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A Novel Similarity Measure on Interval Valued Intuitionistic Fuzzy Numbers and Its Applications

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Abstract

The finding of this paper is to introduce a new similarity measure on interval valued intuitionistic fuzzy numbers (IVIFNs) which computes the distance between intervals valued intuitionistic fuzzy sets. Some properties and some relations of this new measure are also studied. Finally, a new method is proposed for solving pattern recognition problem.
Keywords: Distance Measure; Interval Valued Intuitionistic Fuzzy Number; Pattern Recognition Problem; Similarity Measure;

1. Introduction

Fuzzy set was introduced by Zadeh [19] in the year 1965. This idea is effectively applied in various fields as a result of its convenience. Fuzzy set was summed up to intuitionistic Fuzzy set (IFS) by Atanassov [1] in 1986 and further to vague set developed by Gau and Buehrer [5] in 1993. Both these sets are utilized to process uncertain or dubious data. The IFS describes the degrees of belongingness and non-belongingness by enrollment and non-membership functions individually. Fuzzy set was further developed to interval valued fuzzy sets (IVFS) by Gorzalaczany [6] and Turksen [15] and interval valued intuitionistic fuzzy sets (IVIFS) by Atanassov and Gargov [2].

Various authors have investigated IVFSs and its compatible topics. Cheng and Li [11] contemplated the connection among entropy and closeness proportion of IVFSs.

Similarity measure and distance measure serve as a tool to solve practical applications. Both these measures, being counter parts of IFS, symbolize two expressions of the same measure. The similarity measure estimates the degree of similarity and hence the distance measure between IFSs. Similarity measures between two fuzzy sets have been characterized by numerous creators [8-10], [12]. M.Venkatachalapathy, R.Pandiarajan, S.Ganeshkumar[18], developed a special type of generalized quadratic fuzzy numbers. S.Muthuperumal, P.Titus & M.Venkatachalapathy [17], using an triangular fuzzy numbers.

The approach of this paper is coordinated as follows: The definition of IFSs, IVIFSs, properties of distance measure and similarity measure, comparable, comparable by vagueness, comparable by impreciseness are briefly introduced in section 2. In section 3, novel distance measure for IVIFS is introduced and analyzed. In section 4, novel similarity measure for IVIFNs is introduced and categorized. In section 5, a new method for solving pattern recognition problem is introduced by using the proposed measure. In section 6, conclusion and future scope are given.

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2. Preliminaries

2.1 Definition [2]

AnIVIFS on a nonempty set X is defined as $A = \{(x, \mu_A(x), \vartheta_A(x)) : x \in X\}$, where $\mu_A(x) = [\underline{\mu}_A(x), \overline{\mu}_A(x)]$ and $\vartheta_A(x) = [\underline{\vartheta}_A(x), \overline{\vartheta}_A(x)]$ are closed sub-intervals of $[0, 1]$ which satisfy the condition $0 \le \overline{\mu}_A(x) + \overline{\vartheta}_A(x) \le 1$.The collection of all IVIFS on X is denoted byIVIFS(X). An IVIFS on singleton set is called IVIF Number. The collection of all IVIF Numbers is denoted by IVIFN.

2.2 Definition [13]

Two IVIFNs, $A = ([A_{a_1}, A_{b_1}], [A_{c_1}, A_{d_1}])$ and $B = ([B_{a_2}, B_{b_2}], [B_{c_2}, B_{d_2}]$, are said to be comparable, $\widehat{A} \le_1 \widehat{B}$, if $A_{a_1} \le B_{a_2}$, $A_{b_1} \le B_{b_2}$, $A_{c_1} \ge B_{c_2}$ and $A_{d_1} \ge B_{d_2}$.

2.3 Definition [14]:

Two IVIFNs, $A = ([A_{a_1}, A_{b_1}], [A_{c_1}, A_{d_1}])$ and $B = ([B_{a_2}, B_{b_2}], [B_{c_2}, B_{d_2}]$, are said to be comparable by vagueness, $\widehat{A} \le_2 \widehat{B}$, if $B_{a_2} \le A_{a_1}$, $A_{b_1} \le B_{b_2}$,$A_{c_1} \le B_{c_2}$ and $B_{d_2} \le A_{d_1}$.

2.4 Definition [14]

Two IVIFNs, $A = ([A_{a_1}, A_{b_1}], [A_{c_1}, A_{d_1}])$ and $B = ([B_{a_2}, B_{b_2}], [B_{c_2}, B_{d_2}]$, are said to be comparable by impreciseness $\widehat{A} \le_3 \widehat{B}$, if$B_{a_2} \le A_{a_1}$,$A_{b_1} \le B_{b_2}$, $B_{c_2} \le A_{c_1}$ and $A_{d_1} \le B_{d_2}$.

Distance is a measure of the difference between two elements of a set. In the case of IVIFSs, the distance between two elements must satisfy the following axioms.

2.5 Definition [16]

A mapping $D: \mathrm{IVIFS}(X) \times \mathrm{IVIFS}(X) \to [0,1]$ is called the distance measure onIVIFS(X)if: For any $A, B, C \in \mathrm{IVIFS}(X)$

(D1). $0 \le D(A, B) \le 1$.

(D2). $D(A, B) = 0$ if and only if $A = B$.

(D3). $D(A, B) = D(B, A)$.

(D4). If $A \le_1 B \le_1 C$, then $D(A, B) \le D(A, C)$ and $D(B, C) \le D(A, C)$.

The similarity measure is viewed as a complementary concept of distance measure which is defined as follows.

2.6 Definition [16]

A function $S: \mathrm{IVIFS}(X) \times \mathrm{IVIFS}(X) \to [0,1]$ is called the similarity measure onIVIFS(X) if: For any $\widehat{A}, \widehat{B}, \widehat{C} \in \mathrm{IVIFS}(X)$

(S1). $0 \le S(A, B) \le 1$.

(S2). $S(A, B) = 1$ if and only if $A = B$.

(S3). $S(A, B) = S(B, A)$.

(S4). If, $A \le_1 B \le_1 C$, then $S(A, B) \ge S(A, C)$ and $S(B, C) \ge S(A, C)$.

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2.7 Definition [2-4]

Let $A = ([A_{a_1}, A_{b_1}], [A_{c_1}, A_{d_1}])$ and $\widehat{B} = ([B_{a_2}, B_{b_2}], [B_{c_2}, B_{d_2}] \in \text{IVIFN}$. Now, $A + B$, $A \cap B$, $A \cup B$ and A^c are defined by

1. $A + B = ([A_{a_1} + B_{a_2} - A_{a_1}B_{a_2}, A_{b_1} + B_{b_2} - A_{b_1}B_{b_2}], [A_{c_1}B_{c_2}, A_{d_1}B_{d_2}])$
2. $A \cap B = ([\min\{A_{a_1}, B_{a_2}\}, \min\{A_{b_1}, B_{b_2}\}], [\max\{A_{c_1}, B_{c_2}\}, \max\{A_{d_1}, B_{d_2}\}])$,
3. $A \cup B = ([\max\{A_{a_1}, B_{a_2}\}, \max\{A_{b_1}, B_{b_2}\}], [\min\{A_{c_1}, B_{c_2}\}, \min\{A_{d_1}, B_{d_2}\}])$,
4. $A^c = ([A_{c_1}, A_{d_1}], [A_{a_1}, A_{b_1}])$.

2.8 Definition [7]

A generalized improved accuracy function GIS of IVIFN $A = ([A_a, A_b], [A_c, A_d])$, is expressed by $\text{GIS}(A) = \frac{(A_a + A_b)}{2} + K_1(1 - A_a - A_c) + K_2(1 - A_b - A_d)$, where $\text{GIS} \in [0, 1]$.

3. A New Distance Measure on IVIFNS

3.1 Definition

A map $D: \text{IVIFN} \times \text{IVIFN} \to [0,1]$ between two IVIFNs $A = ([a_1, b_1], [c_1, d_1])$ and $B = ([a_2, b_2], [c_2, d_2])$ is defined by $D(A,B) = \left(\left|\frac{a_1 - a_2}{2}\right| + \left|\frac{b_1 - b_2}{2}\right|\right) + K_1(|a_1(1 -$ a1−c1−a2(1−a2−c2)+K2b11−b1−d1−b2(1−b2−d2) , Where,K1,K2∈0, 1 and $K_1 + K_2 \leq 1$.

3.2 Theorem

$D: \text{IVIFN} \times \text{IVIFN} \to [0,1]$ is a distance measure

Proof:

The conditions in Definition 2.5, (D1), (D2), (D3) and (D4), are obvious.

3.3 Theorem

Let $A = ([a_1, b_1], [c_1, d_1])$, $B = ([a_2, b_2], [c_2, d_2])$ and $C = ([a_3, b_3], [c_3, d_3])$ be an IVIFNs. Then $D(A, C) \leq D(A, B) + D(B, C)$.

Proof:

Now $D(\widehat{A}, \widehat{C}) = \left(\left|\frac{a_1 - a_3}{2}\right| + \left|\frac{b_1 - b_3}{2}\right|\right) + K_1(|a_1(1 - a_1 - c_1) - a_3(1 - a_3 - c_3)|) + K_2(|b_1(1 - b_1 - d_1) - b_3(1 - b_3 - d_3|)$

$$\leq \left(\left|\frac{a_1 - a_2}{2}\right| + \left|\frac{b_1 - b_2}{2}\right|\right) + K_1(|a_1(1 - a_1 - c_1) - a_2(1 - a_2 - c_2)|)$$
$$+ K_2(|b_1(1 - b_1 - d_1) - b_2(1 - b_2 - d_2)|) + \left(\left|\frac{a_2 - a_3}{2}\right| + \left|\frac{b_2 - b_3}{2}\right|\right)$$
$$+ K_1(|a_2(1 - a_2 - c_2) - a_3(1 - a_3 - c_3)|)$$
$$+ K_2(|b_2(1 - b_2 - d_2) - b_3(1 - b_3 - d_3)|)$$
$$= D(\widehat{A}, \widehat{B}) + D(\widehat{B}, \widehat{C}).$$

Hence $D(A, C) \leq D(A, B) + D(B, C)$.

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3.4 Theorem

Let $\widehat{A}, \widehat{B}, \widehat{C} \in$ IVIFN, if $\widehat{A} \leq_1 \widehat{B} \leq_1 \widehat{C}$, then $D(\widehat{A}, \widehat{C}) = D(\widehat{A}, \widehat{B}) + D(\widehat{B}, \widehat{C})$.

3.5 Theorem

Let $A = (a_1, c_1)$and $B = (a_1, c_1)$ be two IFNs. Then $D(A, B) = |a_1 - a_2| + |a_1(1 - a_1 - c_1) - a_2(1 - a_2 - c_2)|(K_1 + K_2)$.

3.6 Theorem

Let$A = [a_1\ , b_1]$,$B = [a_2\ , b_2]$ be two IVFNs.Then $D(A, B) = \left(\left|\frac{a_1 - a_2}{2}\right| + \left|\frac{b_1 - b_2}{2}\right|\right) + K_1(|a_1(b_1 - a_1) - a_2(b_2 - a_2)|) + K_2(|b_1(a_1 - b_1) - b2(a2-b2))$.

3.7 Theorem

Let $A = a_1$ and $B = a_2$ be two Fuzzy numbers defined on singleton set.Then $D(A, B) = |a_1 - a_2|$.

3.8 Theorem

Let $A = ([a_1\ , b_1], [c_1, d_1])$ $B = ([a_2\ , b_2], [c_2, d_2]) \in$ IVIFN . Then

$$D(A \cup B, A) = \left[\frac{|a_1 - \max\{a_1, a_2\}|}{2} + \frac{|b_1 - \max\{b_1, b_2\}|}{2}\right] + K_1[|a_1(1 - a_1 - c_1) - \max\{a_1, a_2\}(1 - \max\{a_1, a_2\} - \min\{c_1, c_2\})|] + K_2\left[\left|b_1(1 - b_1 - d_1) - \max\{b_1, b_2\}(1 - \max\{b_1, b_2\} - \min\{d_1, d_2\})\right|\right].$$

3.9 Theorem

Let $= ([a_1\ , b_1], [c_1, d_1])$, $B = ([a_2\ , b_2], [c_2, d_2]) \in$ IVIFN . Then

$$D(A \cap B, A) = \left[\frac{|a_1 - \min\{a_1, a_2\}|}{2} + \frac{|b_1 - \min\{b_1, b_2\}|}{2}\right] + K_1[|a_1(1 - a_1 - c_1) - \min\{a_1, a_2\}(1 - \min\{a_1, a_2\} - \max\{c_1, c_2\})|] + K_2\left[\left|b_1(1 - b_1 - d_1) - \min\{b_1, b_2\}(1 - \min\{b_1, b_2\} - \max\{d_1, d_2\})\right|\right].$$

3.10 Theorem

The distance between two crisp numbers$A = ([0, 0], [1, 1])$ and $B = ([1, 1], [0, 0])$ is obtained as one $(D(A, B) = 1)$, which supports our existing crisp set theory.

3.11Theorem

Let A, B be two IVIFNs, if $A \leq_1 B$, then (i). $D(A \cup B,\ A) = D(A, B)$, (ii).$D(A \cap B, B = DA, B$.

Proof:

Since $A \leq_1 B$, we have $A \cup B = B$and $A \cap B = A$.

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3.12 Theorem

Let$A = ([a_1, b_1], [c_1, d_1]) \in$ IVIFN.Then (i). $D[0, A] = GIS(A$.

3.13 Theorem

Let$A = ([a_1, b_1], [c_1, d_1]) \in$ IVIFN. Then

$D(A, A^c) = \left|\frac{(a_1 - c_1)}{2}\right| + \left|\frac{(b_1 - d_1)}{2}\right| + K_1(|a_1(1 - a_1) - c_1(1 - c_1)|) + K_2(|b_1(1 - b_1) - d1(1-d1)$.

4. A New Similarity Measure on IVIFNS

4.1 Definition

A map S: IVIFN $\times$ IVIFN$\rightarrow [0,1]$ between two IVIFN,
$A = ([a_1, b_1], [c_1, d_1])$,$B = ([a_2, b_2], [c_2, d_2])$ is defined as $S(A, B) = 1 - D(A, B)$.

4.2 Theorem

S: IVIFN $\times$ IVIFN$\rightarrow [0,1]$ is a distance measure

Proof:

The conditions in Definition 2.6, (S1), (S2), (S3) and (S4), are obvious.

4.3 Theorem

Let$= ([a_1, b_1], [c_1, d_1])$,$B = ([a_2, b_2], [c_2, d_2])$and $C = ([a_3, b_3], [c_3, d_3])$ be an IVIFNs. Then $S(A, C) \geq S(A, B) + S(B, C)$.

Proof:

We know that, $D(A, C) \leq D(A, B) + D(B, C)$, implies that
$1 - D(A, C) \geq 1 - D(A, B) + 1 - D(B, C)$. Hence $S(A, C) \geq S(A, B) + S(B, C)$.

4.4 Theorem

The similarity between two crisp numbers$A = ([0, 0], [1, 1])$ and
$B = ([1, 1], [0, 0])$ is obtained as one ($S(A, B) = 0$), which supports our existing crisp set theory.

5 . Application of the Proposed Similarity Measure to Pattern Recognition Problem

Assume that there are three IFS $X = \{c_1, c_2, c_3\}$ representing three patterns. The three patterns are written as follows:
$P_1 = \{(0.4,0.4), (0.3,0.3), (0.2,0.2)\}$,
$P_2 = \{(0.3,0.3), (0.3,0.3), (0.3,0.3)\}$,
$P_3 = \{(0.5,0.5), (0.5,0.5), (0.5,0.5)\}$.
Assume that a sample $Q = \{(0.4,0.4), (0.3,0.3), (0.2,0.2)\}$ is to be distinguished.

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Table1:

The similarity measure between the known pattern and the unknown pattern in Example(Patterns not discriminated are in bold type)

Existing measure	$\boldsymbol{S(P_1,Q)}$	$\boldsymbol{S(P_2,Q)}$	$\boldsymbol{S(P_3,Q)}$
$S_C = 1 - \sum_{i=1}^{n} \left\lvert \frac{S_A(x_i) - S_B(x_i)}{2n} \right\rvert$	**1**	**1**	**1**
$S_H = 1 - \sum_{i=1}^{n} \left\lvert \frac{(\mu_A(x_i) - \mu_B(x_i)) - (\vartheta_A(x_i) - \vartheta_B(x_i))}{2n} \right\rvert$	**1**	**1**	**1**
$S_{DC} = 1 - \sqrt[p]{\sum_{i=1}^{n} \left\lvert \frac{\varphi_A(x_i) - \varphi_B(x_i)}{n} \right\rvert^p}$	**1**	**1**	**1**
$S_{HB} = \frac{1}{2}\left(\rho_\mu(A,B) + \rho_\vartheta(A,B)\right)$	1	0.93	0.8
$S_e^p = 1 - \sqrt[p]{\sum_{i=1}^{n} \frac{\left(\varphi_t(x_i) + \varphi_f(x_i)\right)^p}{n}}$	1	0.93	0.8
$C_{IFS} = \frac{1}{n} \frac{\sum_{i=1}^{n} \mu_A(x_i)\mu_B(x_i) + \vartheta_A(x_i)\vartheta_B(x_i)}{\sqrt{\mu_A^2(x_i) + \vartheta_A^2(x_i)}\sqrt{\mu_B^2(x_i) + \vartheta_B^2(x_i)}}$	**1**	**1**	**1**

$P = 1$ for S_{HB}, S_e^p

The similarity degrees of $S(P_1,Q)$, $S(P_2,Q)$ and $S(P_3,Q)$ calculated for all similarity measure are shown in table 1.

The proposed similarity measure S can be calculated by above example as:
$S(P_1,Q) = 1, S(P_2,Q) = 0.76(K_1 + K_2), S(P_3,Q) = 0.08(K_1 + K_2)$.

It is clearly that B is equal to A_1, which indicates that sample B should be distinguished to A_1. However, the similarity degree of $S(P_1,Q)$, $S(P_2,Q)$ and $S(P_3,Q)$ are equal to each other when S_C, S_H, S_{DC}, and C_{IFS} are employed. These four similarity measures will not be enough to discriminate the difference between the three patterns. This means that the proposed similarity measure is more applicable and be useful with majority of the existing measures.

6. Conclusions and Future Scope

In this Research paper a novel distance measure between IVIFNs is introduced and is applied to pattern recognition problem. The novel proposed distance measure has been verified. The distance measure proposed in this paper can be extended to any triangular, trapezoidal, IFNs or any two generalized IFNs. Using the distance measure practical fuzziness problem of pattern recognition and clustering can be solved.

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7. References

7.1 Journal Article

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