

Different Phase Retrieval Algorithms: A Review

Himanshu¹, Satinder Bal Gupta², Raj Kumar Yadav², Vikas Poply³

¹ Research Scholar, Indira Gandhi University, Meerpur, Rewari, India

² Associate Professor, Indira Gandhi University, Meerpur, Rewari, India

³ Assistant Professor, K.L.P. college, Rewari, India

Abstract

We think about the issue of stage recovery, in particular, recuperation of a sign from the greatness of its Fourier change, or of some other direct change. Because of the loss of Fourier stage data, this issue is poorly presented. Consequently, earlier data on the sign is required so as to empower its recuperation. Iterative calculations for stage recovery from force information are contrasted with inclination search techniques. As a union, the different picture stage recovery innovation is imagined to acquire more exact intermingling bring about iterative calculation. The various info recovery plans can append new imperatives on assembly as another impediment. As an aberrant estimating technique, it will make it conceivable to remake the dissemination of force and stage in an imaging or estimation framework, where information handling is executed by PC. In addition, the recovery technique has been applied for picture encryption effectively. Stage recovery strategy is viewed as one of the hugest instruments to take care of optical converse issues. A few stage recovery calculations are examined in this audit such as Gerchberg–Saxton, Yang–Gu, Hybrid input-output, Difference map, Encryption with phase retrieval, Double image encryption etc.

Keywords: Inverse problem, Phase measurement Security, Encryption, Watermarking, Phase retrieval algorithm.

INTRODUCTION

Stage recovery is a difficult opposite issue that emerges from various logical applications, for example, X-beam diffractive imaging, galactic imaging and optics. In an old style stage recovery issue, one attempts to recuperate a sign (or picture) from estimations that contain just the size (modulus) of its Fourier change. Since stage data is feeling the loss of, the reverse issue is poorly presented as a rule. Notwithstanding, if the Fourier size estimations are adequately oversampled, it has been demonstrated [1] that stage data can on a basic level be recuperated from an arrangement of quadratic conditions.

Recuperation of a sign from the extent of its Fourier change, otherwise called stage recovery, is of incredible enthusiasm for applications, for example, optical imaging, crystallography, and that's just the beginning. Because of the loss of Fourier stage data, the issue (in 1D) is commonly poorly presented [2]. A typical way to deal with beat this Sick posedness is to abuse earlier data on the sign. An assortment of strategies have been built up that utilization such earlier data, which might be the sign's help (locale in which the sign is nonzero), non-cynicism, or the sign's greatness[3]. A mainstream class of calculations depends on the utilization of substitute projections between the various requirements. So as to expand the likelihood of right recuperation, these techniques require the earlier data to be exact.

Stage Retrieval: The Missing Phase Problem [4].In high-recurrence (for example optical) applications, the (optical) discovery gadgets [examples , CCD camera, photosensitive movies, and the human being eye can't quantify their period of light wave show in Fig. A.

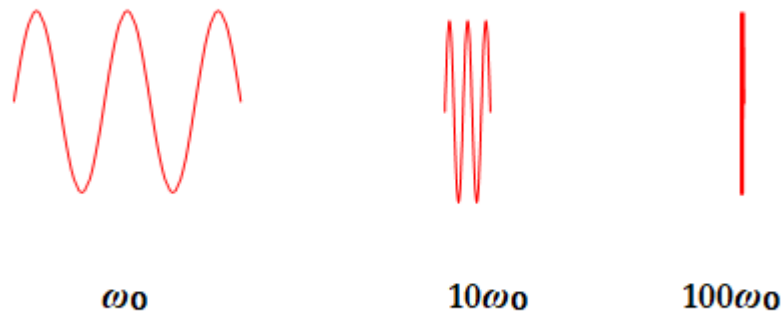


Fig. A: Various Frequency Period of light wave

Optical gadgets measure the photon motion (no. of photons every second per unit territory), which is corresponding to the greatness. This prompts the purported stage recovery issue — induction with just force estimations.

The traditional calculations, which began during the 1970s, were proposed at Gerchberg & Saxton (Error Reduction), and after that refined at Fienup (Hybrid Input-Output) [5]. A ton of late intrigue as a result of

- Modern applications in computational imaging: calculation and detecting co-plan.
- Connections with AI: understanding when non raised issues can be explained in a provable way utilizing straightforward calculations.
- This talk will zero in on iterative calculations: exchanging minimization and angle plunge.

In numerous physical estimation frameworks, one can just quantify the force otherworldly thickness, such that extent square of Fourier change of the basic sign. Such instance, when the optical setting, recognition gadgets similar CCD cameras and photo sensitive's movies can't quantify the period as to the light wave and rather compute these photon motion [6]. Moreover, at a huge enough good ways such that imaging planes the field is given by the Fourier change as the picture. Subsequently, in the far field, optical gadgets basically measure the Fourier change size. Since the stage encodes a ton of the basic substance of the picture, significant data is lost [7]. The issue of reproducing a sign among its Fourier greatness is familiar the stage recovery. This remaking issue is unified with a rich in past and emerges in numerous zones of designing and select material science, together with optics, X-beam crystallography, galactic imaging, discourse preparing, computational science, dazzle deconvolution [8] and that's only the tip of the iceberg.

Reproducing a sign from its Fourier extent alone is commonly an extremely troublesome undertaking. It is notable that Fourier stage is regularly more significant such Fourier' extent in remaking a sign from its Fourier' change [9]. They exhibit this reality, a manufactured model, politeness [10], is given in Fig. B. The fig. shows the consequence of the accompanying mathematical reenactment: Two pictures are Fourier' changed, their Fourier stages are traded and afterward they are converse Fourier changed. The outcome plainly shows the significance of Fourier stage. In this way, essentially disregarding the stage and playing out an opposite Fourier change doesn't prompt good recuperation. Rather, algorithmic stage recovery can be utilized, offering a methods for recuperating the stage from the given size estimations and perhaps extra earlier information, giving an option in contrast to advanced estimation arrangements as in holography which endeavor to legitimately gauge the stage by requiring obstruction with an alternative known state.

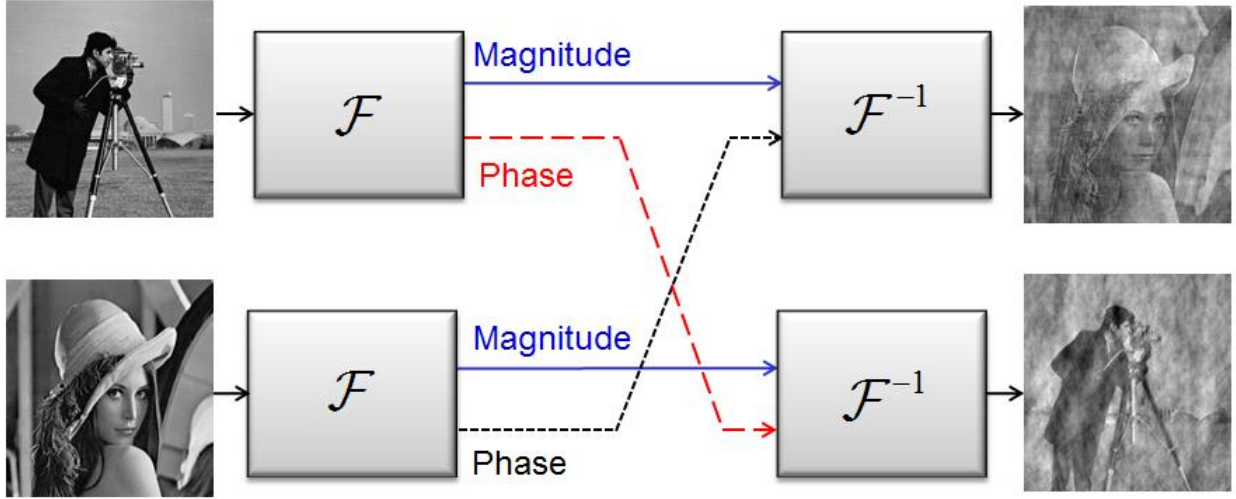


Fig (B): An engineered model showing the significance of Fourier stage in recreating a sign from its Fourier change

Stage recovery is the cycle of algorithmically discovering answers for the stage issue. Given an unpredictable sign $F(k)$, of sufficiency $|F(k)|$, and stage $\psi(k)$:

$$F(k) = |F(k)|e^{i\varphi(k)} = \int_{-\infty}^{+\infty} f(x)e^{-2\pi ikx} dx$$

Where x is a N-dimensional spatial arrange and k is a N-dimensional spatial recurrence facilitate [8]. Stage recovery comprises of finding the stage that fulfills a lot of requirements for estimated plentifulness. To show in fig. C.

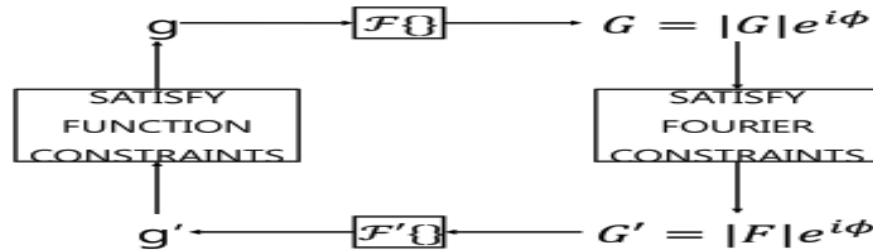


Fig.(C). Square chart of the blunder decrease (Gerchberg-Saxton) calculation

Methods & Algorithms

1. Iterative phase retrieval algorithm's

➤ Gerchberg--Saxton algorithm

These iterative techniques are notable ways to deal with tackle the one-dimensional stage recovery issue. Among them, the mistake decrease calculation is regularly utilized since it can undoubtedly actualize uphold limitations. Lamentably this technique regularly deteriorates [11]. As of late we have defined the all-inclusive type of the 1-D discrete stage recovery issue and we have expected that the stagnation can be stayed away from by oversampling. Recreations have demonstrated that the guess is valid show in Fig. D. The Gerchberg-Saxton calculation was initially concocted regarding the issue of reproducing stage from two force estimations (and for combining stage codes given power limitations in every one of two areas).

The calculation comprises of the accompanying four straightforward advances: (1) Fourier change a gauge of the article; (2) supplant the modulus of the subsequent processed Fourier change with the deliberate Fourier module to shape a gauge of the Fourier change; (3) converse Fourier change the gauge of the Fourier change; and (4) supplant the modulus of the subsequent figured picture with the deliberate item modulus to frame another gauge of the item [12]. Mathematically this can be expressed as equation 1:

$$\phi_{im}(xy) = \sum_l \sum_m A_{lm} \exp(l(\phi_{lm}(x,y) + \phi_{lm})) \dots\dots\dots(1)$$

Where $\phi_{im}(X, Y)$ is the absolute optical state that objective plane, A_{lm} and ϕ_{lm} are the adequacy and period of field there are source-plane at (x_l, y_m) , and $\phi_{lm}(X, Y)$ is the period of the optical field on the objective plane because of the point source situated at (x_l, y_m) on the source-plane.

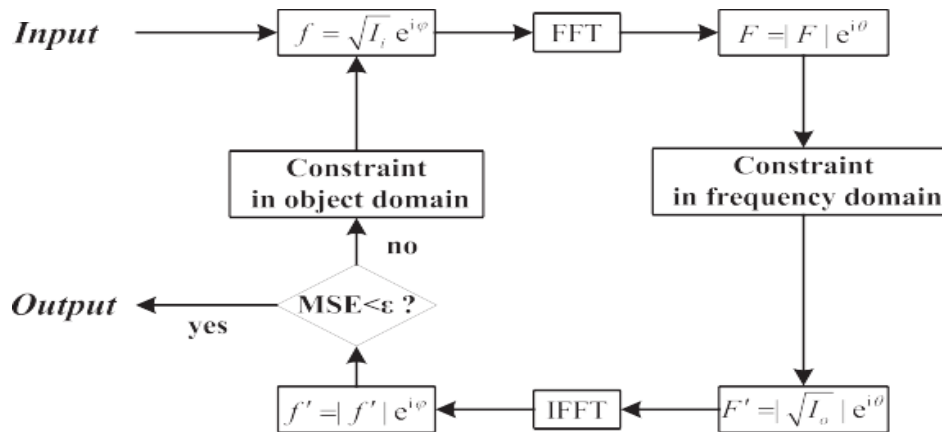


Fig.(D). The chart of Gerchberg--Saxton calculation, quick Fourier' change

➤ Yang Gu algorithm

As Gerchberg–Saxton calculation offers the answer for the stage recovery issue in which the two amplitudes of information and yield plane are known, it is only applied to unitary change framework [13]. Taking into account this, Yang–Gu calculation is designed and is end up being viable with any straight unitary or non-unitary change framework. Yang–Gu calculation requests that the imaging cycle be introduced by a groundbreaking framework G [14]. Particularly, G indicates an administrator of direct change framework, for example, Fresnel change, whirligig change or fragmentary Fourier change. Particularly, this strategy includes two sorts of cycle, specifically internal emphasis and outside cycle. The external cycle is halted when the MSE between the registered plentifulness and the deliberate abundancy meets the surmised condition. Just the same as Gerchberg–Saxton calculation, Yang–Gu calculation is gradually joined and touchy to the underlying estimation of the obscure stage in the recuperation preparing [15]. Its legitimacy is to stretch out Gerchberg–Saxton calculation into any direct frameworks.

➤ Input-Output Algorithm

An answer for the issue of the moderate assembly of the mistake decrease calculation has been the information yield calculation, which has demonstrated to meet quicker for both the issue of two force estimations 6 17 and the issue of a solitary power estimation [16]. The info yield calculation varies from the mistake decrease calculation just in the article space activity. The initial three activities Fourier changing $g(x)$, fulfilling the Fourier-area limitations, and backwards Fourier changing the outcome are the equivalent for the two calculations. Whenever gathered as demonstrated, those three tasks can be thought

of as a nonlinear framework having an information $g(x)$ and a yield $g'(x)$. The helpful property of this framework is that its yield is consistently a picture having a Fourier change that fulfills the Fourier-space limitations [17]. Consequently, if the yield additionally fulfills the article area requirements, it is an answer for the issue. Not at all like the mistake decrease calculation and the angle strategies, the info $g(x)$ not, at this point must be thought of as the current best gauge of the item; rather, it very well may be thought of as the driving capacity for the following yield, $g'(x)$. The information $g(x)$ doesn't really fulfill the item space limitations. This perspective permits one a lot of adaptability and imagination in choosing the following info, and takes into account the creation of calculations that combine all the more quickly to an answer.

- **Hybrid input-output algorithm**

Mixture input-yield calculation for stage recovery to create the figured picture fulfills the article area limitations. By and large the emphasess proceed until the processed Fourier change fulfills the article area limitations if there should be an occurrence of half breed input-yield calculation [18]. The proposed calculation was powerful in the early emphases for stripe example or line and space design pictures. In our investigation we acquired the quick intermingling for the pictures by acquainting the proposed calculation with the ordinary cross breed contribution out calculation. Crossover input-yield (HIO) calculation for stage recovery is a change of the mistake decrease calculation for recovering the stages in Coherent diffraction imaging. Deciding the periods of a diffraction design is critical since the diffraction example of an article is its Fourier change and so as to appropriately converse change the diffraction design the stages must be known [19]. Just the abundance nonetheless, can be estimated from the power of the diffraction design and would thus be able to be known tentatively. This reality along with some sort of help (science) can be utilized so as to iteratively figure the stages. The HIO calculation utilizes negative input in Fourier space so as to logically drive the answer for adjust to the Fourier area imperatives (support) [20]. Unlike the blunder decrease calculation which on the other hand applies Fourier and article limitations the HIO "skips" the item area step and replaces it with negative criticism following up on the past arrangement.

In spite of the fact that it has been indicated that the technique for blunder decrease joins as far as possible (however for the most part not to the right or ideal arrangement) there is no restriction to what amount of time this cycle can require. Additionally, the mistake decrease calculation will more likely than not discover neighbourhood minima rather than the worldwide [21]. The HIO varies from mistake decrease just in one stage yet this is sufficient to diminish this issue altogether. Though the blunder decrease approach iteratively improves arrangements after some time the HIO redesigns the past arrangement in Fourier space applying negative criticism. By limiting the mean square mistake in the Fourier space from the past arrangement, the HIO gives a superior competitor answer for converse changing [22]. Despite the fact that it is both quicker and more impressive than blunder decrease, the HIO calculation has a uniqueness issue. Contingent upon how solid the negative criticism is there can regularly be more than one answer for any arrangement of diffraction information. Albeit an issue, it has been indicated that a large number of these potential arrangements originate from the way that HIO takes into account identical representations taken in any plane to emerge as arrangements [23]. In crystallography, the researcher is only sometimes keen on the nuclear directions comparative with some other reference than the particle itself and is accordingly more than content with an answer that is tops tardy of flipped from the genuine picture. On the drawback, HIO tends to have the option to get away from both worldwide and nearby maxima. This issue likewise relies upon the quality of the input boundary, and a decent answer for this issue is to switch calculation when the mistake arrives at its base [24]. Different strategies for staging a sound diffraction design incorporate contrast map calculation and "loosened up found the middle value of substituting reflections" or RAAR.

So as to quicken the assembly speed of Gerchberg–Saxton calculation, half and half information yield (HIO) calculation has been proposed [25]. In light of the principle technique appeared in, it reinforces the requirement and acquires negative criticism to get the new assessed estimations of stage, whose strategy is delineated

$$\varphi^{k+1}(x, y) = \begin{cases} \theta^k(x, y), & (x, y) \notin \gamma \\ \varphi^k(x, y) - \delta\theta^k(x, y), & (x, y) \in \gamma \end{cases}$$

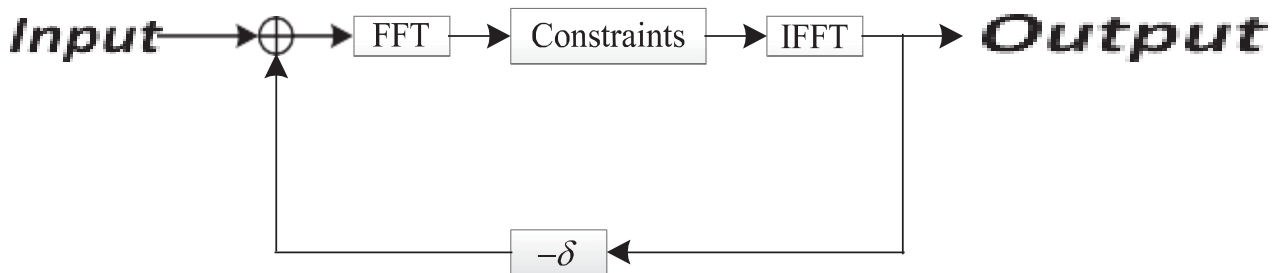


Fig. E: The phase reconstruction for HIO algorithm

Because of utilizing another assessed capacity to take care of the issue where the capacity of HIO calculation isn't continuous. HIO calculation can acquire a more precise outcome with less figuring time [26] show in Fig E. HIO calculation acquires Gerchberg–Saxton calculation's structure, yet it innovatively recovers the stage from a solitary power design. Henceforth HIO calculation is effective to be applied in the portrayal of **HST**. By utilizing estimated point spread capacity, the well-known consistent they essential reflection of the **HST** is determined [27]. (**HST** stands for Hubble space telescope)

➤ *Difference map*

Contrast map calculation is gotten after HIO calculation [28] and is utilized to recover the period of article plane from solitary power estimation. In light of Gerchberg–Saxton calculation, this strategy changes the item limitation and stores the recurrence requirement.

Accordingly the more tight requirements in both spatial and recurrence spaces are intended for this errand, for example, HIO calculation. When all is said in done, these two kinds of techniques expand the primary structure of iterative stage recovery [29].

• **Amplitude replacement**

This progression is the key activity in the established stage recovery it is the cardinal wellspring of estimation blunder, since this method is conceivable to make the iterative calculation inconsistent with the genuine framework. For instance, when the separation between the item plane and the deliberate plane has a dubious mistake with the goal that the separation in calculation isn't equivalent to the real separation, the reproduced picture would be obscured or even guide to a disappointment of recovery.

• **Propagation computation**

Proliferation calculation means to construct a diffraction model to recreate a genuine optical framework to achieve cycle among object plane and recognition level. After abundance substitution, every established engendering calculation is dem onstrated as a mistake decrease strategy, which is the

motivation behind why these sorts of calculations are equipped for remaking a wave plane. In this way it is essential to pick a legitimate engendering capacity [30]. Where is no uncertainty that Fourier change is a successful accomplishment for the calculation of light field, yet it is just applied to awfully field district, all the more solidly, Fraunhofer diffraction locale? With regards to the close state under free proliferation, Fresnel diffraction move work or rakish range hypothesis is the more exact model. Concerning a convoluted framework, it is important to address the fundamental exchange work so as to guarantee the reenactment truer.

- **Support constraint**

The expression 'uphold requirement' has been suggested in HIO calculation by Fienup. It partitions the article picture into 2- sections by fewer uncommon tasks and independently adapts to every part. The motivation behind this thought is to advance the iterative calculation and they remove that repetitive information. For example, when help district S, acquired via autocorrelation work in HIO calculation, contracts the registering locale and kills the excess focuses, which comparably accelerate the union.

In spite of the fact that HIO calculation gives the arrangement of single force picture, its outcome isn't ideal [31]. Thus early iterative stage recovery calculation is constantly utilized on account of twofold force designs. Here we think about the combination speed & the exactness of HIO calculation and GS calculation with twofold estimated force. Specifically, Yang–Gu calculation is simply the expansion of GS calculation in non- absolute change framework. Consequently this reproduction is centered on HIO and GS calculation.

- **Multiple-image phase retrieval algorithm**

The stage recovery matter can be served as the model of a lot of extreme-dimensional poorly adapted conditions in composite area for somewhat straight frameworks, which fundamentally causes equivocallness and low assembly speed for these strategies [32]. Consequently included force information from the estimations by utilizing diverse optical framework boundaries to get numerous deliberate pictures in the yield plane is used to find an inexact arrangement of the conditions. Various picture stage recoveries are viewed as a sort of strategies, which is made out of numerous units and the calculation of every part is the equivalent with the system in. Every part speaks to one of the qualities of optical framework and afterward the picture reproduction is accomplished by the joining of their parts. Here we present a few various picture stage recovery calculations in the accompanying as indicated by various kinds of the mix.

- **Serial computing**

The sequential registering necessitates that the yield of the past picture be utilized as the contribution of the following assessment and proliferation activity is cultivated by changing the highlights of optical framework. The most coherent activity for making numerous power designs is to change the change boundary in relating change space. In the multi-step calculation developed by Rodrigo, the sequential registering is applied to stage recovery in whirling change space. In light of the principle structure of iterative stage recovery calculation, it needs to quantify various pictures with various change points and figures the assessed complex sufficiency of the info plane in chronic request [33].

Where forward spread & back engendering are spinner change ($GT\alpha$) and opposite gyroscope change ($GT-\alpha$), separately. By appear differently in relation to Gerchberg–Saxton calculation, here the wave plane of article is totally obscure and its unpredictable sufficiency is reproduced by various estimated pictures of the yield plane. Expecting that the quantity of estimated pictures is N, in the wake of introducing the unpredictable sufficiency of the article plane arbitrarily, a registered picture is gotten by $GT\alpha$. Supplanting the figured sufficiency with measured abundances and left the processed stage, the new joined complicated adequacy of estimated plane is changed into the assessed picture of article plane

by GT— α . Specifically, every yield assessed output is utilized the following info starting worth. Consequently, after N seasons of this sort of sequential processing, one iterative count of the calculation is done [34]. At the point when iterative figured outcome fulfills the combination edge, emphasis will be halted. Simultaneously, the recouped information of plentifulness and stage is traded. On the other hand, the sequential figuring of stage recovery is accomplished through various enlightenment frequencies.

➤ *Parallel computing*

Equal figuring implies that various estimated pictures partake at the same time in computation and have no effect on one another. Parallel stage recovery calculation has been suggested in; they are called adequacy stage recovery (APR) calculation & iterative wave work recreation (IWFR) calculation, separately. As indicated by the principle system of, here different estimated pictures are utilized to recover the wave data of article by equal processing. These pictures are gotten by modifying the qualities of optical framework. The calculations for this sort of issues are accomplished in spinner area, precise range change space and even fragmentary Fourier space [35]. The proliferation capacities are the relating changes in stage recovery estimation. As of now, the blend of all equal units is done by the activity in this sort of calculation, where the assessed adequacy and stage in the article plane are determined by normal of relating figured information.

Truth be told, the precision of spread capacity is a key to stage recovery. In equal figuring, with the assistance of spatial and fleeting intelligence amendment, more exact proliferation work is introduced. In light of this adjustment, equal iterative stage recovery performs well in stage estimation and transmission electron magnifying lens.

Sequential figuring needs to manage estimated pictures individually. Despite what might be expected, equal registering structure requests that each deliberate picture be independently determined. On the off chance that the separation among info and yield plane contains mistake or estimated pictures have a few commotions, the blunder will be gathered with the cycle proceeding for sequential registering [36]. The mistake, notwithstanding, will be supported as a consistent territory for equal registering, since the normal activity can make the blunder no augmentation as it were or neutralize each other along the channels made out of change and its backwards.

Concerning combination speed and exactness, we utilize a reproduction to think about these characteristics of two kinds of calculations. We expect that the reproduction occurs in the free space and the spread capacity is range rakish capacity.

Encryption with phase retrieval

Stage recovery calculation is utilized for building up an optical change connection between two force pictures. It is valuable to decide the stage cover in optical framework. Along these lines any two individual pictures can be associated by utilizing a couple of assessed stage capacities from stage recovery in picture encryption and pressure [37]. Some new procedures of synchronous pressure and encryption techniques can be investigated, for example. As superficial light modulator is putted for stage and abundances regulation, stage recovery is a fundamental procedure for planning the stage sifting veil in optical data preparing framework.

Double image encryption method

A tale twofold picture encryption calculation is proposed, which can at the same time encode two pictures into a solitary one as the abundance of a fragmentary Fourier change with two unique gatherings of fragmentary requests. The two unique pictures can be recovered autonomously by fragmentary Fourier changes with two various gatherings of partial requests, one public stage cover, and two extraordinary private stage veils. The suggest perspective can augment the key space, accomplish quicker intermingling in the iterative cycle, and keep away

from cross-talk during the two pictures in reproduction. Mathematical recreations are introduced to check its legitimacy and proficiency [38].

➤ ***Encryption with non-uniform beam***

A unique mystery picture is viewed as the yield force of the second gyroscope change. A lucid nonuniform optical bar is changed over into the contribution of the first whirligig change. A Gerchberg-Saxton stage recovery calculation is utilized for getting the pay stages in the 1st gyroscope change couple. The pay stages are viewed as the encoded picture and key in this calculation. The boundaries of the timber bar and whirligig change can fill in as the extra key of encryption technique. The unscrambling cycle of this encryption calculation can be accomplished with an optical framework. Mathematical reproductions are performed to test the legitimacy and capacity of the encryption calculation [39]. As a possible approach in the optical data handling, this framework can be utilized for showing a picture with the assistance of stage regulation. A vortex bar has been presented for picture encryption and is viewed as the light wellspring of optical framework.

➤ ***Watermarking(WM)scheme***

WM innovation has been grown carefully in late twenty years. Optical WM conspire has been explored in the underlying stage. The arrangement of twofold irregular stage encoding is perhaps used to shroud a picture as water-stamping plan. The iterative stage recovery technique is utilized for acquiring the stages F1 and F2, which can be viewed as the key. The WM is shown in the yield plane. This technique is practical and has a decent heartiness. An optical WM strategy has been accounted for by utilizing one-step of whirligig change and delineated, in which a key stage D1 is from stage recovery calculation. Comparing to computerized WM, optical WM approach has a ton of open issues to do, for example, high strength and the ability [40]. Here stage recovery is a helpful device for the plan of the cycle installing WM.

➤ ***Multiple-image encryption***

As appeared in the few pictures are pre-put along the optical hub. The stage dispersion of POM is determined by an altered Gerchberg–Saxton calculation. Different picture encryption is another issue of optical data security procedures and has the reason for information pressure. The nature of recovered pictures, notwithstanding, is restricted by debate commotion in this sort of encryption calculation. The sequential structure has additionally been created in fragmentary Fourier change space [41]. The multi-plane stage recovery and obstruction can be embraced for consolidating a few pictures, wherein the recovery calculation has a decent intermingling conduct through a 100 times of emphases. Under the assistance of obstruction model, a grinding balance has been presented for concealing numerous pictures in WM measure.

➤ ***Color image encryption***

Shading picture cipher calculation is intended to conceal colourous picture in three individual channels. Stage recovery can be utilized to decide stage veil such a channel in the framework. Some dark level picture cipher plans can be used for concealing 3 monochromatic segments in shading picture cipher calculation. To improve the security of shading cipher calculation, pixel disorder activity has been utilized [42]. Multi-picture cipher conspire is additionally a potential strategy concealing shading picture, which is considered as three distinctive dim level pictures for disorder design.

➤ *Multiple-phase retrieval for encryption*

In the twofold arbitrary stage encipher (DRPE) or correlate encipher strategies, two irregular stages are utilized for securing the mystery picture [43]. As a straightforward variant of the calculation of various stage recoveries, twofold stage recovery is utilized for encryption with vortex shaft enlightenment, in which an adequate merged outcome can be gotten after 100 emphases. Despite the fact that this joined structure is intricate in trial framework and estimation of stage circulation, it lead a significant refinement at the part of safety of encryption calculation [44].

Conclusion

The combination of stage recovery calculation has been improved boundlessly by developing limitation situation or presenting criticism control lately. Particularly various picture stage recoveries have uncovered its favorable position in union and exactness. The recovery innovation has additionally been appertain effectively in a few optical imaging framework, for instance, adjusting the mistake of the primary focal point in Hubble scope and the magnifying instrument dependent on PIE. The diffraction imaging framework dependent on stage recovery maintains a strategic distance from the utilization of reference light emission structure in the estimation of stage data. For the problem phase retrieval in past year GS algorithm was introduced, but it has a limitation in the non-unitary cases. So, YG algorithm overcomes this problem by considering the generalization of GS. After that for error reduction in the phase retrieval input output algorithm comes plays an important role in the process of phase retrieval. To overcome the limitation mixed phase retrieval with image and sound, a hybrid input – output algorithm comes in encryption process of phase retrieval and also it has a fast convergence from all of the above algorithms.

REFERENCES

- [1] X. Ji, X. Liu, and B. Zhang, “Inverse acoustic scattering with phaseless far field data: Uniqueness, phase retrieval, and direct sampling methods,” *SIAM J. Imaging Sci.*, vol. 12, no. 2, pp. 1163–1189.
- [2] C. Guo, S. Liu, and J. T. Sheridan, “Iterative phase retrieval algorithms I: optimization,” *Appl. Opt.*, vol. 54, no. 15, p. 4698, 2015.
- [3] D. Y. Alsaka, Ç. Arpali, and S. A. Arpali, “A comparison of iterative Fourier transform algorithms for image quality estimation,” *Opt. Rev.*, vol. 25, no. 5, pp. 625–637, 2018.
- [4] S. Zhang, “Absolute phase retrieval methods for digital fringe projection profilometry: A review,” *Opt. Lasers Eng.*, vol. 107, no. January, pp. 28–37, 2018.
- [5] Y. Chi, “Recent Progress on Algorithmic Phase Retrieval,” no. June, 2017.
- [6] C. Rusu and J. Astola, “Convergence analysis of error-reduction algorithm for solving of the extended one-dimensional discrete phase retrieval problem,” *ISSCS 2017 - Int. Symp. Signals, Circuits Syst.*, pp. 0–3, 2017.
- [7] O. Raz, B. Leshem, J. Miao, B. Nadler, D. Oron, and N. Dudovich, “Direct phase retrieval in double blind Fourier holography,” *Opt. Express*, vol. 22, no. 21, p. 24935, 2014.
- [8] G. Yang, B. Dong, B. Gu, J. Zhuang, and O. K. Ersoy, “Gerchberg–Saxton and Yang–Gu algorithms for phase retrieval in a nonunitary transform system: a comparison,” *Appl. Opt.*, vol. 33, no. 2, p. 209, 1994.
- [9] R. Amézquita-Orozco and Y. Mejía-Barbosa, “Gerchberg-Saxton algorithm applied to a translational-variant optical setup,” *Opt. Express*, vol. 21, no. 16, p. 19128, 2013.
- [10] Z. Wen, C. Yang, X. Liu, and S. Marchesini, “Alternating direction methods for classical and ptychographic phase retrieval,” *Inverse Probl.*, vol. 28, no. 11, 2012.
- [11] Y. Shechtman, A. Beck, and Y. C. Eldar, “GESPAR: Efficient phase retrieval of sparse signals,” *IEEE Trans. Signal Process.*, vol. 62, no. 4, pp. 928–938, 2014.

- [12] J. R. Fienup, “Phase retrieval algorithms: A personal tour [Invited],” *Appl. Opt.*, vol. 52, no. 1, pp. 45–56, 2013.
- [13] J. J. Su and C. H. Tien, “Improved Error Reduction and Hybrid Input Output Algorithms for Phase Retrieval by including a Sparse Dictionary Learning-Based Inpainting Method,” *Int. J. Opt.*, vol. 2020, 2020.
- [14] J. R. Fienup, “Phase retrieval algorithms: a comparison,” *Appl. Opt.*, vol. 21, no. 15, p. 2758, 1982.
- [15] J. G. Jo, S. J. Cho, M. C. Park, Y. M. Jhon, and B. K. Ju, “Modified hybrid input-output algorithm for phase retrieval,” *Tech. Dig. - 25th Int. Vac. Nanoelectron. Conf. IVNC 2012*, pp. 412–413, 2012.
- [16] E. J. R. Pauwels, A. Beck, Y. C. Eldar, and S. Sabach, “On Fienup Methods for Sparse Phase Retrieval,” *IEEE Trans. Signal Process.*, vol. 66, no. 4, pp. 982–991, 2018.
- [17] H. M. Ozaktas, M. Alper Kutay, and Ç. Candan, “Fractional fourier transform,” *Transform. Appl. Handbook, Third Ed.*, vol. 14, no. 10, pp. 14-1-14-28, 2010.
- [18] K. Huang, Y. C. Eldar, and N. D. Sidiropoulos, “Phase Retrieval from 1D Fourier Measurements: Convexity, Uniqueness, and Algorithms,” *IEEE Trans. Signal Process.*, vol. 64, no. 23, pp. 6105–6117, 2016.
- [19] K. Jaganathan, S. Oymak, and B. Hassibi, “Sparse phase retrieval: Uniqueness guarantees and recovery algorithms,” *IEEE Trans. Signal Process.*, vol. 65, no. 9, pp. 2402–2410, 2017.
- [20] J. M. Rodenburg and H. M. L. Faulkner, “A phase retrieval algorithm for shifting illumination,” *Appl. Phys. Lett.*, vol. 85, no. 20, pp. 4795–4797, 2004.
- [21] R. Beinert and G. Plonka, “Sparse Phase Retrieval of One-Dimensional Signals by Prony’s Method,” *Front. Appl. Math. Stat.*, vol. 3, no. April, pp. 1–10, 2017.
- [22] G. Jagatap and C. Hegde, “Fast, sample-efficient algorithms for structured phase retrieval,” *Adv. Neural Inf. Process. Syst.*, vol. 2017-Decem, no. Nips, pp. 4918–4928, 2017.
- [23] C. Guo, Y. Zhao, J. Tan, S. Liu, and Z. Liu, “Multi-distance phase retrieval with a weighted shrink-wrap constraint,” *Opt. Lasers Eng.*, vol. 113, no. July 2018, pp. 1–5, 2019.
- [24] E. Osherovich, “Numerical methods for phase retrieval,” 2012, [Online]. Available: <http://arxiv.org/abs/1203.4756>.
- [25] H. H. Bauschke, P. L. Combettes, and D. R. Luke, “Phase retrieval, error reduction algorithm, and Fienup variants: a view from convex optimization,” *J. Opt. Soc. Am. A*, vol. 19, no. 7, p. 1334, 2002.
- [26] Y. Xiong, R. Kumar, and C. Quan, “Security Analysis on an Optical Encryption and Authentication Scheme Based on Phase-Truncation and Phase-Retrieval Algorithm,” *IEEE Photonics J.*, vol. 11, no. 5, pp. 1–14, 2019.
- [27] C. Zhang, M. Wang, Q. Chen, D. Wang, and S. Wei, “Two-Step Phase Retrieval Algorithm Using Single-Intensity Measurement,” *Int. J. Opt.*, vol. 2018, 2018.
- [28] A. Tripathi, S. Leyffer, T. Munson, and S. M. Wild, “Visualizing and improving the robustness of phase retrieval algorithms,” *Procedia Comput. Sci.*, vol. 51, no. 1, pp. 815–824, 2015.
- [29] H. Wang, C. Luo, L. Zhong, S. Ma, and X. Lu, “Phase retrieval approach based on the normalized difference maps induced by three interferograms with unknown phase shifts,” *Opt. Express*, vol. 22, no. 5, p. 5147, 2014.
- [30] S. Mohammadi, F. Brun, C. Dullin, D. Dreossi, and G. Tromba, “A comparison of free software implementations of phase retrieval algorithms for propagation-based X-ray microtomographic imaging,” *Int. Symp. Image Signal Process. Anal. ISPA*, no. September, pp. 622–626, 2013.
- [31] N. H. Thao, D. R. Luke, O. Soloviev, and M. Verhaegen, “Phase Retrieval with Sparse Phase Constraint,” *SIAM J. Math. Data Sci.*, vol. 2, no. 1, pp. 246–263, 2020.
- [32] J. Wu, J. Wang, Y. Nie, and L. Hu, “Multiple-image optical encryption based on phase retrieval algorithm and fractional Talbot effect,” *Opt. Express*, vol. 27, no. 24, p. 35096, 2019.
- [33] A. Migukin, V. Katkovnik, and J. Astola, “Optimal phase retrieval from multiple observations with Gaussian noise: augmented Lagrangian algorithm for phase objects,” *Opt. Meas. Syst. Ind.*

- Insp. VII*, vol. 8082, p. 80820L, 2011.
- [34] P. L. Aisher, J. Crass, and C. Mackay, “Wavefront phase retrieval with non-linear curvature sensors,” *Mon. Not. R. Astron. Soc.*, vol. 429, no. 3, pp. 2019–2031, 2013.
 - [35] Y. Yang, M. Pesavento, Y. C. Eldar, and B. Ottersten, “Parallel Coordinate Descent Algorithms for Sparse Phase Retrieval,” *ICASSP, IEEE Int. Conf. Acoust. Speech Signal Process. - Proc.*, vol. 2019-May, pp. 7670–7674, 2019.
 - [36] C. Guo, C. Shen, J. Tan, X. Bao, S. Liu, and Z. Liu, “A robust multi-image phase retrieval,” *Opt. Lasers Eng.*, vol. 101, no. October 2017, pp. 16–22, 2018.
 - [37] H. Di, Y. Kang, Y. Liu, and X. Zhang, “Multiple image encryption by phase retrieval,” *Opt. Eng.*, vol. 55, no. 7, p. 073103, 2016.
 - [38] H. Li and Y. Wang, “Double-image encryption by iterative phase retrieval algorithm in fractional Fourier domain,” *J. Mod. Opt.*, vol. 55, no. 21, pp. 3601–3609, 2008.
 - [39] Z. Liu, L. Xu, C. Lin, and S. Liu, “Image encryption by encoding with a nonuniform optical beam in gyrator transform domains,” *Appl. Opt.*, vol. 49, no. 29, pp. 5632–5637, 2010.
 - [40] Z. Liu, L. Xu, Q. Guo, C. Lin, and S. Liu, “Image watermarking by using phase retrieval algorithm in gyrator transform domain,” *Opt. Commun.*, vol. 283, no. 24, pp. 4923–4927, 2010.
 - [41] C. Guo *et al.*, “A review of iterative phase retrieval for measurement and encryption,” *Opt. Lasers Eng.*, vol. 89, pp. 2–12, 2015, doi: 10.1016/j.optlaseng.2016.03.021.
 - [42] M. H. Annaby, M. A. Rushdi, and E. A. Nehary, “Color image encryption using random transforms, phase retrieval, chaotic maps, and diffusion,” *Opt. Lasers Eng.*, vol. 103, no. September 2017, pp. 9–23, 2018.
 - [43] H. T. Chang, W. C. Lu, and C. J. Kuo, “Multiple-phase retrieval for optical security systems by use of random-phase encoding,” *Appl. Opt.*, vol. 41, no. 23, p. 4825, 2002.
 - [44] W. Chen and X. Chen, “Optical image encryption based on multiple-region plaintext and phase retrieval in three-dimensional space,” *Opt. Lasers Eng.*, vol. 51, no. 2, pp. 128–133, 2013.