vertices v_3, v_4,v_5).

Algorithm body:

Build a graph G_3 from G_1,G_2 by connected sum.

1. Attach the vertices v_0 ,v_3 and go to the vertex v_{03} in G_3.
2. Attach the edge e_0 to G_3.
3. Visit v_1.
4. Attach the edge e_1 to G_3.
5. Attach the vertices v_2 ,v_5 and go to the vertex v_{25} in G_3.
6. Removing the edges e_2,e_5.
7. Go to step 1 for the other vertex v_4.
   End while.
8. Output G_3. End
   algorithm.
   It is shown in Fig.(2.9)

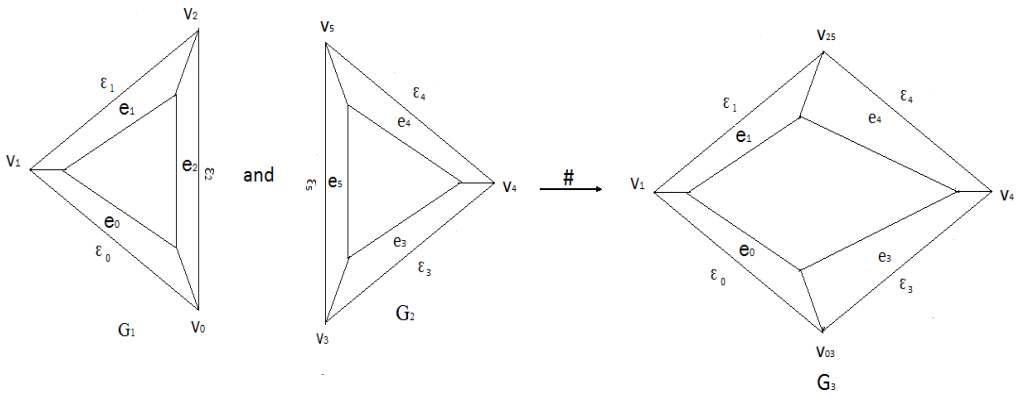

Fig.(2.9)

By using Kruscal's algorithm we find the minimum spanning tree as follows in table (2.5).

Iteration no.	Edge considered	Weight	Action taken
1	v_{03}-v_1	ε_0	Added
2	v_1-v_{25}	ε_1	Added
3	v_{03}-v_4	ε_3	Added
4	v_4-v_{25}	ε_4	Not added

Table (2.5)

The minimum spanning tree is a graph, shown in Fig.(2.10).

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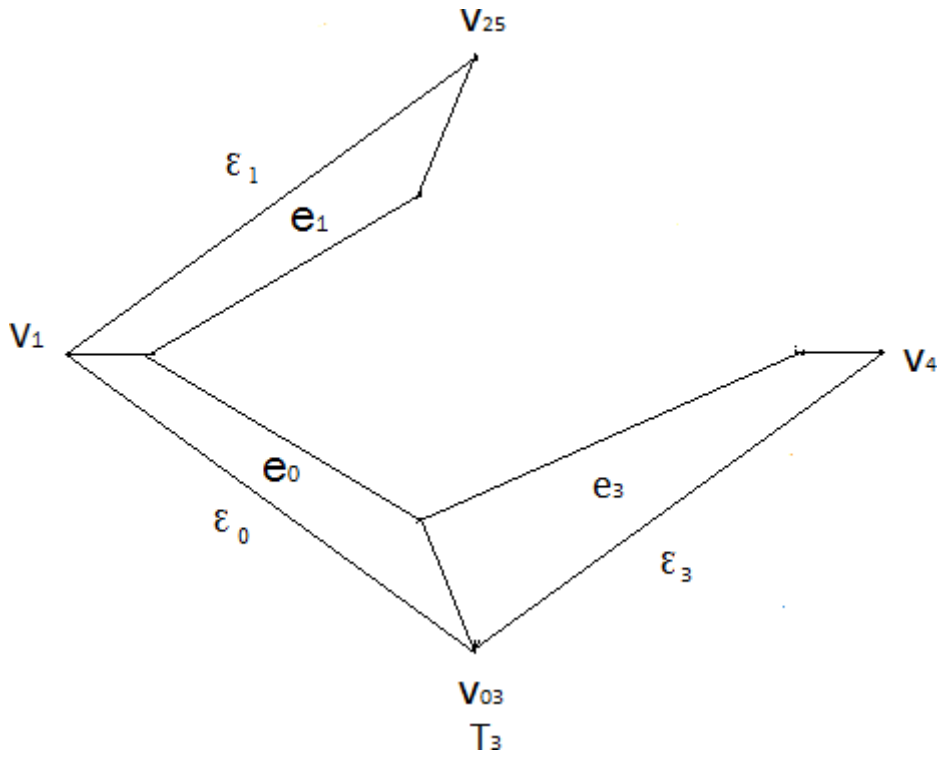

Fig.(2.10)

Second:

Input: G_1 (complete tape graph with vertices v_0,v_1,v_2),G_2 (complete tape graph with vertices v_3,v_4,v_5).

Algorithm body:

Build a graph G_3 from G_1,G_2 by connected sum.

1. Visit v_1.
2. Attach the edge e_0,e_1 to G_3.
3. Go to step 1 for the other vertices v_0,v_2.
4. Attach the edge e_6,e_7 to G_3, where $e_6,e_7 \subset e_2,e_5$.
5. Go to step 1 for the other vertices v_3,v_4,v_5. End while.
6. Output G_3. End algorithm.

When we measure the weight of the graph such as: G_3 ($v_0v_1 = e_0 = \varepsilon_0$, $v_1v_2 = e_1 = \varepsilon_1$, $v_2v_5 = e_6 = \varepsilon_6$, $v_5v_4 = e_4 = \varepsilon_4$, $v_4v_3 = e_3 = \varepsilon_3$,$v_3v_0 = e_7 = \varepsilon_7$), where $\varepsilon_1 > \varepsilon_0$, $\varepsilon_2 > \varepsilon_1$, $\varepsilon_3 > \varepsilon_2$, $\varepsilon_4 > \varepsilon_3$, $\varepsilon_5 > \varepsilon_4$, and $\varepsilon_6, \varepsilon_7 < \varepsilon_2+\varepsilon_5$. it is shown in Fig.(2.11).

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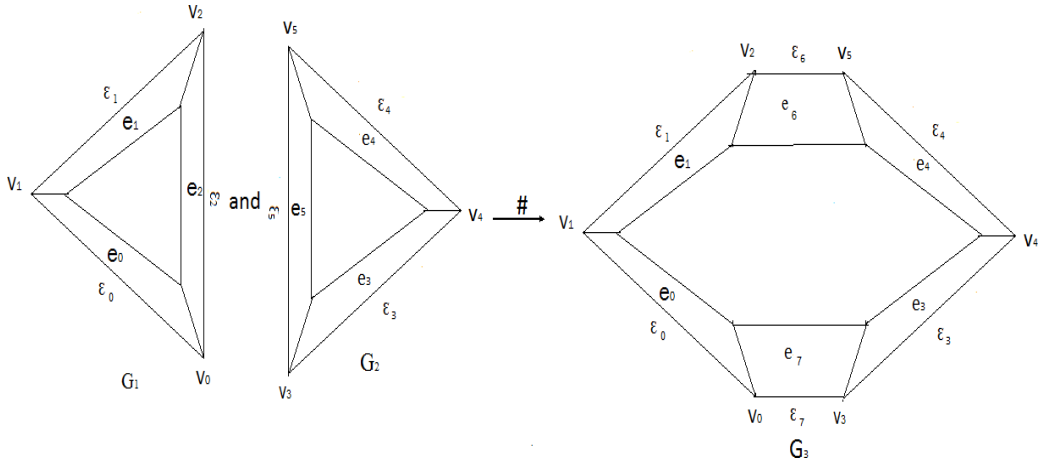

Fig.(2.11)

By using Kruscal's algorithm we find the minimum spanning tree as follows in Table (2.6)

Iteration no.	Edge considered	Weight	Action taken
1	v_0-v_1	ε_0	Added
2	v_1-v_2	ε_1	Added
3	v_3-v_4	ε_3	Added
4	v_4-v_5	ε_4	Added
5	v_5-v_2	ε_6	Added
6	v_3-v_0	ε_7	Not added

Table (2.6)

The minimum spanning tree a graph, shown in Fig.(2.12).

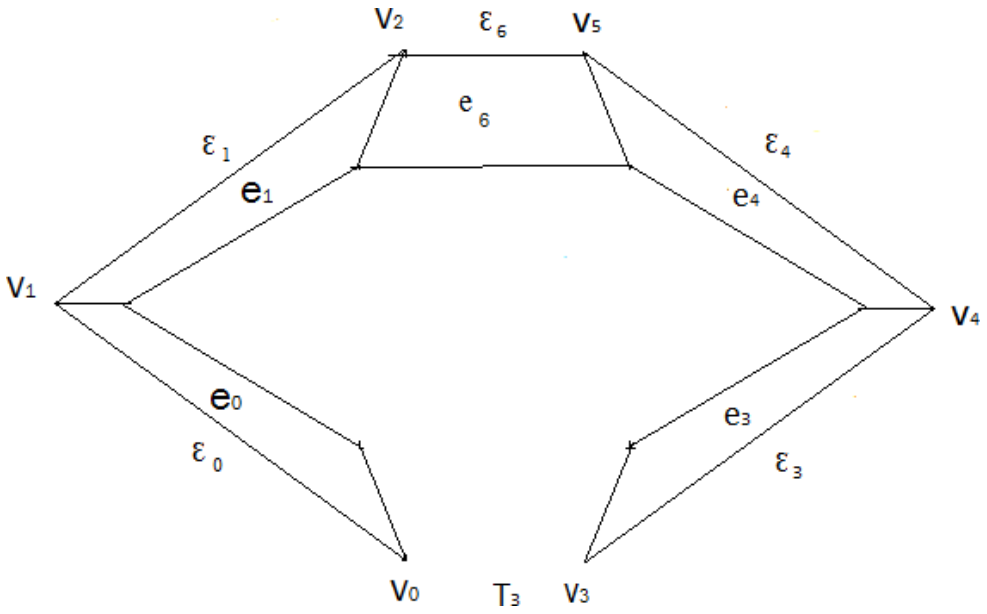

Fig.(2.12)

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Theorem (2):
The result of connected sum of the complete tape graph is a complete tape graph.
Proof:
The proof is clear from the above discussion, See Fig.(2.7) ,Fig. (2.9) and Fig.(2.11).

3. Application in life:
1- Electrical connections, which represent the connected sum of the weighted tape graph in case of removing the vertices.
2- Chemical bonds, which represent the connected sum of the weighted complete tape graph in case of removing the corresponding edges.

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