International Journal of Future Generation Communication and Networking
Vol. 13, No. 4, (2020), pp. 1712–1718

Various Domination Parameters in the Contex of Switching of a Vertex in a Graph

[1]A. Joel Suresh,[2] R. Raja sekar[3]M.Arunkumar
*[1]PG and Research Department of Mathematics,
National College, Tiruchirappalli, India.
[2]Department of Mathematics,
K.Ramakrishnan College of Engineering,
Tiruchirappalli-621112,Tamilnadu, India.
[3]Student, PG and Research Department of Mathematics,
National college, Tiruchirappalli, India.*
Email: [1]ajoelsuresh@gmail.com,[2] jehosekar@gmail.com

Abstract

In this paper we discuss domination parameters like domination number, connected domination number, total domination number for the graph operation switching of a vertex for special graphs.
Keywords: *Domination; Connected Domination; Total Domination;*

1. Introduction

The graph considered here are connected, simple, finite. Let $G(V,E)$ be a graph. Then the number of vertices of G is denoted by $|V| = p$ and number of edges of G is denoted by $|E| = q$. Various result collection on domination can be seen in [1][3,4] Here we focus on vertex domination in graphs. Here are some of the definitions required for this paper.

A dominating set for a graph $G = (V,E)$is a subset D of V such that every vertex not in D is adjacent to at least one member of D. A minimal dominating set is a dominating set in a graph that is not a proper subset of any other dominating set. A dominating set with minimum cardinality is called $\gamma -$ set of G

A set S of vertices in a graph $G = (V,E)$ is called a total dominating set if every vertex $v \in V$ is adjacent to an element of S. A total dominating set with minimum cardinality is called $\gamma_t -$ set of G

A connected dominating set of a graph G is a set D of vertices with two properties:(a)Any node in D can reach any other node in D by a path that stays entirely within D. That is, D induces a connected subgraph of G. (b) Every vertex in G either belongs to D or is adjacent to a vertex in D. That is, D is a dominating set of G. A connected dominating set with minimum cardinality is called $\gamma_c -$ set of G.

[2] A vertex switching G of a graph G' is the graph obtained by taking a vertex v of G', removing all the edges incident at v and adding edges joining v to every other vertex which are not adjacent to v in G'.

ISSN: 2233-7857 IJFGCN

2. Main Results

2.1 Definition

Let P_n denote the path on n vertices. Then the join of K_1 with P_n is defined as fan and is denoted by F_n (i.e) $F_n = K_1 + P_n$.

2.2 Theorem

Let G be the graph obtained by Switching of a vertex with a degree 2 in a fan F_n, ($n \geq 3$,then graph Ghas $\gamma G=2$, $\gamma tG=2$, $\gamma cG=2$.

Proof:

Let $\{v, v_1, v_2, \dots, v_n\}$ be the vertices of F_n. Let Gbe the graph obtained by switching the vertexv_1. Let $\{e_1, e_2, \dots, e_{n-2}, e_{n-1}, \dots, e_{3n-5}\}$be the edges of Gwhich are denoted as in Figure 2.1. We note that $|V(G)| = p = n + 1$ and $|E(G)| = q = 3n - 5$.

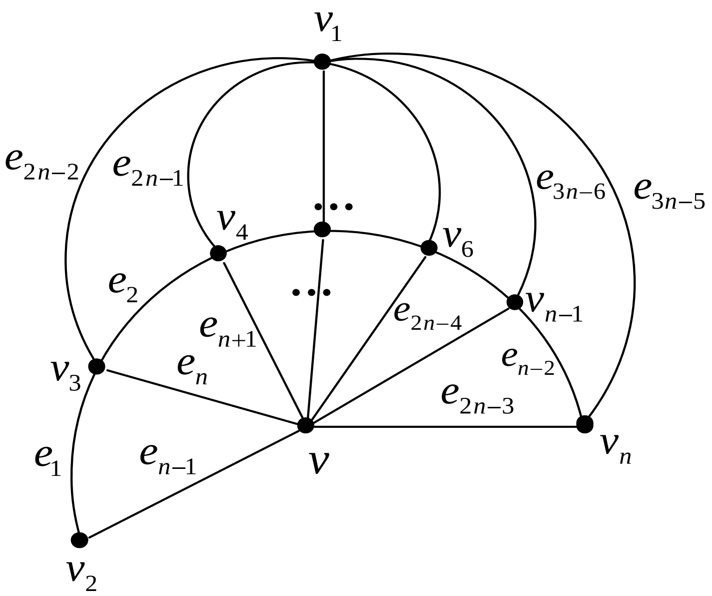

Figure 2.1- Switching of a vertex with a degree 2 in a fan F_n

$\gamma(G) = 1$is possible only if there exist a vertex with the degree = $\text{n} - 1$ here there exist no vertex with this property, Hence $\gamma(G) \geq 2$ We have a set $D = \{v, v_3\}$in$G$ which is a dominating set and also the γ- set, thus $\gamma(G) = 2$. It's clear that the two vertices in D are adjacent and hence it follows that it is a γ_c-set. Hence $\gamma_c(G) = 2$,also it is total, Hence $\gamma_t(G) = 2$.

2.3 Illustration

Here is an illustration for the above theorem. Consider the graph in figure 2.1 the total number of vertices is 8 and number of edges is 16. Here $D = \{G, D\}$.
Hence$\gamma(G) = \gamma_c(G) = \gamma_t(G) = 2$.

ISSN: 2233-7857 IJFGCN

International Journal of Future Generation Communication and Networking
Vol. 13, No. 4, (2020), pp. 1712–1718

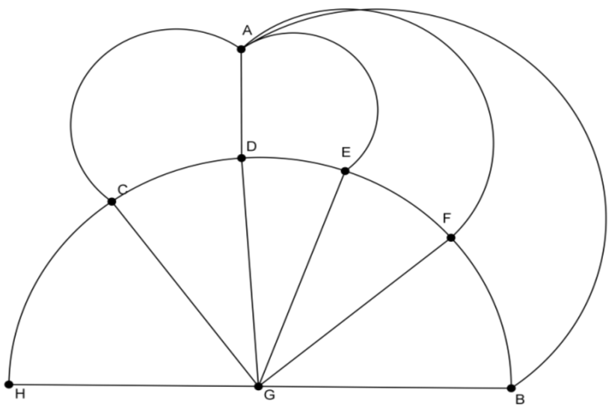

Figure 2.2

2.4 Definition

The wheel W_n is defined as $W_n = C_n + K_1$, where C_n is the cycle of length n. Switching of a rim vertex in a wheel $W_n, (n \geq 4)$ is

2.4 Theorem

Let G be the graph obtained by Switching of a rim vertex in W_n, then for G $\gamma(G) = 2, \gamma_t(G) = 2, \gamma_c(G) = 2$.

Proof :

Let $\{v, v_1, v_2, \dots, v_n\}$ be the vertices of W_n. Let G be the graph obtained by switching the vertex v_1. Let $\{e_1, e_2, \dots, e_{n-2}, e_{n-1}, \dots, e_{3n-6}\}$be the edges of G which are denoted as in Figure 2.2. We note that $|V(G')| = p = n + 1$ and $|E(G')| = q = 3n - 6$.

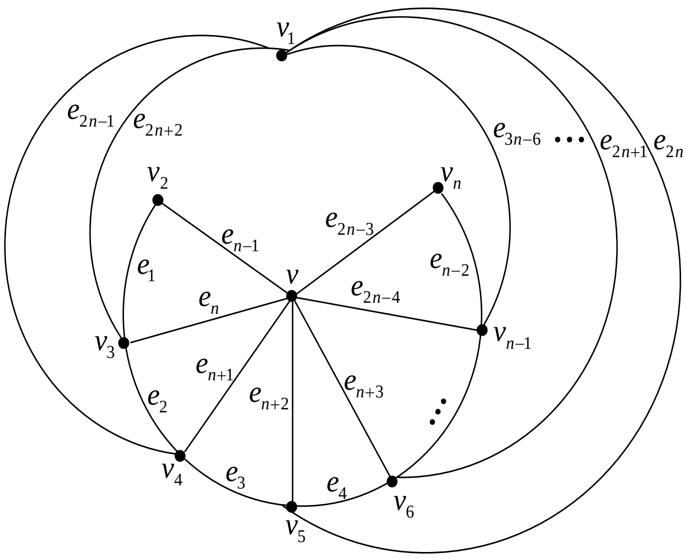

Figure 2.3 - Switching of a rim vertex in W_n

ISSN: 2233-7857 IJFGCN

$\gamma(G) = 1$is possible only if there exist a vertex with the degree = $n - 1$ here there exist no vertex with this property ,Hence $\gamma(G) \geq 2$ We have a set$D_1 = \{v, v_1\}$ in G which is a dominating set .

The above domination D_1 set is a γ –set.

$$\therefore \gamma(G) = 2$$

It's clear that the two vertices in $D_2 = \{v, v_{n-1}\}$ are adjacent and hence it is a γ_c-set.

$$\therefore \gamma_c(G) = 2 ,$$

There is no isolated vertex in D, hence it is total ,$D_3 = \{v, v_{n-2}\}$

$$\therefore \gamma_t(G) = 2$$

2.6 Illustration

Here is an illustration for the above theorem. Consider the graph in figure 2.4 the total number of vertices is 9 and number of edges is

18.$D_1 = \{I, J\}$, $D_2 = \{J, G\}$, $D_3 = \{J, H\}$.

Hence $\gamma(G) = \gamma_c(G) = \gamma_t(G) = 2$.

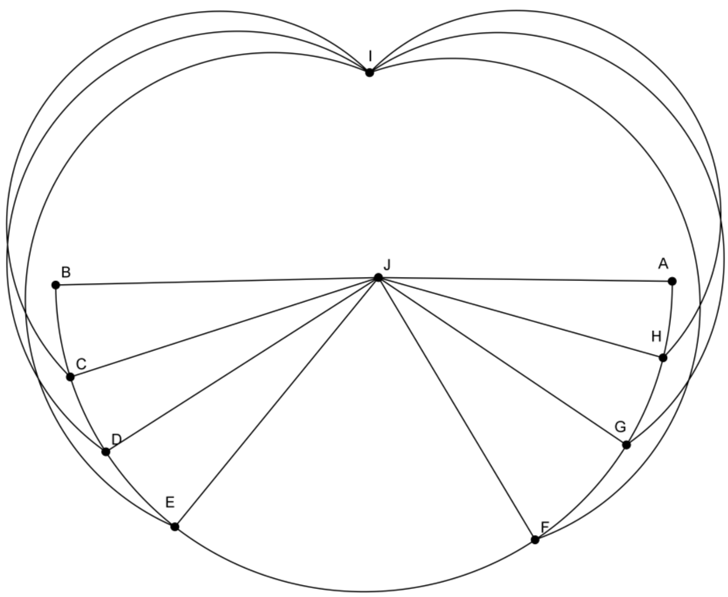

Figure 2.4

2.7 Definition

The helm H_n is the graph obtained from a wheel W_n by attaching a pendant edge to every rim vertex.

ISSN: 2233-7857 IJFGCN

2.8 Theorem

Let G be the graph obtained by Switching of an apex vertex in a helm H_n, $(n \geq$ 3has γGv=$n3$.

Proof :

Let $\{v, v_1, v_2, \dots, v_n, v_1^{'}, v_2^{'}, \dots, v_n^{'}\}$ be the vertices of H_n. Let G be the graph obtained by switching of an apex vertex v and $\{e_1, e_2, \dots, e_n, e_1^{'}, e_2^{'}, \dots, e_{2n}^{'}\}$be the edges of G which are denoted as in Figure 2.5. We note that$|V(G)| = p = 2n + 1$ and $|E(G)| = q = 3n$.

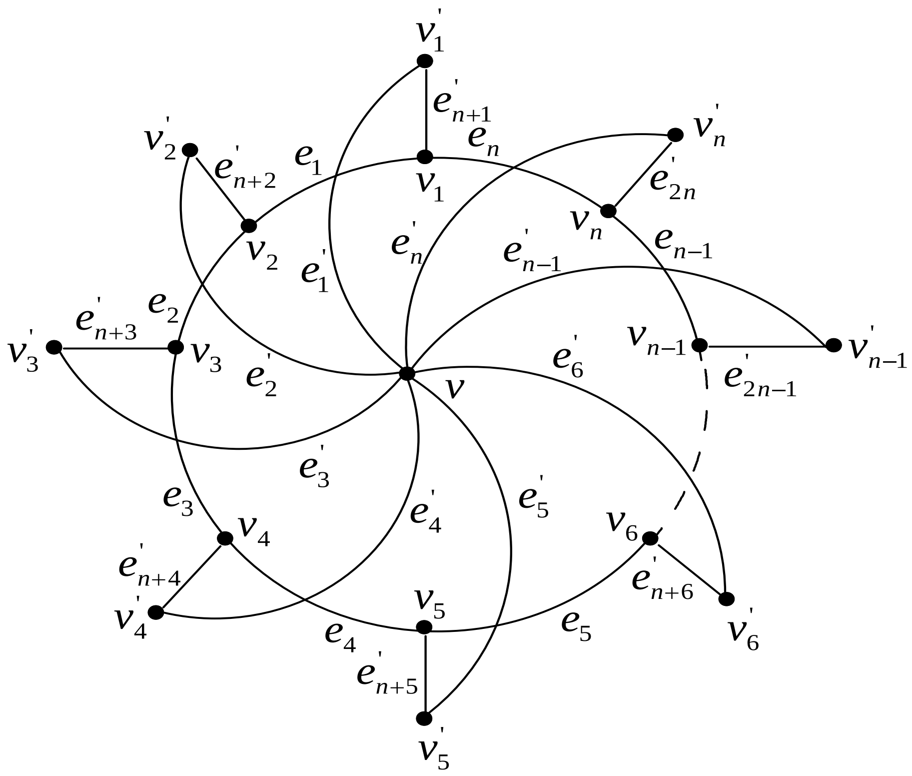

Figure 2.5 - Switching of an apex vertex in a helm H_n

$V$is the maximum degree vertex ,Hence we include V in γ-set of G.
Now consider the graph $G - n[V_1]$ which is a cycle on n-vertices.

Case(1): $n \equiv 0(mod\ 3)$

We know that for a cycle each vertex dominating two other vertices. Hence for each there consecutive vertices the middle vertex will dominate the neighbours. Hence for three vertices we need one vertex in the three for dominating Therefore $\gamma(G - n[V_1]) = \frac{n}{3}$

$$\gamma(G) = \frac{n}{3} + 1 = \frac{n+3}{3}$$

Case(ii) :$n \equiv 1(mod\ 3)$

By the same argument in case (i)

ISSN: 2233-7857 IJFGCN

$$\gamma(G - n[V_1]) = \frac{n-1}{3} + 1$$

$$= \frac{n+2}{3}$$

$$\therefore \gamma(G) = \frac{n+2}{3} + 1 = \frac{n+5}{3}$$

Case (iii):$n \equiv 2(mod\ 3)$

$$\gamma(G - n[V_1]) = \frac{n-2}{3} + 1$$

$$= \frac{n+1}{3}$$

$$\therefore \gamma(G) = \frac{n+1}{3} + 1 = \frac{n+4}{3}$$

2.9 Theorem

Let G be the graph obtained by Switching of an apex vertex in a helm $H_n(n \geq 3)$, has $\gamma_c(G) = n$.

Proof:

Consider the subgraph C_n of GForC_n we need $n-2$ vertices for minimum connected dominating Hence $\gamma_c(C_n) = n-2$ .In the graph $G$for the remaining vertices from$G, v.$ is enough to dominate but it will not implies connectedness so, we include the vertexV_i which is adjacent to any vertex in the $\gamma_c(C_n)$-set
$\therefore \gamma_c(H_n) = n - 2 + 1 + 1 = n.$

2.10 Illustration

Here is an illustration for the above theorem. The total number of vertices is 17 and number of edges is $22. \gamma$ −set = $\{0,13,21,3\}$;
$\gamma_c = \{0,24,25,13,23,15,21,17\}$. Hence $\gamma(G) = 4; \gamma_c(G) = 8.$

ISSN: 2233-7857 IJFGCN

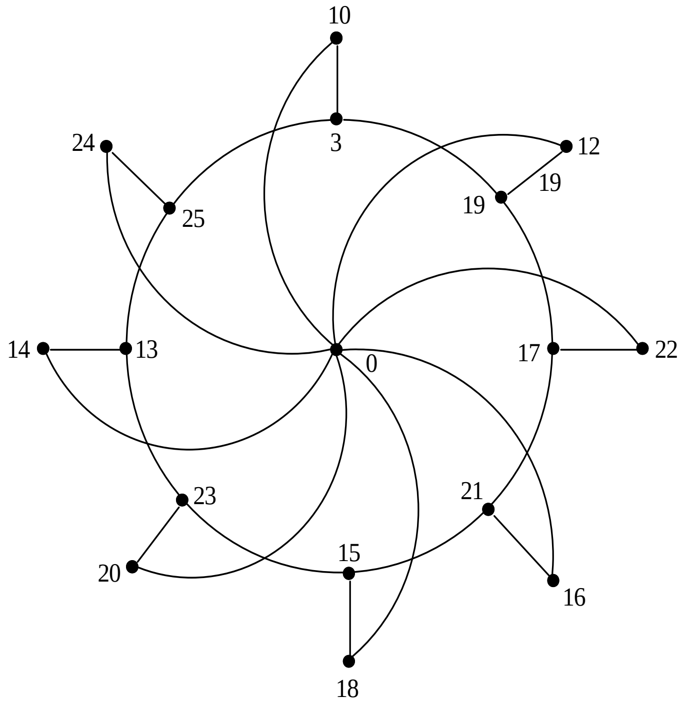

Figure 2.6

Acknowledgments

Authors would like to thank referees for their helpful comments.

3. References

3.1. Journal Article

[1] Cockayne C.J, Dawes R.M. and Hedetniemi S.T., Total domination in graphs, Networks, 10 (1980), 211-219.

[2] Gayathri. B and Joel Suresh. A, Strong Edge-Graceful Labeling In The Context of a Switching of a Vertex, Aryabhatta Journal of Mathematics and informatics Vol. 8, Issue 2,July-Dec,2016, P. No. 107-113

[3] Hedetniemi S.T. and R.C. Laskar, Connected domination in graphs, In B. Bollobas, Editor, Graph Theory and Combinatorics, Academic Press, London (1984), 209-218

[4] C. D. Scott and R. E. Smalley, "Diagnostic Ultrasound: Principles and Instruments", Journal of Nanosci. Nanotechnology., vol. 3, no. 2, **(2003)**, pp. 75-80.

3.2. Book

[1] O.Ore, "Theory of Graphs", Amer.Math.Soc. Colloq.Publ.,38, Providence, (1962).

[2] V. R. Kulli, "Theory of Domination in Graph",(2010).