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On Elemental Algebraic Characteristic of ω −Fuzzy Subring, Normal Subring and Ideal

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Abstract

In this paper, introduced the new notion of on elemental Algebraic characteristic of $\omega - \mathrm{FSR}$ and $\omega - \mathrm{FI}$ are defined and discussed. The "homomorphism" of $\omega - \mathrm{FSR}$, $\omega - \mathrm{FNSR}$ and $\omega - \mathrm{FI}$ and their inverse images has been found. A few related results are deliberated.

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1. Introduction

The pioneering work of L A Zadeh on fuzzy subsets of a set in 1965[9]. A. B Chakranarty et al. invented the speculation of fuzzy "homomorphism" and algebraic structures in 1993[1]. The concept of Prime fuzzy ideals in the ring was established by T.K. Mukhrjee et al. in 1989[5]. V.N. Dixit et al. Presented the idea of Fuzzy rings in 1992[2].

Prasannaet.al.[6] introduced the concept of Fundamental Algebraic characteristics of χ −Fuzzy Subring, Normal Subring, and Ideal.In 1982, Wang-Jin Liu[8],the concept of fuzzy invariant subgroups and fuzzy ideals.D.S. Malik etal.derived from the extension of fuzzy subrings and fuzzy ideals in 1992[3]. V. Veeramani et al. derived from the Some Properties of Intuitionistic Fuzzy Normal Subrings in 2010[7]. T.K. Mukhrjee et al. proposed by the concept of on fuzzy ideals of a ring in 1987[4].

In this paper is arranged as follows; section 2 contains the elementary basic concept of definitions associated with the results, which are thoroughly crucial to the current research. In section 3, we introduce an elemental algebraic characteristic of $\omega -$fuzzy subring $(\omega - FSR)$ and ideal $(\omega - FI)$ and section 4,describe the algebraic

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aspect on "homomorphism" of $\omega-$fuzzy subrings $(\omega - FSR)$, normal subrings $(\omega - FSNR)$ and ideals$(\omega - FI)$.

2. Preliminaries

2.1 Definition [4]

Let R be a ring. A function $A: R \rightarrow [0,1]$ is said to be a FSR of R if
$(i) A(x-y) \geq min\{\mu_A(x), \mu_A(y)\}$
$(ii) A(xy) \geq min\{\mu_A(x), \mu_A(y)\}, \forall\, x, y \in R$.

2.2 Definition [7]

A $FSRA$ of a ring R is said to be a $FNSR$ of R if
$$A(xy) = A(yx), \forall\, x, y \in R.$$

2.3 Definition [8]

Let R be a ring. A function $A: R \rightarrow [0,1]$ is said to be a

(a) FLI of R if
$(i) A(x-y) \geq min\{A(x), A(y)\}$
$(ii) A(xy) \geq A(y), \forall\, x, y \in R$

(b) FRI of R if
$(i) A(x-y) \geq min\{A(x), A(y)\}$
$(ii) A(xy) \geq A(x), \forall\, x, y \in R$

(c) FI of R if
$(i) A(x-y) \geq min\{A(x), A(y)\}$
$(ii) A(xy) \geq max\{A(x), A(y)\}, \forall\, x, y \in R$

2.4 Theorem [3]

If A be a FSR of the ring R then
(i) $A(0) \geq A(x)$
$(ii) A(-x) = A(x), \forall\, x \in R$
(iii) If R is a ring with unity 1, then $A(1) \geq A(x), \forall\, x \in R$.

2.5 Definition [1]

Let X and Y be two non-empty sets and $f: X \rightarrow Y$b e a mapping. Let A and B be FS of X and Y, respectively. Then the image of A under the map f is signified by $f(A)$ and is well-defined as $f(A)(y) = \begin{cases} Sup\{A(x): x \in f^{-1}(y)\} \\ 0: otherwise \end{cases}, \forall\, y \in Y$
Also, the pre-image of B under f is denoted by $f^{-1}(B)$ and defined
$$f^{-1}(B)(x) = Bf(x), \forall\, x \in X.$$

2.6 Definition[1]

The mapping $f: R_1 \rightarrow R_2$ from the ring R_1 into a ring R_2 is called a ring "homomorphism" if
$(i) f(x+y) = f(x) + f(y)$
$(ii) f(xy) = f(x)f(y), \forall\, x, y \in R_1$.

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3. On Elemental Algebraic Characteristic of $\omega - FSR$ and $\omega - $ FI

3.1 Definition

Let M be a FS of a ring τ. Let $\omega \in [0,1]$. Then the FSM^{ω} of τ is termed the $\omega - FSb$ of τconcerningFSM and is defined by
$M^{\omega}(\theta) = \{M(\theta) \wedge \omega\}, \ \forall \omega \in [0,1]$.

3.2 Definition

Let M be a FS of a ring $\tau$and $\in [0,1]$. Then M is termed$\omega - FSR$ of τ if M^{ω} is FSR of τ, i.e.if the subsequent conditions hold

$$(i) M^{\omega}(\theta - \varphi) \geq \{M^{\omega}(\theta) \wedge M^{\omega}(\varphi)\}$$
$$(ii) M^{\omega}(\theta\varphi) \geq \{M^{\omega}(\theta) \wedge M^{\omega}(\varphi)\}, \forall \theta, \varphi \in \tau$$

If otherwise, M is $\omega - FSR$ of τ if M^{ω} is FSR of τ.

3.3 Definition

Let M be a FS of a ring τ. Let $\omega \in [0,1]$. Then M is termed$\omega - FLI$ of τ if
$(i) M^{\omega}(\theta - \varphi) \geq \{M^{\omega}(\theta) \wedge M^{\omega}(\varphi)\}$
$(ii) M^{\omega}(\theta\varphi) \geq M^{\omega}(\varphi), \forall \theta, \varphi \in \tau$.

3.4 Definition

Let M be a FS of a ring τ. Let $\omega \in [0,1]$. Then M is termed$\omega - FRI$ of τ if
$(i) M^{\omega}(\theta - \varphi) \geq \{M^{\omega}(\theta) \wedge M^{\omega}(\varphi)\}$
$(ii) M^{\omega}(\theta\varphi) \geq M^{\omega}(\theta), \forall \theta, \varphi \in \tau$.

3.5 Definition

Let M be a FS of a ring τ. Let $\omega \in [0,1]$. Then M is termed$\omega - FI$of if
$(i) M^{\omega}(\theta - \varphi) \geq \{M^{\omega}(\theta) \wedge M^{\omega}(\varphi)\}$
$(ii) M^{\omega}(\theta\varphi) \geq \{M^{\omega}(\theta) \wedge M^{\omega}(\varphi)\}, \forall \theta, \varphi \in \tau$.

3.6 Proposition

Let M^{ω} and N^{ω} be two $\omega - FS$ of a ring τ. Then $(M \cap N)^{\omega} = M^{\omega} \cap N^{\omega}$.

Proof:

Let $\theta \in \tau$ be any element, then

$$\begin{aligned}(M \cap N)^{\omega}(\theta) &= \{(M \cap N) \wedge \omega\} \\ &= (\{M(\theta) \wedge N(\varphi)\} \wedge \omega) \\ &= (\{M(\theta) \wedge \omega\} \wedge \{N(\varphi) \wedge \omega\}) \\ &= \{M^{\omega}(\theta) \wedge N^{\omega}(\varphi)\} \\ &= (M^{\omega} \cap N^{\omega})(\theta)\end{aligned}$$

Hence $(M \cap N)^{\omega}(\theta) = (M^{\omega} \cap N^{\omega})(\theta)$.

3.7 Proposition

Let $g: \alpha \to \beta$ be a mapping. Let M and N are two FS of α andβ respectively, then $(i) g^{-1}(N^{\omega}) = (g^{-1}(N))^{\omega}$
$(ii)\ g(M^{\omega}) = (g(M))^{\omega}, \forall \omega \in [0,1]$.

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Proof:

Let $g: \alpha \to \beta$ be a mapping.

Let M and N are two FS of α and β, respectively.

$$(i) g^{-1}(N^{\omega})(\varphi) = N^{\omega}\big(g(\varphi)\big)$$
$$= \{N(g(\varphi))\wedge\omega\}$$
$$= \{g^{-1}(N)(\varphi)\wedge\omega\}$$
$$= (g^{-1}(N))^{\omega}\ (\varphi)$$
$$\Rightarrow g^{-1}(N^{\omega})(\varphi) = (g^{-1}(N))^{\omega}$$

$$(ii)\ g(M^{\omega})(\theta) = Sup\{M^{\omega}(\varphi): g(\varphi) = \theta\}$$
$$= sup(\{M(\varphi)\wedge\omega\}: g(\varphi) = \theta)$$
$$= (Sup\{M(\varphi): g(\varphi) = \theta\}\wedge\omega)$$
$$= \{g(M)(\theta)\wedge\omega\}$$
$$= \big(g(M)\big)^{\omega}(\theta)$$
$$\Rightarrow g(M^{\omega}) = \big(g(M)\big)^{\omega}.$$

3.8 Proposition

If M is FSR of a ring τthen M is additionally $\omega - FSR$ of τ.

Proof:

Let $\theta, \varphi \in \tau$ be every element of the ring τ.

Now,

$$(i)\ M^{\omega}(\theta - \varphi) = \{M^{\omega}(\theta - \varphi)\wedge\omega\}$$
$$\geq (\{M^{\omega}(\theta)\wedge M^{\omega}(\varphi)\}\wedge\omega)$$
$$= (\{M^{\omega}(\theta)\wedge\omega\}\wedge\{M^{\omega}(\varphi)\wedge\omega\})$$
$$= \{M^{\omega}(\theta)\wedge M^{\omega}(\varphi)\}$$
$$\Rightarrow M^{\omega}(\theta - \varphi) \geq \{M^{\omega}(\theta)\wedge M^{\omega}(\varphi)\}$$
$$(ii) M^{\omega}(\theta\varphi) = \{M(\theta\varphi)\wedge\omega\}$$
$$\geq (\{M(\theta)\wedge M(\varphi)\}\wedge\omega)$$
$$= (\{M(\theta)\wedge\omega\}\wedge\{M(\varphi)\wedge\omega\})$$
$$= \{M^{\omega}(\theta)\wedge M^{\omega}(\varphi)\}$$
$$\Rightarrow M^{\omega}(\theta\varphi) \geq \{M^{\omega}(\theta)\wedge M^{\omega}(\varphi)\}$$

Therefore, M is $\omega - FSR$ of τ.

3.9 Proposition

The above proposition [3.8] proof it will be needn't be real.

3.9.1 Numerical Example 1

Let the ring $(Z_5, +_5, \times_5)$, where $Z_5 = \{0,1,2,3,4,5\}$.

Define the fuzzy set M of Z_5 by

$$M(\theta) = \begin{cases} 0.7;\ \ if\ x = 0 \\ 0.5; if\ x = 1,3 \\ 0.2; if\ x = 2,4 \end{cases}$$

It is easy to verify that M isn'tFSR of Z_5.

However, if we take $\omega = 0.1$, then $M^{\omega}(\theta) = 0.1, \forall\ \theta \in Z_5$.

Now, it will be easily proved that M^{ω} is FSR of Z_5

$\therefore\ M$is$\omega - FSR$ of Z_5.

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3.10 Lemma

Let M be a FS of the ring τ. Let $\omega \leq L$, where $L = \{M(\theta): \forall\, \theta \in \tau\}$.Then M is $\omega - FSR$ of τ.

Proof:

Let M be a FS of the ring τ and $\omega \leq L$
Since$\omega \leq L \Rightarrow L \geq \omega$
Implies that $\{M(\theta): \forall\, \theta \in \tau\} \geq \omega$
$\Rightarrow M(\theta) \geq \omega, \forall\, \theta \in \tau$
$\therefore M^{\omega}(\theta - \varphi) \geq \{M^{\omega}(\theta) \wedge M^{\omega}(\varphi)\}$and $M^{\omega}(\theta\varphi) \geq \{M^{\omega}(\theta) \wedge M^{\omega}(\varphi)\}$,
Hence M is $\omega - FSR$ of τ.

3.11 Theorem

If intersection of two $\omega - FSR's$ of a ring τ is additionally $\omega - FSR$ of τ.

Proof:

Let $\theta, \varphi \in \tau$ be every element of the ring τ.
Then,

$$
\begin{aligned}
(i)(M \cap N)^{\omega}(\theta - \varphi) &= \{(M \cap N)(\theta - \varphi) \wedge \omega\} \\
&= (\{M(\theta - \varphi) \wedge N(\theta - \varphi)\} \wedge \omega) \\
&= (\{M(\theta - \varphi) \wedge \omega\} \wedge \{N(\theta - \varphi) \wedge \omega\}) \\
&= \{M^{\omega}(\theta - \varphi) \wedge N^{\omega}(\theta - \varphi)\} \\
&\geq (\{M^{\omega}(\theta) \wedge M^{\omega}(\varphi)\} \wedge \{N^{\omega}(\theta) \wedge N^{\omega}(\varphi)\}) \\
&= (\{M^{\omega}(\theta) \wedge N^{\omega}(\theta)\} \wedge \{M^{\omega}(\varphi) \wedge N^{\omega}(\varphi)\}) \\
&= \{(M^{\omega} \cap N^{\omega})(\theta) \wedge (M^{\omega} \cap N^{\omega})(\varphi)\} \\
&= \{(M \cap N)^{\omega}(\theta) \wedge (M \cap N)^{\omega}(\varphi)\}
\end{aligned}
$$

$\Rightarrow (M \cap N)^{\omega}(\theta - \varphi) \geq \{(M \cap N)^{\omega}(\theta) \wedge (M \cap N)^{\omega}(\varphi)\}$

$$
\begin{aligned}
(ii)(M \cap N)^{\omega}(\theta\varphi) &= \{(M \cap N)(\theta\varphi) \wedge \omega\} \\
&= (\{M(\theta\varphi) \wedge N(\theta\varphi)\} \wedge \omega) \\
&= (\{M(\theta\varphi) \wedge \omega\} \wedge \{N(\theta\varphi) \wedge \omega\}) \\
&= \{M^{\omega}(\theta\varphi) \wedge N^{\omega}(\theta\varphi)\} \\
&\geq (\{M^{\omega}(\theta) \wedge M^{\omega}(\varphi)\} \wedge \{N^{\omega}(\theta) \wedge N^{\omega}(\varphi)\}) \\
&= (\{M^{\omega}(\theta) \wedge N^{\omega}(\theta)\} \wedge \{M^{\omega}(\varphi) \wedge N^{\omega}(\varphi)\}) \\
&= \{(M^{\omega} \cap N^{\omega})(\theta) \wedge (M^{\omega} \cap N^{\omega})(\varphi)\} \\
&= \{(M \cap N)^{\omega}(\theta) \cap (M \cap N)^{\omega}(\varphi)\}
\end{aligned}
$$

$\Rightarrow (M \cap N)^{\omega}(\theta\varphi) \geq \{(M \cap N)^{\omega}(\theta) \cap (M \cap N)^{\omega}(\varphi)\}$
Therefore $M \cap N$ is $\omega - FSR$ of τ.

3.12 Theorem

Let M be $FNSR$ of a ring τ. Then M is additionally $\omega - FNSR$ of τ.

Proof:

Let $\theta, \varphi \in \tau$ be any element of the ring τ.

$$
\begin{aligned}
\text{Then } M^{\omega}(\theta\varphi) &= \{M(\theta\varphi) \wedge \omega\} \\
&= \{M(\varphi\theta) \wedge \omega\} \\
&= M^{\omega}(\varphi\theta)
\end{aligned}
$$

Therefore M is $\omega - FNSR$ of τ.

3.13 Lemma

Let M is FLI of a ring τ, then M is additionally $\omega - FLI$ of τ.

Proof:

During this Proposition [3.8], we'd like only to prove that

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$$M^{\omega}(\theta\varphi) \geq M^{\omega}(\varphi), \forall\, \theta, \varphi \in \tau$$
$$M^{\omega}(\theta\varphi) = \{M(\theta\varphi), \omega\}$$
$$\geq \{M(\varphi) \wedge \omega\}$$
$$= M^{\omega}(\varphi)$$

Implies that $M^{\omega}(\theta\varphi) \geq M^{\omega}(\varphi)$

$\therefore M$ is $\omega - FLI$ of τ.

3.14 Lemma

Let M is FRI of a ring τ, then M is additionally $\omega - FRI$ of τ.

Proof:

During this Proposition [3.8], we'd like only to prove that

$$M^{\omega}(\theta\varphi) \geq M^{\omega}(\theta), \forall\, \theta, \varphi \in \tau$$
$$M^{\omega}(\theta\varphi) = \{M(\theta\varphi), \omega\}$$
$$\geq \{M(\theta) \wedge \omega\}$$
$$= M^{\omega}(\theta)$$

Implies that $M^{\omega}(\theta\varphi) \geq M^{\omega}(\theta)$

$\therefore M$ is $\omega - FRI$ of τ.

4. On Algebraic Aspect on "homomorphism" of $\omega - FSR$, $\omega - FNSR$ and $\omega - FI$

4.1 Theorem

Let $g: \tau_1 \rightarrow \tau_2$ be a ring "homomorphism" from the ring τ_1 into a ring τ_2. Let N be $\omega - FSR$ of τ_2. Then $g^{-1}(N)$ is $\omega - FSR$ of τ_1.

Proof:

Let $N\omega - FSR$ of τ_2.

Let $\theta_1, \theta_2 \in \tau_1$ be any element.

Then

(i) $g^{-1}(N^{\omega})(\theta_1 - \theta_2) = N^{\omega}\big(g(\theta_1 - \theta_2)\big)$
$$= N^{\omega}\big(g(\theta_1) - g(\theta_2)\big)$$
$$\geq \{N^{\omega}(g(\theta_1)) \wedge N^{\omega}(g(\theta_2))\}$$
$$= \{g^{-1}(N^{\omega})(\theta_1) \wedge g^{-1}(N^{\omega})(\theta_2)\}$$
$$\Rightarrow g^{-1}(N^{\omega})(\theta_1 - \theta_2) \geq \{g^{-1}(N^{\omega})(\theta_1) \wedge g^{-1}(N^{\omega})(\theta_2)\}$$

(ii) $g^{-1}(\theta_1\theta_2) = N^{\omega}\big(g(\theta_1\theta_2)\big)$
$$= N^{\omega}\big(g(\theta_1) - g(\theta_2)\big)$$
$$\geq \big\{N^{\omega}\big(g(\theta_1)\big) \wedge N^{\omega}\big(g(\theta_2)\big)\big\}$$
$$= \{g^{-1}(N^{\omega})(\theta_1) \wedge g^{-1}(N^{\omega})(\theta_2)\}$$
$$\Rightarrow g^{-1}(\theta_1\theta_2) \geq \{g^{-1}(N^{\omega})(\theta_1) \wedge g^{-1}(N^{\omega})(\theta_2)\}$$

Therefore $g^{-1}(N^{\omega}) = (g^{-1}(N))^{\omega}$ is FSR of τ_1

$\therefore g^{-1}(N)$ is $\omega - FSR$ of τ_1.

4.2 Lemma

Let $g: \tau_1 \rightarrow \tau_2$ be a ring "homomorphism" from the ring τ_1 into a ring τ_2. Let N be $\omega - FNSR$ of τ_2. Then $g^{-1}(N)$ is $\omega - FNSR$ of τ_1.

Proof:

Let N be $\omega - FNSR$ of τ_2 and $\theta_1, \theta_2 \in \tau_1$ be any element.

$$g^{-1}(N^{\omega})(\theta_1\theta_2) = N^{\omega}\big(g(\theta_1\theta_2)\big)$$
$$= N^{\omega}\big(g(\theta_1)g(\theta_2)\big)$$

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$$= N^{\omega}\big(g(\theta_2)g(\theta_1)\big)$$
$$= N^{\omega}(g(\theta_1\theta_2))$$
$$= g^{-1}(N^{\omega}(\theta_1\theta_2))$$
$$\Rightarrow g^{-1}(N^{\omega}) = (g^{-1}(N))^{\omega} \text{ is } FNSR \text{ of} \tau_1$$
$$\therefore g^{-1}(N) \text{is} \omega - FNSR \text{ of } \tau_1.$$

4.3 Theorem

Let $g: \tau_1 \to \tau_2$ be a ring "homomorphism" from the ring τ_1 into a ring τ_2. Let N be $\omega - FLI$ of τ_2. Then $g^{-1}(N)$ is $\omega - FLI$ of τ_1.

Proof:

Let N be $\omega - FLI$ of τ_2 and $\theta_1, \theta_2 \in \tau_1$ be any element.
Then, invisible of the theorem [4.1],
We have only proved that

$$g^{-1}(N^{\omega})(\theta_1\theta_2) \geq g^{-1}(N^{\omega})(\theta_2)$$

Now, $g^{-1}(N^{\omega})(\theta_1\theta_2) = N^{\omega}(g(\theta_1\theta_2))$
$$= N^{\omega}\big(g(\theta_1)g(\theta_2)\big)$$
$$\geq N^{\omega}(g(\theta_2))$$
$$= g^{-1}(N^{\omega})(\theta_2)$$

Thus implies that $g^{-1}(N^{\omega})(\theta_1\theta_2) \geq g^{-1}(N^{\omega})(\theta_2)$
Thus implies also $g^{-1}(N^{\omega}) = (g^{-1}(N))^{\omega}$ is FLI of τ_1
$\therefore g^{-1}(N)$is$\omega - FLI$ of τ_1.

4.4 Theorem

Let $g: \tau_1 \to \tau_2$ be a ring "homomorphism" from the ring τ_1 into a ring τ_2. Let N be $\omega - FRI$ of τ_2. Then $g^{-1}(N)$ is $\omega - FRI$ of τ_1.

Proof:

It will be easier to prove that will be got the same as theorem[4.3].

4.5 Theorem

Let $g: \tau_1 \to \tau_2$ be a ring "homomorphism" from the ring τ_1 into a ring τ_2. Let N be $\omega - FI$ of τ_2. Then $g^{-1}(N)$ is $\omega - FI$ of τ_1.

Proof:

It will be obtained the same as theorem[4.3] and [4.4].

4.6 Theorem

Let $g: \tau_1 \to \tau_2$ be "surjective" ring "homomorphism" and M be $\omega - FSR$ of τ_1.Then$g(M)$ is $\omega - FSR$ of τ_2.

Proof:

Let M be $\omega - FSR$ of τ_1.
Let $\varphi_1, \varphi_2 \in \tau_2$ be any element. Then $\exists$ some $\theta_1, \theta_2 \in \tau_1$ such $g(\theta_1) = \varphi_1$ and $g(\theta_2) = \varphi_2$. (Since that θ_1, θ_2 needn't be unique)

$$(i) g(M^{\omega})(\varphi_1 - \varphi_2) = (g(M))(\varphi_1 - \varphi_2)$$
$$= \{g(M)(g(\theta_1) - g(\theta_2)) \wedge \omega\}$$
$$= \{g(M)(g(\theta_1 - \theta_2)) \wedge \omega\}$$
$$\geq \{M(\theta_1 - \theta_2) \wedge \omega\}$$
$$= M^{\omega}(\theta_1 - \theta_2)$$
$$\geq \{M^{\omega}(\theta_1) \wedge M^{\omega}(\theta_2)\},$$

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For all $\theta_1, \theta_2 \in \tau_1$ such that $(\theta_1) = \varphi_1$ and $g(\theta_2) = \varphi_2$

$$= \{\{M^\omega(\theta_1): g(\theta_1) = \varphi_1\} \wedge \{M^\omega(\theta_2): g(\theta_2) = \varphi_2\}\}$$
$$= \{g(M^\omega)(\varphi_1) \wedge g(M^\omega)(\varphi_2)\}$$

Thus implies that $g(M^\omega)(\varphi_1 - \varphi_2) \geq \{g(M^\omega)(\varphi_1) \wedge g(M^\omega)(\varphi_2)\}$.

$$(ii)\ g(M^\omega)(\varphi_1\varphi_2) = (g(M))^\omega(\varphi_1\varphi_2)$$
$$= \{g(M)(g(\theta_1)g(\theta_2)) \wedge \omega\}$$
$$= \{g(M)(g(\theta_1\theta_2)) \wedge \omega\}$$
$$\geq \{M(\theta_1\theta_2) \wedge \omega\}$$
$$= M^\omega(\theta_1\theta_2)$$
$$\geq \{M^\omega(\theta_1) \wedge M^\omega(\theta_2)\}$$

For all $\theta_1, \theta_2 \in \tau_1$ such that $(\theta_1) = \varphi_1$ and $g(\theta_2) = \varphi_2$

$$= \{\{M^\omega(\theta_1): g(\theta_1) = \varphi_1\} \wedge \{M^\omega(\theta_2): g(\theta_2) = \varphi_2\}\}$$
$$= \{g(M^\omega)(\varphi_1) \wedge g(M^\omega)(\varphi_2)\}$$

Thus implies that $g(M^\omega)(\varphi_1\varphi_2) \geq \{g(M^\omega)(\varphi_1) \wedge g(M^\omega)(\varphi_2)\}$.

Thus $g(M^\omega) = (g(M))^\omega$ is FSR of τ_2 and hence $g(M)$ is $\omega - FSR$ of τ_2.

4.7 Theorem

Let $g: \tau_1 \to \tau_2$ be "surjective" ring "homomorphism" and M be $\omega - FNSR$ of τ_1.Then $g(M)$ is $\omega - FNSR$ of τ_2.

Proof:

Let M be $\omega - FNSR$ of τ_1.
Let $\varphi_1, \varphi_2 \in \tau_2$ be any element.
Then $\exists$ some $\theta_1, \theta_2 \in \tau_1$ such $g(\theta_1) = \varphi_1$ and $g(\theta_2) = \varphi_2$.
(Since that θ_1, θ_2 needn't be unique)
In this view of the theorem [4.6],
we want only to prove that $(g(M))^\omega(\varphi_1\varphi_2) = g(M^\omega)(\varphi_2\varphi_1)$

$$(g(M))^\omega(\varphi_1\varphi_2) = g(M^\omega)(g(\theta_1)g(\theta_2))$$
$$= g(M^\omega)(g(\theta_1\theta_2))$$
$$= \{M^\omega(\theta_1\theta_2): g(\theta_1\theta_2) = \varphi_1\varphi_2\}$$
$$= \{M^\omega(\theta_2\theta_1): g(\theta_1\theta_2) = \varphi_1\varphi_2\}$$
$$= g(M^\omega)(g(\theta_2)g(\theta_1))$$
$$= (g(M))^\omega(\varphi_2\varphi_1)$$
$$\Rightarrow (g(M))^\omega \text{ is } FNSR \text{ of } \tau_2$$
$$\therefore g(M) \text{ is } \omega - FNSR \text{ of } \tau_2.$$

4.8 Theorem

Let $g: \tau_1 \to \tau_2$ be "bijective" ring "homomorphism" and M be $\omega -$FLI of τ_1. Then $g(M)$ is $\omega -$FLI of τ_2.

Proof:

Let M be $\omega -$FLI of τ_1.
Let $\varphi_1, \varphi_2 \in \tau_2$ be any element.
Then $\exists$ some $\theta_1, \theta_2 \in \tau_1$ such $g(\theta_1) = \varphi_1$ and $g(\theta_2) = \varphi_2$.
In this view of the theorem [4.6],
we want only to prove that $(g(M))^\omega(\varphi_1\varphi_2) \geq (g(M))^\omega(\varphi_2)$

$$(g(M))^\omega(\varphi_1\varphi_2) = \{g(M)(g(\theta_1)g(\theta_2)) \wedge \omega\}$$
$$= \{g(M)(g(\theta_1\theta_2)) \wedge \omega\}$$
$$= \{M(\theta_1\theta_2) \wedge \omega\}$$
$$= M^\omega(\theta_1\theta_2)$$

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$\geq M^{\omega}(\theta_2)$
$= \{M^{\omega}(\theta_2) \wedge \omega\}$
$= \{g(M)(g(\theta_2)) \wedge \omega\}$
$= \{g(M)(\varphi_2) \wedge \omega\}$
$= (g(M))^{\omega}(\varphi_2)$

Thus implies that $(g(M))^{\omega}(\varphi_1\varphi_2) \geq (g(M))^{\omega}(\varphi_2)$

Hence $(g(M))^{\omega}$ is FLI of τ_2

$\therefore g(M)$ is $\omega - FLI$ of τ_2.

4.9 Theorem

Let $g: \tau_1 \to \tau_2$ be "bijective" ring "homomorphism" and M be $\omega - FRI$ of τ_1.Then $g(M)$ is $\omega - FRI$ of τ_2.

Proof:

During this proof, it will be obtained the same as theorem[4.8].

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