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On Algebraic Characteristic on Normalization of Anti Q −Fuzzy B – Ideals in B – Algebra

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Abstract

In this communication of this paper described are algebraic characteristic on normalization of anti $\mathrm{Q}-$ fuzzy B – sub algebra (NAQFBSA) and also derived various algebraic aspects of normalization of anti $\mathrm{Q}-$fuzzy B – ideal (NAQFBI). and discuss number of their results.

Keywords: Fuzzy Set; Fuzzy B-Ideal(FBI),;Fuzzy B-Sub algebra(FBSA), ; Fuzzy B-Algebra(FBA),;Anti $\mathrm{Q}-$fuzzy subset (AQFSb),; Anti $\mathrm{Q}-$fuzzy B-Ideal (AQFBI),; Anti $\mathrm{Q}-$fuzzy B Sub algebra (AQFBSA), ; Normal anti $\mathrm{Q}-$ fuzzy B- ideal (NAQFBI) and Normal anti $\mathrm{Q}-$ fuzzy B- sub algebra(NAQFBSA).;

1. Introduction

In 1954, described the new idea of fuzzy subsets by L A Zadeh[9] R Biswas [1] developed the new concept of fuzzy subgroups and anti fuzzy subgroups in 1990.Sharma P K [5], defined the new concept of anti $\mathrm{Q}-$ fuzzy subgroups in 2012. J R Choet.al..[2] Introduced the B-algebras and quasi-groups in 2001. In 2001, depicted the notion of Quadratic B-algebras by H K Parket.al.[3]. KeumseongBanget.al.[7] introduced the fuzzy sub-algebra in B-algebra in 2003.In 2018, Prasanna A et al..[4], normalization of Fuzzy B – Ideals in B – Algebra.In 2014,develop the concept of Fuzzy B-ideals by S Kailasavalliet al.[8].In 2009,Solairaju A and Nagarajan R [6], derived the new structure and construction of Q- fuzzy groups.

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In the short communication of this paper, developed the new notion of algebraic structures on $NAQFBSA$ of B – algebras and $NAQFBI$ of B – algebras and establish a number of its properties in intimately.

2. Preliminaries

2.1 Definition [2]

A B-algebra is a non-empty set M with a constant 0 and a binary operation '$*$' satisfying the following axioms:

(1) $a * a = 0$
(2) $a * 0 = a$
(3) $(a * b) * c = a * (c * (0 * b)), for\ all\ a, b, c \in M$

For brevity, we also call M a B-algebra. In M we can define a binary relation "≤" by $a \leq b$ if and only if $a * b = 0$.

2.2 Definition [2]

A non-empty subset M of a B-algebra M is called a sub-algebra of M if $a * b \in M$ for any $a, b \in M$.

2.3 Definition [3]

A non-empty subset N of a B-algebra M is called a B-ideal of M if it satisfies for $a, b, c \in M$

(1) $0 \in N$
(2) $(a * b) \in N$ and $(c * a) \in N$ implies $(b * c) \in N$

2.4 Definition [9]

Let $(M,* ,0)$ be a B-algebra, a $FS\tau$ in M is called a FBI of M if it satisfies the following axioms z

(1) $\tau(0) \geq \tau(x)$
(2) $\tau(y * z) \geq \tau(x * y) \wedge \tau(z * x), for\ all\ x, y, z \in M$

*	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

The fuzzy set τ given by
$\tau(0) = 0.8, \tau(1) = 0.5, \tau(2) = 0.2$ is a fuzzy B-ideal.

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2.5 Definition [7]

Let Q and H be a set and group respectively. Then the mapping $A: H \times Q \rightarrow [0,1]$ is called a Q-fuzzy set in H.

2.6 Definition [7]

Let the mapping $f: H_1 \rightarrow H_2$ be a homomorphism . Let A and B be QFS of H_1 and H_2 respectively, then $f(A)$ and $f^{-1}(B)$ are image of A and the inverse image of B respectively, defined as

i. $f(A)(v,q) = \begin{cases} \sup\{A(u,q): u \in f^{-1}(v)\}, & \text{if } f^{-1}(v) \neq \emptyset \\ 0, & \text{if } f^{-1}(v) = \emptyset \end{cases}$, for every $v \in H_2$ and $q \in Q$.

ii. $f^{-1}(B)(u,q) = B(f(u),q)$, for every $u \in H_1$ and $q \in Q$.

2.7 Definition [1]

Let A be fuzzy subset of a group H. Then A is said to an anti-fuzzy subgroup if $A(u^{-1}v) \leq \max\{A(u), A(v)\}$, for all $u, v \in H$.

2.8 Definition [6]

A function $A : H \times Q \longrightarrow [0,1]$ is a anti Q-fuzzy subgroup of a group H if $A(uv^{-1}, q) \leq \max\{A(u,q), A(v,q)\}$,for all $u, v \in H$ and $q \in Q$.

3. Algebraic characteristic of Normalization of $AQFBI$ in B-Algebra

3.1 Definition

Let $AQFBI\xi$ of M is said to be normal if there exists $\theta \in M \; and \; q \in Q$ such that $\xi(\theta, q) = 1$.

3.2 Lemma

A $AQFBI\xi$ of M is normal, if and only if $\xi(0,q) = 1$.

3.3 Theorem

For any $AQFBI\xi$ of M we can generate a $NAQFBI$ of M which contains ξ.

Proof:

Let ξ be a $AQFBI$ of M

Define a $AQFS$ ξ^+ of M as

$$\xi^+(\theta, q) = \xi(\theta) + \xi^c(0,q) \, , \forall \, \theta \in M \,\&\, q \in Q$$

Let $\theta, \varphi, \omega \in M \,\&\, q \in Q$

(i) $\xi^+(0,q) = \xi(0,q) + \xi^c(0,q)$

$\leq \xi(\theta, q) + \xi^c(0,q)$

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$$= \xi^{+}(\theta, q)$$

(ii) $\xi^{+}(\varphi * \omega, q) = \xi(\varphi * \omega, q) + \xi^{c}(0, q)$

$$\leq \{\xi(\theta * \varphi, q) \vee \xi(\omega * \theta, q)\} + \xi^{c}(0, q)$$

$$= \{\xi(\theta * \varphi, q) + \xi^{c}(0, q)\} \vee \{\xi(\omega * \theta, q) + \xi^{c}(0, q)\}$$

$$= \xi^{+}(\theta * \varphi, q) \vee \xi^{+}(\omega * \theta, q)$$

$$\Longrightarrow \xi^{+}(\theta, q) \leq \xi^{+}(\theta * \varphi, q) \vee \xi^{+}(\omega * \theta, q)$$

and

$$\xi^{+}(0, q) = \xi(0, q) + \xi^{c}(0, q)$$

$$= \xi(0, q) + 1 - \xi(0, q) = 1$$

$$\Longrightarrow \xi^{+}(0, q) = 1$$

$\therefore \xi^{+}$ is a $NAQFBI$ of M.

Clearly $\xi \subset \xi^{+}$

Thus ξ^{+} is a $NAQFBI$ of M which contains ξ.

3.4 Theorem

Let ξ be $AQFBI$ of M. If ξ contains a NBI of M generated by any other $AQFBI$ of M then ξ is normal.

Proof:

Let θ be a $AQFBI$ of M & $q \in Q$.

By theorem 2.7,

ϑ^{+} is a $NAQFBI$ of M.

$$\therefore \vartheta^{+}(0, q) = 1$$

Let ξ be a $AQFBI$ of M such that $\vartheta^{+} \subset \xi$

$$\Longrightarrow \xi(\theta, q) \leq \vartheta^{+}(\theta, q), \forall\, \theta \in X \;\&\; q \in Q$$

Put $\theta = 0$,

$$\Longrightarrow \xi(0, q) \leq \vartheta^{+}(0, q) = 1$$

$$\Longrightarrow \xi(0, q) \leq 1$$

Hence ξ is normal.

3.5 Lemma

Define a set $M_{\theta} = \{\theta \in M \;\&\; q \in Q : \xi(\theta, q) = \xi(0, q)\}$ and let ξ and ϑ be $NAQFBIs$ of M. If $\xi \subset \vartheta$, then $M_{\xi} \subset M_{\vartheta}$.

Proof:

Let $\vartheta \in M_{\xi} \;\&\; q \in Q$

Then

$$\vartheta(\theta, q) \leq \xi(\theta, q) = \xi(0, q) = 1 = \theta(0, q)$$

$$\Longrightarrow \theta \in M_{\vartheta}$$

$$\Longrightarrow M_{\xi} \subset M_{\vartheta}$$

3.6 Theorem

Let ξ be a $AQFBI$ of M. Let $\square : [0, \xi(0, q)] \to [0,1]$ be an increasing function. Define a $AQFS\,\xi^{\square} : M \to [0,1]$ by $\xi^{\square} = \square(\xi(\theta, q)), \forall\, \theta \in M \;\&\; q \in Q$. Then,

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(i) $\xi^{\square}$is a $AQFBI$ of M.
(ii) If $\square\big(\xi(0,q)\big)=1$, then $\xi^{\square}$ is normal.
(iii)If $\square(t,q)\leq t$, $\forall t\in[0,\xi(0,q)]$& $q\in Q$ then $\xi\subset\xi^{\square}$

Proof:

(i) $\xi^{\square}(0,q) = \square\big(\xi(0,q)\big)$
$\leq \square\big(\xi(\theta,q)\big)$
$= \xi^{\square}(\theta,q)$
$\Longrightarrow \xi^{\square}(0,q)\leq \xi^{\square}(\theta,q)$

Also

$\xi^{\square}(\varphi*\omega,q) = \square\big(\xi(\varphi*\omega,q)\big)$
$\leq \square\big(\xi(\theta*\varphi,q)\vee\xi(\omega*\theta,q)\big)$
$= \square\big(\xi(\theta*\varphi,q)\big)\vee\square\big(\xi(\omega*\theta,q)\big)$
$= \xi^{\square}(\theta*\varphi,q)\vee\xi^{\square}(\omega*\theta,q)$
$\Longrightarrow \xi^{\square}(\theta*\varphi,q)\leq \xi^{\square}(\theta*\varphi,q)\vee\xi^{\square}(\omega*\theta,q)$
$\Longrightarrow \xi^{\square}$is a $AQFBI$.

(ii) If $\square\big(\xi(0,q)\big)= 1$
$\Longrightarrow \xi^{\square}(0,q)=1$
$\Longrightarrow \xi^{\square}$is normal

(iii) Let $\square(t,q)\leq t, \forall\, t\in[0,\xi(0,q)]$

Then

$\xi^{\square}(\theta,q)=\square\big(\xi(\theta,q)\big)$
$\leq \xi(\theta,q), \quad \forall\,\theta\in M\ \&\ q\in Q$

Hence $\xi\subset\xi^{\square}$.

4. Algebraic Properties on Normalization Of $AQFBSA$ in B-Algebra

4.1 Definition

Let ξ be a $AQFS$ in B - Algebra. Then ξ is called a $AQFBSA$ of M if $\xi(\theta*\varphi,q)\leq\xi(\theta,q)\vee\xi(\varphi,q)$, $\forall\,\theta,\varphi\in M$ & $q\in Q$.

4.2 Definition

A $AQFBSA$ ξ of M is said to be normal if there exist $\theta\in M$& $q\in Q$ such that $\xi(\theta,q)=1$.

4.3 Lemma

Let $AQFBSA\xi$ of M is normal if and if only $\xi(0,q)=1$.

4.4 Theorem

For any $AQFBSA$ ξ of M, we can generate a $NAQFBSAM$ which contains ξ.

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Proof:

Let ξ be a $AQFBSA$ of M.

Define a $AQFSb$ ξ^{+} of M as

$$\xi^{+}(\theta, q) = \xi(\theta, q) + \xi^{c}(0, q), \forall\, \theta \in M \;\&\; q \in Q.$$

Let $\theta, \varphi \in M \& \, q \in Q$.

$$\begin{aligned} \xi^{+}(\theta * \varphi, q) &= \xi(\theta * \varphi, q) + \xi^{c}(0, q) \\ &\leq \{\xi(\theta, q) \vee \xi(\varphi, q)\} + \xi^{c}(0, q) \\ &= \{\xi(\theta, q) + \xi^{c}(0, q)\} \vee \{\xi(\varphi, q) + \xi^{c}(0, q)\} \\ &= \xi^{+}(\theta, q) \vee \xi^{+}(\varphi, q) \\ &\Longrightarrow \xi^{c}(\theta * \varphi, q) \leq \xi^{+}(\theta, q) \vee \xi^{+}(\varphi, q) \end{aligned}$$

Also

$$\begin{aligned} \xi^{+}(0, q) &= \xi(0, q) + \xi^{c}(0, q) \\ &= \xi(0, q) + 1 - \xi(0, q) \\ &= 1 \end{aligned}$$

$\therefore$ $\xi^{+}$is $NAQFBA$ of M.

Clearly $\xi \subset \xi^{+}$

Thus $\xi^{+}$is a $NAQFBA$ of M which contains ξ.

4.5 Theorem

Let ξ be $AQFBSA$ of M. If ξ contains a $NAQFBSA$ of M generated by any other $AQFBSA$ a of M then ξ is normal.

Proof:

Let ϑ be a $AQFBSA$of M. By theorem 3.4,ϑ^{+} is a $NAQFBSA$ of M.

$$\therefore \vartheta^{+}(0, q) = 1$$

Let ξ be a $AQFBA$of M such that $\vartheta^{+} \subset \xi$

$$\Longrightarrow \xi(\theta, q) \leq \vartheta^{+}(\theta, q)\,, \forall\, \theta \in M \;\&\; q \in Q.$$

Put $\theta = 0$,

$$\begin{aligned} &\Longrightarrow \xi(0, q) \leq \vartheta^{+}(0, q) = 1 \\ &\Longrightarrow \xi(0, q) \leq 1 \\ &\Rightarrow \xi \text{ is normal.} \end{aligned}$$

4.6 Theorem

Let ξ and ϑ be $NAQFBSA$ of M. If $\xi \subset \vartheta$ then $M_{\xi} \subset M_{\vartheta}$.

4.7 Theorem

Let ξ be a $AQFBSA$ of M.

Let $\square: [0, \xi(0, q)] \rightarrow [0,1]$ be an increasing function.

Define a $AQFS\xi^{\square}: X \rightarrow [0,1]$by $\xi^{\square} = \square\big(\xi(\theta, q)\big), \forall\, \theta \in M \;\&\; q \in Q$.

Then,

(i) $\xi^{\square}$is a $AQFBSA$ of M.

(ii) If $\square\big(\xi(0, q)\big) = 1$, then $\xi^{\square}$ is normal.

(iii)If $\square(t, q) \leq t,\ \forall\, t \in [0, \xi(0, q)] \& \; q \in Q$, then $\xi \subset \xi^{\square}$.

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l.Proof:

(i) $\xi^{\square}(\theta * \varphi, q) = \square(\xi(\theta * \varphi, q))$
$\leq \square\{\xi(\theta, q) \vee \xi(\varphi, q)\}$
$= \square(\xi(\theta, q)) \vee \square(\xi(\varphi, q))$
$= \xi^{\square}(\theta, q) \vee \xi^{\square}(\varphi, q)$
$\Longrightarrow \xi^{\square}(\theta * \varphi, q) \leq \xi^{\square}(\theta, q) \vee \xi^{\square}(\varphi, q)$
$\Longrightarrow \xi^{\square}$is a $AQFBI$.

(ii) If $\square(\xi(0, q)) = 1$
$\Longrightarrow \xi^{\square}(0, q) = 1$
$\Longrightarrow \xi^{\square}$is normal
(iii) Let $\square(t, q) \leq t$, $\forall\, t \in [0, \xi(0, q)]$, & $q \in Q$.
Then
$\xi^{\square}(\theta, q) = \square(\xi(\theta, q))$
$\leq \xi(\theta, q), \ \forall\, \theta \in M$ & $q \in Q$.
$\therefore \xi \subseteq \xi^{\square}$.

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