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On Algebraic Structures on κ-Intuitionistic Q −Fuzzy Quotient Group

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Abstract

In this paper, we introduced of$\kappa -$ Intuitionistic Q −fuzzy quotient group, $\kappa -$ Intuitionistic Q −fuzzy cosets of an Intuitionistic Q −fuzzy normal subgroup are defined and discussed. A homomorphism from a given group onto the set of all $\kappa -$ intuitionistic Q −fuzzy quotient group is established. Some related results has been derived.

Keywords: Intuitionistic Fuzzy Set (IFS); Intuitionistic Fuzzy Subset (IFSb); Intuitionistic Fuzzy Subgroup (IFSG) ; Intuitionistic Fuzzy Normal Subgroup (IFNSG); $\kappa -$ Intuitionistic Q− fuzzy Set ($\kappa -$IQFS;Pκ−Intuitionistic Q−fuzzy subset κ−IQFSb;κ−Intuitionistic Q − fuzzy subgroup (κ − IQFSG); κ − intuitionistic Q − fuzzy normal subgroup (κ − IQFNSG); $\kappa -$ Intuitionistic Q − fuzzy coset, $\kappa -$ intuitionisticQ −fuzzy quotient group.

1. Introduction

The fundamental concept of fuzzy sets was initiated by Zadeh L[21] in 1965 .Since then these ideas have been applied to other algebraic structures such as groups, rings, modules, vector spaces and topologies. In 1986, introduced the new notation on Intuitionistic Fuzzy Sets by Atanassov K T[1]. Barbhuiya S R[4], introduced the concept of t- Intuitionistic Fuzzy Sub algebra of BG-Algebras in 2015.

Atanassov K T[2], introduced the new notation on Intuitionistic Fuzzy Versions of L. Zadeh's extension principle in 2006. In 1994, developed the concept of new Operations defined over the intuitionistic fuzzy sets by Atanassov K T[3]. Szmidt E and KacprzykJ[19], introduced the concept of Intuitionistic Fuzzy Sets in group decision making in 1996. In 2015, developed the concept of Doubt Intuitionistic Fuzzy Ideals in BCK/BCI-Algebras by TriptiBej and MadhumangalPal[20].BavanariSatyanarayana, BinduMadhavi and Durga Prasad R[5], introduced the new notation On Intuitionistic Fuzzy H-Ideals in BCK-Algebras in 2010. Sharma P K[15], developed the concept of Intuitionistic Fuzzy Module over intuitionistic FuzzyRing in 2012.In 2012, initiated by the concept of t- Intuitionistic Fuzzy Quotient Group by Sharma P K[14]. MuhammdAkram[9], introduced the new notation of Intuitionistic Fuzzy Closed Ideals in BCI-algebras in 2006. In 2012, initiated by the concept of t- Intuitionistic Fuzzy Quotient modules by Sharma P K[13]. Sharma P K[10], developed the notation of $(\alpha, \beta)-$ Cut of Intuitionistic Fuzzy Groups in 2011.In 2009, introduced the new concept of A new structure and Construction of Q-fuzzy Groups by Solairaju A and NagarajanR[16],BodinKesorn, KhanrudeeMaimun, WatcharaRatbandan and AiyaredIampan[6],introduced the concept of Intuitionistic Fuzzy Sets in Up-Algebras in 2015. In 2015, develop the concept of Intuitionistic Fuzzy Filters On-β algebras by Sujatha K Chandramouleeswaran M and Muralikrishna P[18]. Sharma P K[12] initiated by the concept On the direct product of Intuitionistic Fuzzy Groups in 2012. In 2014, introduced the concept of An Overview On Intuitionistic Fuzzy Sets by Ejegwa P A, Akowe SO, Otene P M and Ikyule J M[8]. Sharma P K[11],develop the notation of Homomorphism of Intuitionistic Fuzzy Groups in 2011.In 2001, initiated by the concept of An application of Intuitionistic Fuzzy Sets in Medical diagnosis by SupriyaK.De, Biswas R and Roy A R[17]. Dengfeng Li, Cheng Chutian[7],introduced the concept of New Similarly Measure of IFS and Application of Pattern recognition in 2002.

In this paper, we introduce the notion of κ-Intuitionistic $\mathrm{Q}-$Fuzzy cosets of an intuitionistic $\mathrm{Q}-$fuzzy normal subgroup and κ-Intuitionistic $\mathrm{Q}-$Fuzzy Quotient Groupand discuss some of their properties.

2. Preliminaries

In this section,we site the fundamental definitions that will be used in the sequel.

2.1 Definition [ZadehL A (20)]

Let X be a non-empty set .A FSb of the set X is a mapping μ: $X\rightarrow$ [0, 1].

2.2. Definition [Atanassov K T(1)]

Let X be a fixed non-empty set. An IFS A of X is an object of the following from $A = \{< x, \mu_A(x),\ \nu_A(x) >: x \in X\}$, where $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ define the degree of membership and degree of non-

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membership of the element $x \in X$ respectively and for any $x \in X$, we have $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

2.2.1 Remark

(i) When $\mu_A(x) + \nu_A(x) = 1$, i,e., when $\nu_A(x) = 1 - \mu_A(x) = {\mu_A}^c(x)$. Then A is called FS.

(ii) We use the notation $A = (\mu_A, \nu_A)$ to denote the IFS A of X.

2.3 Definition [Sharma P K (11)]

Let G be a group. An $IFSbA = (\mu_A, \nu_A)$ of G is called $IFSG$ of G if

(i) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$

(ii) $\mu_A(x^{-1}) = \mu_A(x)$

(iii) $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$

(iv) $\nu_A(x^{-1}) = \nu_A(x), \forall\, x, y \in G$

or Equivalently A is $IFSG$ of G if and only if $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$.

2.4 Definition [Sharma P K (11)]

An IFSG $A = (\mu_A, \nu_A)$ of a group G is said to be $IFNSG$ of G if

(i) $\mu_A(xy) = \mu_A(yx)$

(ii) $\nu_A(xy) = \nu_A(yx)$, for all $x, y \in G$

Or Equivalently A is an $IFNSG$ of a group G is normal if and only if $\mu_A(y^{-1}xy) = \mu_A(x)$ and $\nu_A(y^{-1}xy) = \nu_A(x), \forall\, x, y \in G$.

2.5 Definition [Sharma P K (10)]

Let A be IFS of a universe set X. Then (α, β)-cut of A is a crisp subset $C_{\alpha,\beta}(A)$ of the IFS A is given by $C_{\alpha,\beta}(A) = \{x : x \in X \text{ such that } \mu_A(x) \geq \alpha,\ \nu_A(x) \leq \beta\}$, where $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$.

2.6 Theorem

If A is IFS of a group G. Then A is $IFSG(IFNSG)$ of G if and only if $C_{\alpha,\beta}(A)$ is a subgroup (normal) of group G, for all $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$.

2.7 Definition[Solairaju A and Nagarajan R (16)]

Let Q and G a set and a group respectively. A mapping $\mu: G \times Q \to [0,1]$ is named $Q - FS$ in G. For any $Q - FS$μ in G and t ∈ [0,1] we define the set $U(\mu; t) = \{x \in G \ \ / \ \mu(x, q) \geq t, q \in Q\}$ which is named an upper cut of "μ" and may be use to the characterization of μ.

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2.8 Definition [Solairaju A and Nagarajan R (16)]

A $Q-FS\mu$ is named $Q-FSG$ of G if

1. $\mu(xy,q) \geq min\{\mu(x,q),\mu(y,q)\}$
2. $\mu(x^{-1},q)=\mu(x,q)$
3. $\mu(x,q)=1,$ for all $x,y \in G$ and $q \in Q$.

3. On algebraic structures on κ-Intuitionistic Q −Fuzzy Quotient Group

3.1 Definition

Let A be an $IQFSNG$ of a group H. Let $\boldsymbol{\kappa} \in [0,1]$ and $q \in Q$.For any $m \in H$ define an $IQFSA^{\kappa}{}_{m}$ of H is called $\boldsymbol{\kappa}$-Intuitionistic $q-$fuzzy Coset of A in H as follows $A^{\kappa}{}_{m}(g,q)=\{\theta_{A^{\kappa}{}_{m}}(g,q),\emptyset_{A^{\kappa}{}_{m}}(g,q)\}$,
where
$\theta_{A^{\kappa}{}_{m}}(g,q)=\{\theta_A(gm^{-1},q)\wedge\kappa\}$ and $\emptyset_{A^{\kappa}{}_{m}}(g,q)=\{\emptyset_A(gm^{-1},q)\wedge 1-\kappa\}$, $\forall m,g \in H$ and $q \in Q$.

3.2 Proposition

Let S and Q be the set of all $\boldsymbol{\kappa}$-Intuitionistic $q-$fuzzy cosets of an $IQFNSGA$ in $H.i.e.,S=\{A_m{}^{\kappa}: m \in H$ and $q \in Q\}$. Then the binary operations $\otimes$ defined on the set S as follows: $A^{\kappa}{}_{m} \otimes A^{\kappa}{}_{n}=A^{\kappa}{}_{mn}, \forall$ $m,n \in H$ and $q \in Q$ is a well defined operation.

Proof:

Let $A^{\kappa}{}_{m}=A^{\kappa}{}_{m'}$ and $A^{\kappa}{}_{n}=A^{\kappa}{}_{n'}$, for some $m,n,m',n' \in H$ and $q \in Q$.

Let $g \in Hand$ $q \in Q$ ne any element, then

$$[A_m{}^{\kappa} \otimes A_n{}^{\kappa}](g,q)=(A^{\kappa}{}_{mn})(g,q)=(\theta_{A^{\kappa}{}_{mn}}(g,q),\emptyset_{A^{\kappa}{}_{mn}}(g,q))$$

Now

$$\begin{aligned}
\theta_{A^{\kappa}{}_{mn}}(g,q) &= \{\theta_A g((mn)^{-1},q)\wedge\kappa\}=\{\theta_A\big((gn^{-1})m^{-1},q\big)\wedge\kappa\} \\
&= \theta_{A^{\kappa}{}_{m}}(gn^{-1},q)=\theta_{A^{\kappa}{}_{m'}}(gn^{-1},q) \\
&= \left\{\theta_A\left((gn^{-1})m'^{-1},q\right)\wedge\kappa\right\}=\left\{\theta_A\left((gm'^{-1})n^{-1},q\right)\wedge\kappa\right\} \\
&= \theta_{A^{\kappa}{}_{n}}\big(m'^{-1}g,q\big)=\theta_{A^{\kappa}{}_{n'}}(m'^{-1}g,q) \\
&= \left\{\theta_A\left((gm'^{-1})n'^{-1},q\right)\wedge\kappa\right\}=\{\theta_A n'^{-1}\big(gm'^{-1},q\big)\wedge\kappa\} \\
&= \{\theta_A(n'^{-1}m'^{-1},q)g\wedge\kappa\} \\
&= \{\theta_A((m'n')^{-1}g,q)\wedge\kappa\}=\{\theta_A(g(m'n')^{-1},q)\wedge\kappa\} \\
&= \theta_{A^{\kappa}{}_{m'n'}}(g,q)
\end{aligned}$$

Similarly, we can show that

$$\emptyset_{A^{\kappa}{}_{mn}}(g,q)=\emptyset_{A^{\kappa}{}_{m'n'}}(g,q), \forall g \in H \ and \ q \in Q.$$

Therefore $\otimes$ is well defined operation on S.

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3.3 Lemma

If A is $IQFNSG$ of a group G. Then $A^{\kappa}{}_{m} = A^{\kappa}{}_{m'} \Leftrightarrow Nm = Nm', \forall\, m, m' \in H\ and\ q \in Q$. where $N = C_{\kappa,1-\kappa}(A, q)$.

Proof:

Let $A^{\kappa}{}_{m} = A^{\kappa}{}_{m'}$ for $m, m' \in H\ and\ q \in Q$

then $A^{\kappa}{}_{m}(m', q)$=$A^{\kappa}{}_{m'}(m', q)$

$$\left(\{\theta_A(m'm'^{-1}, q)\wedge\kappa\}, \{\emptyset_A(m'm'^{-1}, q)\vee 1-\kappa\}\right)$$
$$= (\{\theta_A(m'm'^{-1}, q)\wedge\kappa\}, \{\emptyset_A(m'm'^{-1}, q)\vee 1-\kappa\})$$
$$= (\{\theta_A(e, q)\wedge\kappa\}, \{\emptyset_A(e, q)\vee 1-\kappa\}$$
$$= \{\kappa, 1-\kappa\}$$

$\Rightarrow\{\theta_A(m'm'^{-1}, q)\wedge\kappa\} = \kappa$and$\{\emptyset_A(m'm'^{-1}, q)\vee 1-\kappa\} = 1-\kappa$

$\Rightarrow\theta_A(m'm'^{-1}, q) \geq \kappa$and$\emptyset_A(m'm'^{-1}, q) \leq 1-\kappa$

and so $m'm'^{-1} \in C_{\kappa,1-\kappa}(A) = N$

$\Rightarrow Nm = Nm'$ ....(1)

Now, we show that if $Nm = Nm'$, then $A^{\kappa}{}_{m} = A^{\kappa}{}_{m'}$.

Let for some $n \in H\ and\ q \in Q, A^{\kappa}{}_{m}(n, q) \neq A^{\kappa}{}_{m'}(n, q)$

$i.e., (\{\theta_A(mn^{-1}, q)\wedge\kappa\}, \{\emptyset_A(mn^{-1}, q) \vee \quad -\kappa\}) \neq$
$(\{\theta_A(m'n^{-1}, q)\wedge\kappa\}, \{\emptyset_A(m'n^{-1}, q)\wedge 1-\kappa\})$

Suppose $\theta_A(mn^{-1}, q) < \kappa$ and $\theta_A(m'n^{-1}, q) \geq \kappa$

Therefore $\emptyset_A(mn^{-1}, q) \leq 1-\kappa$ and $\emptyset_A(m'n^{-1}, q) \leq 1-\kappa$

$$\Rightarrow m'n^{-1} \in C_{\kappa,1-\kappa}(A) = N$$

$\Rightarrow Nm'n^{-1} = N \Rightarrow Nmn^{-1} = N$ (Using 1) $\Rightarrow\ mn^{-1} \in N$
and so $\theta_A(mn^{-1}, q) \geq \kappa$, a contradiction,
Similarly, if $\theta_A(mn^{-1}, q) \geq \kappa$ and $\theta_A(m'n^{-1}, q) < \kappa$ also leads to contradiction. Therefore either $\theta_A(mn^{-1}, q) \geq \kappa$ and $\theta_A(m'n^{-1}, q) \geq \kappa\ i.e., \emptyset_A(mn^{-1}, q) \leq 1-\kappa$ and $\emptyset_A(m'n^{-1}, q) \leq 1-\kappa\ or\ \theta_A(mn^{-1}, q) < \kappa\ i.e., \emptyset_A(mn^{-1}, q) \leq 1-\kappa\ and\ \emptyset_A(m'n^{-1}, q) \leq 1-\kappa$.

In First Part:

$\{\theta_A(mn^{-1}, q)\wedge\kappa\} = \kappa$ and $\{\emptyset_A(mn^{-1}, q)\wedge 1-\kappa\} = 1-\kappa$

and so $A^{\kappa}{}_{m}(n, q) = (\kappa, 1-\kappa)$ and also
$\Rightarrow\{\theta_A(m'n^{-1}, q)\vee\kappa\} = \kappa$ and $\{\emptyset_A(m'n^{-1}, q)\wedge 1-\kappa\} = 1-\kappa$
and so
$\Rightarrow\ A^{\kappa}{}_{m'}(n, q) = (\kappa, 1-\kappa)$. Thus $A^{\kappa}{}_{m}(n, q) = A^{\kappa}{}_{m'}(n, q), \forall\, n \in H\ and\ q \in Q$.

$$\therefore A^{\kappa}{}_{m} = A^{\kappa}{}_{m'}.$$

In Second Part:

$$\{\theta_A(mn^{-1}, q)\wedge\kappa\} = \theta_A(mn^{-1}, q) < \kappa$$

and$\{\emptyset_A(mn^{-1}, q)\vee 1-\kappa\} = 1-\kappa$

and also

$\Rightarrow\{\theta_A(m'n^{-1}, q)\wedge\kappa\} = \theta_A(m'n, q) < \kappa$

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and$\{\emptyset_A(m^{'}n^{-1},q)\vee 1-\kappa\}=1-\kappa$
Now since $Nm=Nm^{'}$,
therefore let$m=Nm^{'}$, where $a\in N\ and\ q\in Q$.
So that $\theta_A(a,q)\geq\kappa$ and $\emptyset_A(a,q)\leq 1-\kappa$

$$
\begin{aligned}
A^{\kappa}{}_{m'}(n,q) &= (\{\theta_A(nm^{'-1},q)\wedge\kappa\},\{\emptyset_A(nm^{'-1},q)\vee 1-\kappa\}) \\
&= (\theta_A(nm^{-1}a,q),1-\kappa) \\
&= (\theta_A(anm^{-1},q),1-\kappa) \\
&\geq (\theta_A(a,q)\cap\theta_A(nm^{-1},q),1-\kappa) \\
&= (\theta_A(nm^{-1},q),1-\kappa) \\
&= (\{\theta_A(nm^{-1},q)\wedge\kappa\},\{\emptyset_A(nm^{-1},q)\vee 1-\kappa\} \\
&= A^{\kappa}{}_{m}(n,q)
\end{aligned}
$$

Thus $A^{\kappa}{}_{m'}(n,q)\geq A^{\kappa}{}_{m}(n,q),\forall\ n\in H\ and\ q\in Q$..

$$
\begin{aligned}
\text{Similarly } A^{\kappa}{}_{m}(n,q) &= (\{\theta_A(nm^{-1},q)\wedge\kappa\},\{\emptyset_A(nm^{-1},q)\vee 1-\kappa\}) \\
`&= (\theta_A(nm^{-1},q)1-\kappa) \\
&= \left(\theta_A(nm^{'-1}a^{-1},q\right),1-\kappa \\
&\geq (\theta_A(a,q)\cap\theta_A\left(nm^{'-1},q\right),1-\kappa) \\
&= \left(\theta_A\left(nm^{'-1},q\right),1-\kappa\right) \\
&= \left(\left\{\theta_A\left(nm^{'-1},q\right)\wedge\kappa\right\},\{\emptyset_A(nm^{-1},q)\vee 1-\right. \\
\kappa \\
&= A^{\kappa}{}_{m'}(n,q)
\end{aligned}
$$

Thus $A^{\kappa}{}_{m}(n,q)\geq A^{\kappa}{}_{m'}(n,q),\forall n\in H\ and\ q\in Q$.

$$\therefore A^{\kappa}{}_{m}(n,q)=A^{\kappa}{}_{m'}.$$

3.4 Proposition

The set $Q\ and\ S$ of all $\boldsymbol{\kappa}$-Intuitionistic $q-$fuzzy cosets of an $IQFNSG$ A of a group $H\ and\ q\in Q$, from a group the well-defined operations $\otimes$.

Proof:

It is easy to check that the identity element of S is $A^{\kappa}{}_{e,}$ where e is the identity element of group $H$and , and the inverse of an element $A^{\kappa}{}_{m}$ is $A^{\kappa}{}_{m^{-1}}$.

3.5 Proposition

A mapping $f:G\to S$, where G is a group and S is the set of all $\boldsymbol{\kappa}$-Intuitionistic $q-$fuzzy cosets of the $IQFNSGA$ of G defined by $f(m,q)=A^{\kappa}{}_{m}$, is an onto homomorphism with ker $f=N(=C_{\kappa,1-\kappa}(A,q))$, where $\kappa\in[0,1]and\ q\in Q$.

Proof:

Clearly f is an onto homomorphism
Let $m\in kerfand\ q\in Q$, then $f(m,q)=$identity element of $S=A^{\kappa}{}_{e}$
Therefore $A^{\kappa}{}_{m}=A^{\kappa}{}_{e}$ so $Nm=Ne=N\Rightarrow m\in N$
$\Rightarrow kerf\subseteq N$
Conversely, let $m\in N\Rightarrow Nm=N$ so that

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$Nmg^{-1} = Ng^{-1} \; \forall \; g \in G \; and \; q \in Q$.
If possible let $m \notin kerf$
$i.e., A^{\kappa}{}_{m} \neq A^{\kappa}{}_{e}$there fore there exists $g \in G \; and \; q \in Q$
such that $A^{\kappa}{}_{m}(g,q) \neq A^{\kappa}{}_{e}(g,q)$.
Suppose $\theta_A(mg^{-1},q) < \kappa$ and $\theta_A(g^{-1},q) \geq \kappa, i.e., \emptyset_A(mg^{-1},q) \leq 1-\kappa$ and $\emptyset_A(g^{-1},q) \leq 1-\kappa$
Therefore $\theta_A(g^{-1},q) \geq \kappa$ and $\emptyset_A(g^{-1},q) \leq 1-\kappa \Rightarrow g^{-1} \in N \; so \; Ng^{-1} = N$
$\therefore Nmg^{-1} = N \Rightarrow mg^{-1} \in N$and so $\theta_A(mg^{-1}) \geq \kappa$, a contradiction.
Similarly, $\theta_A(mg^{-1},q) \geq \kappa$ and $\theta_A(g^{-1},q) < \kappa$ is not possible.
$\therefore$either$\theta_A(mg^{-1},q) \geq \kappa, \theta_A(g^{-1},q) < \kappa$ i.e., $\emptyset_A(mg^{-1},q) \leq 1-\kappa$
and $\emptyset_A(g^{-1},q) \leq 1-\kappa$ or
$\theta_A(mg^{-1},q) < \kappa, \theta_A(g^{-1},q) < \kappa$ i.e., $\emptyset_A(mg^{-1},q) \leq 1-\kappa$
and $\emptyset_A(g^{-1},q) \leq 1-\kappa$.

In First Part:

$\{\theta_A(mg^{-1},q) \wedge \kappa\} = \kappa$and$\{\emptyset_A(mg^{-1},q) \vee 1-\kappa\} = 1-\kappa$
and so $A^{\kappa}{}_{m}(g,q) = (\kappa, 1-\kappa)$
Similarly we get, $A^{\kappa}{}_{m}(g,q) = (\kappa, 1-\kappa)$. Thus $A^{\kappa}{}_{m}(g,q) = A^{\kappa}{}_{e}(g,q)$.

In Second Part:

$\{\theta_A(mg^{-1},q) \wedge \kappa\} = \theta_A(mg^{-1},q) < \kappa$
and$\{\emptyset_A(mg^{-1},q) \vee 1-\kappa\} = 1-\kappa$

$$A^{\kappa}{}_{m}(g,q) = (\{\theta_A(mg^{-1},q) \wedge \kappa\}, \{\emptyset_A(mg^{-1},q) \vee 1-\kappa\})$$
$$= (\theta_A(mg^{-1},q), 1-\kappa)$$
$$\geq (\theta_A(m,q) \cap \theta_A(g,q), 1-\kappa)$$
$$= (\theta_A(g,q), 1-\kappa) \; [\therefore m \in N \; and \; q \in Q \; \therefore \theta_A(m,q) \geq \kappa$$
and $\theta_A(g,q) = \theta_A(g^{-1},q) < \kappa]$
$$= (\{\theta_A(eg,q) \wedge \kappa\}, \{\emptyset_A(eg^{-1},q) \vee 1-\kappa\}$$
$$= A^{\kappa}{}_{e}(g,q)$$
$$= (\theta_A(g^{-1},q), 1-\kappa)$$
$$\geq (\theta_A(mg^{-1}m^{-1},q), 1-\kappa)$$
[As A is $IQFNSG$ of G so$\theta_A(mg^{-1}m^{-1},q) = \theta_A(g^{-1},q)$]
$$\geq(\theta_A(mg^{-1},q) \cap \theta_A(m,q), 1-\kappa)$$
$$= (\theta_A(mg^{-1},q), 1-\kappa)$$
$$= A^{\kappa}{}_{m}(g,q)$$
i.e., $f(m,q) =$ identity element of S and so $m \in \ker f \; and \; q \in Q$.
$N \subseteq kerf$so$kerf = N$.

3.6 Proposition

If $f: G \to S$, is an onto homomorphism, then $f(A,q) = B$, where A is $IQFS$ of G and B is $IQFS$ of S.

Proof:

Let $A^{\kappa}{}_{x} \in S$ be any element of S, where $x \in Hand \; q \in Q$ such that $f(x,q) = A^{\kappa}{}_{x}$

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Let A be $IQFS$of G, then

$$f(A,q)(A^{\kappa}{}_x) = \begin{cases} (Sup\{\theta_A(m,q): m \in f^{-1}(A^{\kappa}{}_x)\}, Inf\{\emptyset_A(m,q): m \in f^{-1}(A^{\kappa}{}_x)\}) \\ (\theta_A(e,q), \emptyset_A(e,q)) \end{cases}$$

$$= \begin{cases} (Sup\{\theta_A(m,q): A^{\kappa}{}_m = A^{\kappa}{}_x\}, Inf\{\emptyset_A(m,q): A^{\kappa}{}_m = A^{\kappa}{}_x\}) \\ (\theta_A(e,q), \emptyset_A(e,q)) \end{cases}$$

$$= B(A^{\kappa}{}_x)$$

Hence $f(A,q) = B$

3.7 Theorem

Let A be a $IQFNSG$ of G and B be a $IQFSG$of S, then $C_{\kappa,1-\kappa}(B,q) = \{A^{\kappa}{}_e\}$.

Proof:

Now $B(A^{\kappa}{}_e) = \{\theta_B(A^{\kappa}{}_e), \emptyset_B(A^{\kappa}{}_e)\}$,

where $\theta_B(A^{\kappa}{}_e) = Sup\{\theta_A(m,q): Nm = N\}$

$= Sup\{\theta_A(m,q): m \in N\}$

$\geq \theta_A(a,q), \forall a \in N and\ q \in Q = C_{\kappa,1-\kappa}(A,q)$

$\geq \kappa$

Similarly, we can show that $\emptyset_A(A^{\kappa}{}_e) \leq 1-\kappa$.

Thus $A^{\kappa}{}_e \in C_{\kappa,1-\kappa}(B,q)$

Let $A^{\kappa}{}_x \in C_{\kappa,1-\kappa}(B,q) \Rightarrow \theta_B(A^{\kappa}{}_x) \geq \kappa$ and $\emptyset_B(A^{\kappa}{}_x) \leq 1-\kappa$

Let $\alpha_1 = \theta_B(A^{\kappa}{}_x) = Sup\{\theta_A(m,q): Nm = Nx\}$

and $\alpha_2 = \emptyset_B(A^{\kappa}{}_x) = Inf\{\emptyset_A(m,q): Nm = Nx\}$.

Therefore $\alpha_1 \geq \kappa$and $\alpha_2 \leq 1-\kappa$.

Let $\varepsilon > 0$ be given there exist such that $m, n \in H and\ q \in Q$ such that $Nm = Nx$ so that $mx^{-1} = a_1 \in N$ and $\theta_A(m,q) > \alpha_1 - \varepsilon \geq \kappa - \varepsilon$ and $Nn = Nx$ so that $nx^{-1} = a_2 \in N$ and $\emptyset_A(n,q) < \alpha_2 + \varepsilon \leq (1-\kappa) + \epsilon$

$\theta_A(x,q) = \theta_A(xa_1{a_2}^{-1}, q) \geq \theta_A(xa_1,q) \cap \emptyset_A(a_1,q)$

$$= \begin{cases} \geq \kappa\ if \theta_A(xa_1,q) \geq \kappa\ and \\ = \theta_A(xa_1,q)\ if \theta_A(xa_1,q) < \kappa \end{cases}$$

$\emptyset_A(x,q) = \emptyset_A(xa_2{a_2}^{-1}, q) \leq \emptyset_A(xa_2,q) \cup \emptyset_A(a_2,q)$

$$= \begin{cases} \leq 1-\kappa\ if \emptyset_A(xa_2,q) \leq 1-\kappa \\ = \emptyset_A(xa_2,q)\ if\ \emptyset_A(xa_2,q) > 1-\kappa \end{cases}$$

Thus in any case $\theta_A(x,q) > \kappa - \varepsilon$ and $\emptyset_A(x,q) < (1-\kappa) + \varepsilon$, for all $\varepsilon > 0$

$\Rightarrow \theta_A(x,q) \geq \kappa$and$\emptyset_A(x,q) \leq (1-\kappa) \Rightarrow x \in C_{\kappa,1-\kappa}(A)$

$\Rightarrow Nx = N$so$A^{\kappa}{}_x = A^{\kappa}{}_e$

Hence $C_{\kappa,1-\kappa}(B,q) = \{A^{\kappa}{}_e\}$.

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4.1. Journal Article

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