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Nano Semi c(s) Generalized Closed Sets In Nano Topological Spaces

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Abstract:

The purpose of this paper is to define and study a new class of sets called Nano semi c(s) generalized closed sets in Nano topological spaces. Basic properties of Nano semi c(s) - generalized closed sets are analyzed. Union and Intersection of any Nsc(s) g-closed sets need not be Nsc(s) g-closed set in Nano topological spaces. Also its relation with already existing well known sets are investigated.
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1. Introduction

M.Lellis Thivagar and Carmel Richard [11] introduced nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. The elements of Nano topological space are called Nano open sets. He has defined nano closed sets, nano interior and nano closure of a set in nano topological space. He has also introduced the weaker form of nano open sets namely nano α - open sets, nano semi open sets and nano pre open sets in nano topological space.

In 1970, Levine [12] introduced the concept of generalized closed sets were defined and investigated. Hatir et al [10] introduced t - sets and α^{*}- sets in topological spaces. A.Pushpalatha and R.Nithyakala introduced a concept of scg - closed, $sc^{*}g$ - closed, sc(s)g - closed sets in topological spaces.

In this paper we have introduced a new class of set called Nsc(s)g - closed sets and obtain some of its properties in nano topological spaces. Throughout this paper (U, $\tau_R(X)$) is nano topological space with respect to X, where $X \subseteq U$, R is an equivalence relation on U, U/R denotes the the family of equivalence classes of U by R.

In this paper we compare and find the relationship of Nsc(s)g – closed set with the following sets.

a) nano closed set
b) nano semi closed set
c) nano α - closed (briefly Nα – closed set)
d) nano regular closed(briefly Nr – closed set)
e) nano generalized closed(briefly Ng – closed set)
f) nano semi generalized closed(briefly Nsg – closed set)
g) nano generalized semi closed set(briefly Ngs – closed set)
h) nano α-generalized closed set (briefly Nαg – closed set)
i) nano generalized- α closed set (briefly Ngα – closed set)
j) nano regular generalized closed set (briefly Nrg – closed set)

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k) nano generalized pre closed set (briefly Ngp – closed set)
l) nano generalized pre regular closed set (briefly Ngpr – closed set)
m) nano weakly generalized closed set (briefly Nwg – closed set)
n) nano generalized* - closed set (briefly Ng*- closed set)
o) nano strongly nano generalized*- closed set (briefly strongly nano g*- closed set)
p) nano b - closed sets (briefly Nb – closed sets)
q) nano generalized b - closed set (briefly Ngb – closed set)
r) nano regular generalized b - closed set (briefly Nrgb – closed set)
s) nano generalized semi pre closed set (briefly Ngsp – closed set)
t) nano semi pre generalized closed set (briefly Nspg – closed set)
u) nano pre generalized closed set (briefly Npg – closed set)
v) nano pre closed set
w) nano generalized*semi - closed set (briefly Ng*s - closed set)
x) nano B closed set (briefly NB – closed sets)
y) nano β closed set(briefly Nβ – closed sets)

with nano semi c(s) generalized closed set(briefly Nsc(s)g – closed set).

2. Preliminaries

In this section we recall some definitions and properties which are useful in this study.

Definition 2.1: A subset of a topological space (X,τ) is called

a) generalized closed [12] if *cl*(A) $\subseteq$ U, whenever A $\subseteq$ U, and U is open in X.
b) semi generalized - closed [3] if *scl*(A) $\subseteq$ U, whenever A $\subseteq$ U, and U is semi open in X.
c) t - set [10] if *int*(A) = *int*(*cl*(A)).
d) c(s) - set [19] if A = G $\cap$ F, where G is g–open and F is t-set.
e) semi c(s) generalized-closed [16] if scl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is c(s)-set in X.
f) semi open [13] if A $\subseteq$ *cl (int* (A)).
g) pre open [14] if A $\subseteq$ *int (cl* (A)).
h) α - open [15] if A $\subseteq$ *int (cl* (*int (*A))).
i) regular open [15] if A = *int (cl (* A)).

Definition 2.2 :

Let U be a non- empty finite set of objects is called the universe and R be an equivalence relation on U named as the indicernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U , R) is said to be the approximation space. Let X $\subseteq$ U.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by L_R (X). That is $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by U_R (X). That is $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not- X with respect to R and it is denoted by B_R (X). That is, $B_R(X) = U_R(X) – L_R$ (X).

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Property 2.3: If (U , R) is an approximation space and X ,Y $\subseteq$U, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$;

(ii) $L_R(\phi) = U_R(\phi)$ *and* $L_R(U) \subseteq U_R(U) = U$;

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;

(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;

(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;

(vi) $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;

(vii) $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$;

(viii) $U_R(X^C) = [L_R(X)]^C$ *and* $L_R(X^C) = [U_R(X)]^C$;

(ix) $U_R U_R(X) = L_R L_R(X) = U_R(X)$;

(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.4: Let U be the universe, R be an equivalence relation on U $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where X $\subseteq$ U. Then by property 2.3, $\tau_R(X)$ satisfies the following axioms:

(i) U and ϕ are in $\tau_R(X)$

(ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$ That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. The space (U, $\tau_R(X)$) is the nano topological space. The elements of $\tau_R(X)$ are called as nano – open sets of (U, $\tau_R(X)$) .

Remark 2.5: If $\tau_R(X)$ is the nano topology on U with respect to X, then the set B = $\{U, L_R(x), B_R(x)\}$is the basis for $\tau_R(X)$.

Definition 2.6 : If (U , $\tau_R(X)$) is a nano topological space with respect to X where X $\subseteq$ U and if A $\subseteq$ U, then the nano interior of A is defined as the union of all nano – open subsets of A and it is denoted by N*int* (A). That is N*int*(A) is the largest nano- open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by Nc*l*(A). That is, N*Cl*(A) is the smallest nano closed set containing A.

Definition 2.7: A subset H of a nano topological space (U , $\tau_R(X)$ is called;

a) nano g - closed set [5] if Nc*l*(H) $\subseteq$ G, whenever H$\subseteq$G and G is nano open set in $(U, \tau_R(X))$.

b) nano sg – closed set [4] if N*scl*(H) $\subseteq$G, whenever H$\subseteq$G and G is nano semi open set in $(U, \tau_R(X))$.

c) nano gs – closed set [4] if N*scl*(H) $\subseteq$G, whenever H$\subseteq$G and G is nano open set in $(U, \tau_R(X))$.

d) nano gα - closed set [26] if Nα*cl*(H) $\subseteq$G, whenever H$\subseteq$G and G is nano α – open set in $(U, \tau_R(X))$.

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e) nano αg – closed set [26] if Nα*cl*(H) $\subseteq$G, whenever H$\subseteq$G and G is nano open set in $(U,\tau_R(X))$.
f) nano rg – closed set [25] if Nc*l*(H) $\subseteq$ G, whenever H$\subseteq$G and G is nano regular open set in $(U,\tau_R(X))$.
g) nano gp – closed set [6] if Np*cl*(H) $\subseteq$G, whenever H$\subseteq$G and G is nano open set in $(U,\tau_R(X))$.
h) nano gpr – closed set [18] if N*pcl*(H) $\subseteq$G, whenever H$\subseteq$G and G is nano regular open set in $(U,\tau_R(X))$.
i) nano wg – closed set [17] if Nc*l(* N*int* (H)) $\subseteq$ G, whenever H$\subseteq$G and G is nano open set in $(U,\tau_R(X))$.
j) nano g*- closed set [20] if Nc*l*(H) $\subseteq$ G, whenever H$\subseteq$G and G is nano g – open set in $(U,\tau_R(X))$.
k) strongly nano g*- closed set [20] if Nc*l(* N*int* (H)) $\subseteq$ G, whenever H$\subseteq$G and G is nano g – open set in $(U,\tau_R(X))$.
l) nano rgb – closed [8] if Nb*cl*(H) $\subseteq$G, whenever H$\subseteq$G and G is nano regular open set in $(U,\tau_R(X))$.
m) nano gb – closed [7] if Nb*cl*(H) $\subseteq$G, whenever H$\subseteq$G and G is nano open set in $(U,\tau_R(X))$.
n) nano b – closed [7] if [N*cl (* N*int (*A))] $\cap$[N*int (* N*cl* (A)] $\subseteq$A.
o) nano gsp – closed [24] if Nsp*cl*(H) $\subseteq$G, whenever H$\subseteq$G and G is nano open set in $(U,\tau_R(X))$.
p) nano spg – closed [24] if Nsp*cl*(H) $\subseteq$G, whenever H$\subseteq$G and G is nano semi open set in $(U,\tau_R(X))$.
q) nano g*s - closed set [23] if Nsc*l*(H) $\subseteq$ G, whenever H$\subseteq$G and G is nano g – open set in $(U,\tau_R(X))$.
r) nano g*p - closed set [22] if Npc*l*(H) $\subseteq$ G, whenever H$\subseteq$G and G is nano g – open set in $(U,\tau_R(X))$.
s) Nano pg closed set [6] if Npc*l*(H) $\subseteq$ G, whenever H$\subseteq$G and G is nano pre – open set in $(U,\tau_R(X))$.
t) nano β – closed [9] if *Nint (* N*cl* (*Nint (*A))) $\subseteq$ A.
u) nano B closed[27] if A = G $\cap$F, G is nano open set and F is nano t – set.

The complement of the above mentioned closed sets are their respective open sets in (U , $\tau_R(X)$).

3. Nano semi c(s) generalized - closed sets in nano topological spaces

In this section we define and study the forms of Nano semi c(s) generalized closed sets in (U , $\tau_R(X)$).

Definition 3.1: A subset A of $(U,\tau_R(X))$ is called an Nano c(s) - set if A = G$\cap$ F, where G is Ng–open and F is N t-set in $(U,\tau_R(X))$.

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Definition 3.2: A subset A of $(U,\tau_R(X))$ is called an Nano semi c(s) generalized closed set if Nscl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is Nc(s)- set in $(U,\tau_R(X))$. The complement of nano semi c(s) generalized closed set is nano semi c(s) generalized open set in $(U,\tau_R(X))$.

Example – 3.3: Let U = { a, b, c, d } with U/R = { { a }, { c }, { b, d } } and X= { a, b } then $\tau_R(X)=\{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$ and $\tau_R{}^c(X)=\{\phi, U, \{c\}, \{b, c, d\}, \{a, c\}\}$. A subset H of a nano topological space $(U,\tau_R(X))$ is called a

a) nano semi closed if
H = {{ ϕ, U, { a }, { c }, { a, c }, { b, d }, { b, c, d } } in $(U,\tau_R(X))$.

b) nano g – closed if
H = {{ ϕ, U, { c }, { a, c }, { b, c }, { c, d }, { a, b, c }, { a, c, d }, { b, c, d } } in $(U,\tau_R(X))$

c) nano sg – closed if
H = {{ ϕ, U, { a }, { b }, { c }, { d }, { a, c }, { b, c }, { b, d }, { c, d }, { a, b, c }, { a, c, d }, { b, c, d } } in $(U,\tau_R(X))$.

d) nano gs – closed if
H ={{ ϕ, U, { a }, { b }, { c }, { d }, { a, c }, { b, c }, { b, d }, { c, d }, { a, b, c }, { a, c, d }, { b, c, d }} in $(U,\tau_R(X))$.

e) nano gα – closed if
H = {{ ϕ, U, { c }, { a, c }, { b, c }, { c, d }, { a, b, c }, {a, c, d },{ b, c, d } } in $(U,\tau_R(X))$.

f) nano αg – closed if
H = {{ ϕ, U, { c }, { a, c }, { b, c }, { c, d }, { a, b, c }, { a, c, d }, { b, c, d } } in $(U,\tau_R(X))$.

g) nano rg – closed if
H = {{ ϕ, U, { c }, { a, b }, { a, c }, { a, d }, { b, c }, { c, d }, { a, b, c }, { a, b, d }, { a, c, d }, { b, c, d } } in $(U,\tau_R(X))$.

h) nano wg – closed if
H = {{ ϕ, U, { b },{ c },{ d },{ a, c },{ b, c },{ c, d },{ a, b, c },{a, c, d },{b, c, d }} in $(U,\tau_R(X))$.

i) nano gp – closed if
H = {{ ϕ, U, { b },{ c },{ d },{ a, c },{ b, c },{ c, d },{a, b, c },{a, c, d },{ b, c, d }} in $(U,\tau_R(X))$.

j) nano gpr – closed if
H = {{ ϕ,U, { b },{ c },{ d },{ a, b },{ a, c },{ a, d },{ b, c },{ c, d },{ a, b, c }, { a, b, d },{ a, c, d },{ b, c, d }} in $(U,\tau_R(X))$.

k) nano g^*- closed if
H = {{ ϕ, U,{ c },{ a, c },{ b, c },{ c, d },{ a, b, c },{ a, c, d } ,{ b, c, d }} in $(U,\tau_R(X))$

l) nano strongly g^* - closed if
H = {{ ϕ, U, { b },{ c },{ d },{ a, c },{ b, c },{ c, d },{ a, b, c },{ a, c, d },{ b, c, d }} in $(U,\tau_R(X))$.

m) nano g^*p - closed if

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H = {{ ϕ,U, { b },{ c },{ d },{ a, c },{ b, c },{ c, d },{ a, b, d },{ a, c, d },{ b, c, d }} in $(U,\tau_R(X))$.

n) nano g^*s - closed if
H = {{ ϕ, U, { a },{ c },{ a, c },{ b, c },{ b, d },{ c, d },{ a, b, d },{ a, c, d },{ b, c, d }} in $(U,\tau_R(X))$.

o) nano gsp – closed if
H ={{ ϕ, U, { a }, { b }, { c }, { d }, {a, b}, { a, c }, {a, d}, { b, c }, { b, d }, { c, d }, { a, b, c },{ a, c, d }, { b, c, d }} in $(U,\tau_R(X))$.

p) nano spg – closed if
H ={{ ϕ, U, { a }, { b }, { c }, { d }, {a, b}, { a, c }, {a, d}, { b, c }, { b, d }, { c, d }, { a, b, c },{ a, c, d }, { b, c, d }} in $(U,\tau_R(X))$.

q) nano b – closed if
H ={{ ϕ, U, { a }, { b }, { c }, { d }, { a, c }, { b, c }, { b, d }, { c, d }, { a, b, c }, { a, c, d }, { b, c, d }} in $(U,\tau_R(X))$.

r) nano gb – closed if
H ={{ ϕ, U, { a }, { b }, { c }, { d }, { a, c }, { b, c }, { b, d }, { c, d }, { a, b, c }, { a, c, d }, { b, c, d }} in $(U,\tau_R(X))$.

s) nano rgb – closed if
H ={{ ϕ, U, { a }, { b }, { c }, { d }, {a, b}, { a, c },{a, d}, { b, c }, { b, d }, { c, d }, { a, b, c }, {a, b, d }, { a, c, d }, { b, c, d }} in $(U,\tau_R(X))$.

t) nano pre closed if
H ={{ ϕ, U, { b }, { c }, { d }, {a, c}, { b, c }, { c, d }, { a, b, c },{ a, c, d }, { b, c, d }} in $(U,\tau_R(X))$.

u) nano α – closed if
H = {{ ϕ, U, { c },{ a, c },{ b, c, d } } in $(U,\tau_R(X))$.

v) nano r – closed if
H = {{ ϕ, U, { a, c },{ b, c, d } } in $(U,\tau_R(X))$.

w) nano B – closed if
H = {{ ϕ, U, { a },{ b, d } } in $(U,\tau_R(X))$.

x) nano β – closed if
H ={{ ϕ, U, { a }, { b }, { c }, { d }, {a, b}, { a, c }, {a, d}, { b, c }, { b, d }, { c, d }, { a, b, c },{ a, c, d }, { b, c, d }} in $(U,\tau_R(X))$.

y) nano sc(s)g - closed if
H = {{ ϕ, U, { a },{ c },{ a, c },{ b, c },{ b, d },{ c, d },{ a, b, c },{ a, c, d },{ b, c, d }} in $(U,\tau_R(X))$.

Theorem 3.4: Every nano closed set is a nano semi c(s) generalized closed set but not conversely.

Proof: Assume that A is a nano closed set in $(U,\tau_R(X))$. Let G be a nano c(s) set such that A $\subseteq$ G. Since A is nano closed, N*cl*(A) = A. Therefore N*cl*(A) $\subseteq$ G .But N*scl*(A) $\subseteq$ N*cl*(A) $\subseteq$ G. Therefore N*scl*(A) $\subseteq$ G where G is Nc(s) - set. Hence A is a nano semi c(s) generalized closed set in $(U,\tau_R(X))$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.5: Let U = { a, b, c, d } with U/R = {{ a },{ c }, { b, d }} and X= { a, b } then $\tau_R(X) = \{\phi, U, \{a\},\{a, b, d\},\{ b, d \}\}$. In Example 3.2 the sets { a }, { b, c }, { b, d }, { c, d }, { a, b, d },and { a, c, d } are Nsc(s)g - closed set but not nano closed set in $(U, \tau_R(X))$. Let A = { a } be a Nsc(s)g-closed set in $(U, \tau_R(X))$, but A is not a nano closed set because { a } ∈ $\tau_R(X)$.

Theorem 3.6: Every nano semi closed set is Nsc(s)g – closed set but not conversely.

Proof: Assume that A is nano semi closed set in $(U, \tau_R(X))$. Let A ⊆ V here V is nano c(s) set. Since A is nano semi closed, N*scl* (A) = A we have N*scl*(A) ⊆ V. Therefore A is Nsc(s)g-closed set in $(U, \tau_R(X))$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.7: Let U = { a, b, c, d } with U/R = {{ a },{ c }, { b, d }} and X= { a, b } then $\tau_R(X) = \{\phi, U, \{a\},\{a, b, d\},\{ b, d \}\}$. In example 3.2, the set { b, c }, { c, d }, { a, b, c } and { a, c, d } are Nsc(s)g –closed set but not nano semi closed set. Let A = { b, c } be a Nsc(s)g-closed set in $(U, \tau_R(X))$.But N*int*(*Ncl*(*A*)) = { b, d } ⊄ { b, c } = A. Therefore N*int*(*Ncl*(*A*)) ⊄ A.Therefore A is not a nano semi closed set in $(U, \tau_R(X))$.

Theorem 3.8: Every nano regular closed set is a nano semi c(s) generalized closed set but not conversely.

Proof: Assume that A is a nano regular closed set in $(U, \tau_R(X))$. Let G be a nano c(s) set such that A ⊆ G. Since A is nano regular closed, Nr*cl*(A) = A. Therefore Nr*cl*(A) ⊆G. But N*scl*(A) ⊆ Nr*cl*(A)⊆G. Therefore N*scl*(A) ⊆ G where G is Nc(s) - set. Hence A is a nano semi c(s) generalized closed set in $(U, \tau_R(X))$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.9: Let U = { a, b, c, d } with U/R = {{ a },{ c }, { b, d }} and X= { a, b } then $\tau_R(X) = \{\phi, U, \{a\},\{a, b, d\},\{ b, d \}\}$. In Example 3.2 the sets { a }, { c }, { b, c }, { b, d }, { c, d }, { a, b, c },and { a, c, d } are Nsc(s)g - closed set but not nano regular closed set in $(U, \tau_R(X))$.Let A = { b, d } be a Nsc(s)g-closed set in $(U, \tau_R(X))$, but not a nano regular closed set. N*cl*(N*int*(A)) = { b, c, d } ≠ { b, d } = A. Therefore N*cl*(N*int*(A)) ≠ A. Hence A is not a nano regular closed set in $(U, \tau_R(X))$.

Theorem 3.10: Every nano α - closed set is a nano semi c(s) generalized closed set but not conversely.

Proof: Assume that A is a nano α - closed set in $(U, \tau_R(X))$. Let G be a nano c(s) set such that A ⊆ G. Since A is nano α - closed, Nα*cl*(A) = A. Therefore Nα*cl*(A) ⊆ G. But N*scl*(A) ⊆ Nα*cl*(A)⊆G. Therefore N*scl*(A) ⊆ G where G is Nc(s)- set. Hence A is a nano semi c(s) generalized closed set in $(U, \tau_R(X))$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.11: Let U = { a, b, c, d } with U/R = {{ a },{ c }, { b, d }} and X= { a, b } then $\tau_R(X) = \{\phi, U, \{a\},\{a, b, d\},\{ b, d \}\}$. In Example 3.2 the sets { a }, { b, c }, { b, d }, { c, d }, { a, b, c },and { a, c, d } are Nsc(s)g - closed set but not nano α - closed set in $(U, \tau_R(X))$.

Let A = { b, d } be a Nsc(s)g-closed set in $(U, \tau_R(X))$, but not a nano α - closed set.

But *Ncl*(N*int*(*Ncl*(*A*))) = { b, c, d } ⊄ { b, d } = A. Therefore *Ncl*(N*int*(*Ncl*(*A*))) ⊄ A.

Hence A is not a nano α - closed set in $(U, \tau_R(X))$.

Theorem 3.12: Every nano g- closed set is Nsc(s)g –closed set but not conversely.

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Proof: Assume that A is Ng – closed set in $(U, \tau_R(X))$. Let A $\subseteq$ V where V in nano c(s) set. Since A is nano g- closed, N*cl*(A) $\subseteq$G whenever A$\subseteq$ G and G is nano open. Since N*scl* (A) $\subseteq$ N*cl*(A), we have N*scl* (A) $\subseteq$ V, whenever A $\subseteq$ G and V is nano c(s) set in $(U, \tau_R(X))$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.13: Let U = { a, b, c, d } with U/R = {{ a },{ c }, { b, d }} and X= { a, b } then $\tau_R(X) = \{\phi, U, \{a\},\{a, b, d\},\{ b, d \}\}$. In example 3.2, the sets { a }, and { b, d } are Nsc(s)g – closed set but not Nsg- closed in $(U, \tau_R(X))$.Let A = { a } be a Nsc(s)g-closed set and also A is a open set in $(U, \tau_R(X))$. Ncl(A) = { a, c } $\not\subseteq$ { a } = A. Ncl(A) $\not\subseteq$ A. So A is not a Ng-closed set in $(U, \tau_R(X))$.

Theorem 3.14: Every Nsc(s)g – closed set is Ngs –closed set but not conversely.

Proof: Assume that A is Nsc(s)g –closed set in $(U, \tau_R(X))$. So N*scl*(A) $\subseteq$ V whenever A $\subseteq$ V, where V is Nc(s) set. Then V can be written as V= $G \cap F$, where G is nano g- open and F is nano t – set. Since A is Nsc(s)g – closed set and every nano open set is nano g – open, therefore N*scl*(A) $\subseteq$ G where G is nano open. Hence A is nano gs – closed set in $(U, \tau_R(X))$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.15: Let U = { a, b, c, d } with U/R = {{ a },{ c }, { b, d }} and X= { a, b } then $\tau_R(X) = \{\phi, U, \{a\},\{a, b, d\},\{ b, d \}\}$. In Example 3.2, the sets { b } and { d } are nano generalized semi closed set but not a nano semi c(s) generalized closed set in $(U, \tau_R(X))$. Let A = { b } be a Ngs-closed set and also A be a Nc(s)- set in $(U, \tau_R(X))$. Now $Nscl(A) = A \cup N\text{int}(Ncl(A)) = \{b\} \cup \{b, d\} = \{b, d\} \not\subseteq \{b\} = A.$ (ie) $Nscl(A) \not\subseteq A.$ So A is not a Nsc(s)g-closed set in $(U, \tau_R(X))$.

Theorem 3.16: Every Nsc(s)g – closed set is Nsg – closed set but not conversely.

Proof: Assume that A is Nsc(s)g –closed set in $(U, \tau_R(X))$. So N*scl*(A) $\subseteq$ V whenever A $\subseteq$ V, where V is Nc(s) set. But every nano semi open set is Nc(s) - set, therefore N*scl*(A) $\subseteq$ V where G is nano semi open. Hence A is nano sg – closed set in $(U, \tau_R(X))$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.17: Let U = { a, b, c, d } with U/R = {{ a },{ c }, { b, d }} and X= { a, b } then $\tau_R(X) = \{\phi, U, \{a\},\{a, b, d\},\{ b, d \}\}$. In Example 3.2, the sets { b } and { d } are nano semi generalized closed set but not a nano semi c(s) generalized closed set in $(U, \tau_R(X))$.

Let A = { b } be a Nsg - closed set and also A be a Nc(s) - set in $(U, \tau_R(X))$. Now $Nscl(A) = A \cup N\text{int}(Ncl(A)) = \{b\} \cup \{b, d\} = \{b, d\} \not\subseteq \{b\} = A.$ (ie) $Nscl(A) \not\subseteq A.$ So A is not a Nsc(s)g-closed set in $(U, \tau_R(X))$.

Theorem 3.18: Every Ngα –closed set is Nsc(s) g –closed set but not conversely.

Proof: Assume that A is Ngα – closed set in $(U, \tau_R(X))$. Let V be a Nc(s) set such that A $\subseteq$ V. Since A is Ngα – closed set, N*αcl*(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano α – open. Since every nano α – open set is Nc(s) set and N*scl*(A) $\subseteq$ N*αcl* (A), we have N*scl*(A) $\subseteq$ G whenever A $\subseteq$ G and G is Nc(s) - set. Therefore A is Nsc(s)g – closed set in $(U, \tau_R(X))$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.19: Let U = { a, b, c, d } with U/R = {{ a },{ c }, { b, d }} and X= { a, b } then $\tau_R(X) = \{\phi, U, \{a\},\{a, b, d\},\{ b, d \}\}$. In example 3.2, the sets {a} and {b,d} are Nano semi c(s) generalized closed set but not Ngα – closed set in $(U, \tau_R(X))$. Let A = { a } be a Nano semi c(s)

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generalized -closed set and also A is N α - open set in $(U,\tau_R(X))$. $N\alpha cl(A)= A\cup Ncl(N\text{int}(Ncl(A)))=\{a\}\cup\{a,c\}=\{a,c\}\not\subseteq\{a\}=A.$ $(ie)\ N\alpha cl(A)\not\subseteq A.$
So A is not a Ngα – closed set in $(U,\tau_R(X))$.

Theorem 3.20: Every Nαg –closed set is Nsc(s) g –closed set but not conversely.

Proof: Assume that A is Nαg – closed set in $(U,\tau_R(X))$. Let V be a Nc(s) set such that A $\subseteq$ V. Since A is Nαg – closed set, N*αcl*(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano open. Since every nano open set is nano Nc(s) set and N*scl*(A) $\subseteq$ N*αcl* (A), we have N*scl*(A) $\subseteq$ G whenever A $\subseteq$ G and G is Nc(s) set. Therefore A is Nsc(s) – closed set in $(U,\tau_R(X))$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.21: Let U = { a, b, c, d } with U/R = {{ a },{ c }, { b, d }} and X= { a, b } then $\tau_R(X)=\{\phi, U, \{a\},\{a, b, d\},\{b, d\}\}$. In example 3.2, the sets {a} and {b,d} are Nsc(s) closed set but not Nαg – closed set in $(U,\tau_R(X))$. Let A = { a } be a Nsc(s)g-closed set and also A is N open set in $(U,\tau_R(X))$. $N\alpha cl(A)= A\cup Ncl(N\text{int}(Ncl(A)))=\{a\}\cup\{a,c\}=\{a,c\}\not\subseteq\{a\}=A.$ $(ie)\ N\alpha cl(A)\not\subseteq A.$ So A is not a Ngα – closed set in $(U,\tau_R(X))$.

Theorem 3.22: Every Ng* -closed set is Nsc(s)g – closed set but not conversely.

Proof: Let A be Ng* - closed set in $(U,\tau_R(X))$. Let V be a Nc(s) – set such that A $\subseteq$ V. Since A is Ng* - closed set, N*cl* (A) $\subseteq$ G whenever A $\subseteq$ G and G is nano g –open . Since N*scl*(A) $\subseteq$ N*cl*(A) and every nano g – open set is Nc(s) set, we have N*scl* (A) $\subseteq$ G whenever A $\subseteq$ G and G is Nc(s) – set. Therefore A is Nsc(s)g – closed set in $(U,\tau_R(X))$..

The converse of the above theorem need not be true as seen from the following example.

Example 3.23: Let U = { a, b, c, d } with U/R = {{ a },{ c }, { b, d }} and X= { a, b } then $\tau_R(X)=\{\phi, U, \{a\},\{a, b, d\},\{b, d\}\}$. In example 3.2, {a}, {b,d} are Nsc(s)g – closed set but not Ng*-closed set in $(U,\tau_R(X))$. Let A = { b, d } be a Nsc(s)g-closed set and A is also a Ng- open set in $(U,\tau_R(X))$. Ncl(A) = { b, c, d } $\not\subseteq$ { b, d } = A. Ncl(A) $\not\subseteq$ A. So A is not a Ng*-closed set in $(U,\tau_R(X))$.

Theorem 3.24: Every Nsc(s)g – closed set is Nb – closed set but not conversely.

Proof: Assume that A is Nsc(s)g –closed set in $(U,\tau_R(X))$. So N*scl*(A) $\subseteq$ V whenever A $\subseteq$ V, where V is Nc(s) set. That is N*scl*(A) $\subseteq$ V we have N*int*(*Ncl*(*A*)) $\subseteq$ V. Therefore N*int*(*Ncl*(*A*)) $\cap$ *Ncl*(N*int*(A) $\subseteq$ V. Therefore V is Nb – closed set and A $\subseteq$ V. Hence A is Nb – closed set in $(U,\tau_R(X))$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.25: Let U = { a, b, c, d } with U/R = {{ a },{ c }, { b, d }} and X= { a, b } then $\tau_R(X)=\{\phi, U, \{a\},\{a, b, d\},\{b, d\}\}$. In Example 3.2, the sets { b }, { d }, { a, b } and { a, d} are nano b - closed set but not a nano semi c(s) generalized closed set in $(U,\tau_R(X))$.

Let A = { b } be a Nb - closed set and also A be a Nc(s)- set in $(U,\tau_R(X))$.

Now $Nscl(A)= A\cup N\text{int}(Ncl(A))=\{b\}\cup\{b,d\}=\{b,d\}\not\subseteq\{b\}=A.$ $(ie)\ Nscl(A)\not\subseteq A.$

So A is not a Nsc(s)g-closed set in $(U,\tau_R(X))$.

Theorem 3.26: Every Nsc(s)g – closed set is Ngb – closed set but not conversely.

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Proof: Assume that A is Nsc(s)g –closed set in $(U, \tau_R(X))$. So N*scl*(A) $\subseteq$ V whenever A $\subseteq$ V, where V is Nc(s) set. Since N*bcl*(A) $\subseteq$ N*scl*(A) $\subseteq$ V and every nano open set is Nc(s) - set we have N*bcl*(A) $\subseteq$ V where V is nano open. Therefore A is Ngb – closed set in $(U, \tau_R(X))$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.27: Let U = { a, b, c, d } with U/R = {{ a },{ c }, { b, d }} and X= { a, b } then $\tau_R(X) = \{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$. In Example 3.2, the sets { b }, { d }, { a, b } and { a, d} are nano gb - closed set but not a nano semi c(s) generalized closed set in $(U, \tau_R(X))$.

Let A = { b } be a Ngb - closed set and also A be a Nc(s)- set in $(U, \tau_R(X))$.

Now $Nscl(A) = A \cup N\text{int}(Ncl(A)) = \{b\} \cup \{b,d\} = \{b,d\} \not\subseteq \{b\} = A.$ (ie) $Nscl(A) \not\subseteq A.$

So A is not a Nsc(s)g-closed set in $(U, \tau_R(X))$.

Theorem 3.28: Every Nsc(s)g – closed set is Nrgb – closed set but not conversely.

Proof: Assume that A is Nsc(s)g –closed set in $(U, \tau_R(X))$. So N*scl*(A) $\subseteq$ V whenever A $\subseteq$ V, where V is Nc(s) set. Since N*bcl*(A) $\subseteq$ N*scl*(A) $\subseteq$ V and every nano regular open set is Nc(s) - set we have N*bcl*(A) $\subseteq$ V where V is nano regular open. Therefore A is Nrgb – closed set in $(U, \tau_R(X))$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.29: Let U = { a, b, c, d } with U/R = {{ a },{ c }, { b, d }} and X= { a, b } then $\tau_R(X) = \{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$. In Example 3.2, the sets { b }, { d }, { a, b } { a,d }and { a, b, d} are nano rgb - closed set but not a nano semi c(s) generalized closed set in $(U, \tau_R(X))$.

Let A = { a, b } be a Nrgb - closed set and also A be a Nc(s)- set in $(U, \tau_R(X))$.

Now $Nscl(A) = A \cup N\text{int}(Ncl(A)) = \{a,b\} \cup U = U \not\subseteq \{a,b\} = A.$ (ie) $Nscl(A) \not\subseteq A.$

So A is not a Nsc(s)g-closed set in $(U, \tau_R(X))$.

Theorem 3.30: Every NB - closed set is Nsc(s)g – closed set but not conversely.

Proof: Assume that A is NB –closed set in $(U, \tau_R(X))$. So A = $G \cap F$, where G is nano open and F is nano t – set. Since every nano open set is nano g – open set we have A = $G \cap F$, where G is nano g - open and F is nano t – set, therefore A is Nc(s) - set and also N*scl*(A) $\subseteq$ A . Hence A is Nsc(s)g-closed set in $(U, \tau_R(X))$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.31: Let U = { a, b, c, d } with U/R = {{ a },{ c }, { b, d }} and X= { a, b } then $\tau_R(X) = \{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$. In example 3.2, the sets { c }, { a, c}, { b, c }, { c, d }, { a, b, c } { a, c, d } and { b, c, d } are Nsc(s)g –closed set but not nano B closed set in $(U, \tau_R(X))$. Let A = { c } be a Nsc(s)g-closed set in $(U, \tau_R(X))$. This A is a Nt – set but not nano open set, So A is not NB – closed set in $(U, \tau_R(X))$.

Theorem 3.32: Every Nsc(s)g – closed set is Nβ – closed set but not conversely.

Proof: Assume that A is Nsc(s)g –closed set in $(U, \tau_R(X))$. So N*scl*(A) $\subseteq$ V whenever A $\subseteq$ V, where V is Nc(s) set. That is N*int*(*Ncl(A)*) $\subseteq$ V. But N*int*(*Ncl(*N*int*(A))) $\subseteq$ N*int*(*Ncl(A)*) $\subseteq$ V, then we have N*int*(*Ncl(*N*int*(A))) $\subseteq$ V. Therefore A $\subseteq$V and A is Nβ – closed set in $(U, \tau_R(X))$.

The converse of the above theorem need not be true as seen from the following example.

Example 3.33: Let U = { a, b, c, d } with U/R = {{ a },{ c }, { b, d }} and X= { a, b } then $\tau_R(X) = \{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$. In example 3.2, the sets { b }, { d }, { a, b } and { a, d } are

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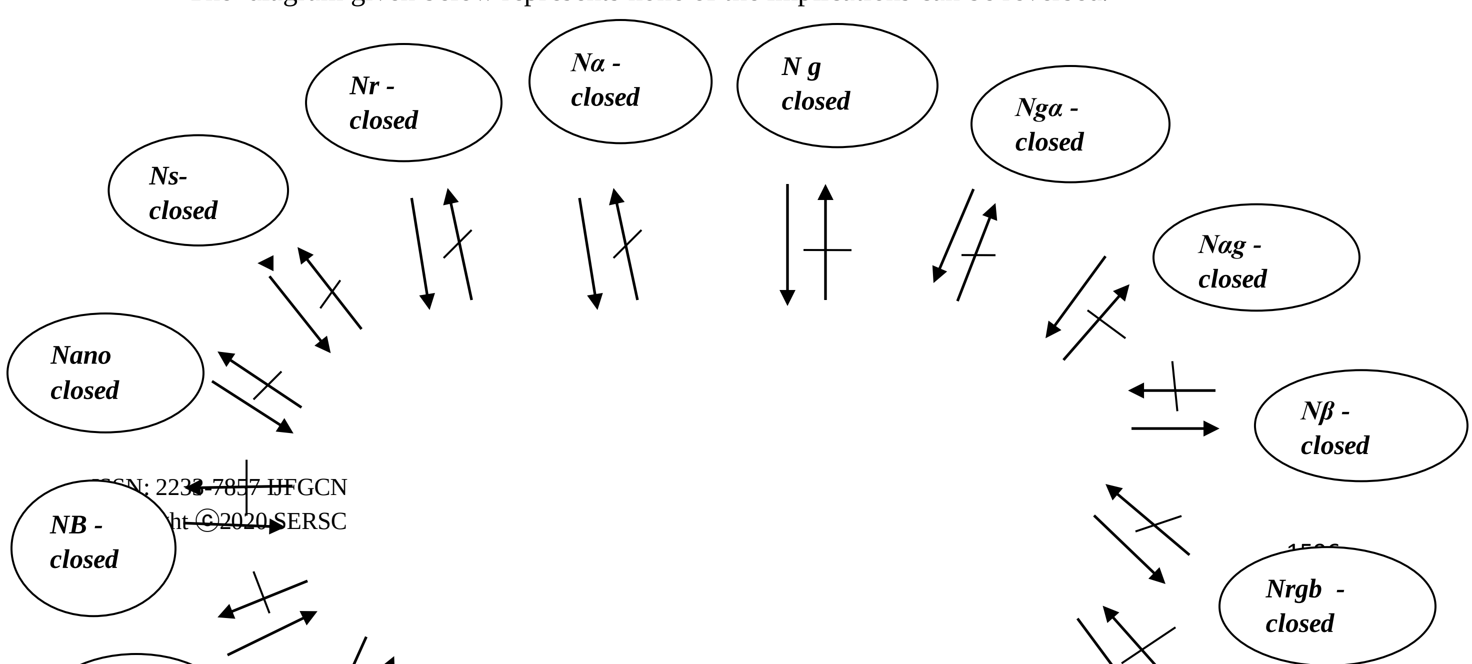
N β – closed set but not Nsc(s)g – closed set in $(U, \tau_R(X))$.Let A = { a, d } be a Nrgb - closed set and also A be a Nc(s)- set in $(U, \tau_R(X))$. Now $Nscl(A) = A \cup N\text{int}(Ncl(A)) = \{a,d\} \cup U = U \not\subseteq \{a,d\} = A.$ (ie) $Nscl(A) \not\subseteq A.$ So A is not a Nsc(s)g-closed set in $(U, \tau_R(X))$.

Theorem 3.34: Every Nsc(s)g – closed set is Ngsp – closed set but not conversely.
Proof: Assume that A is Nsc(s)g –closed set in $(U, \tau_R(X))$. So N*scl*(A) $\subseteq$ V whenever A $\subseteq$ V, where V is Nc(s) set. Since Nsp*cl*(A) $\subseteq$ N*scl*(A) $\subseteq$ V and every nano open set is Nc(s) - set we have Nsp*cl*(A) $\subseteq$ V where V is nano open. Therefore A is Ngsp – closed set in $(U, \tau_R(X))$.
The converse of the above theorem need not be true as seen from the following example.
Example 3.35: Let U = { a, b, c, d } with U/R = {{ a },{ c }, { b, d }} and X= { a, b } then $\tau_R(X) = \{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$. In Example 3.2, the sets { b }, { d }, { a, b } and { a, d} are nano gsp - closed set but not a nano semi c(s) generalized closed set in $(U, \tau_R(X))$.
Let A = { a, b } be a Ngsp - closed set and also A be a Nc(s)- set in $(U, \tau_R(X))$.
Now $Nscl(A) = A \cup N\text{int}(Ncl(A)) = \{a,b\} \cup U = U \not\subseteq \{a,b\} = A.$ (ie) $Nscl(A) \not\subseteq A.$
So A is not a Nsc(s)g-closed set in $(U, \tau_R(X))$.

Theorem 3.36: Every Nsc(s)g – closed set is Nspg – closed set but not conversely.
Proof: Assume that A is Nsc(s)g –closed set in $(U, \tau_R(X))$. So N*scl*(A) $\subseteq$ V whenever A $\subseteq$ V, where V is Nc(s) set. Since Nsp*cl*(A) $\subseteq$ N*scl*(A) $\subseteq$ V and every nano semi open set is Nc(s) - set we have Nsp*cl*(A) $\subseteq$ V where V is nano semi open. Therefore A is Nspg – closed set in $(U, \tau_R(X))$.
The converse of the above theorem need not be true as seen from the following example.
Example 3.37: Let U = { a, b, c, d } with U/R = {{ a },{ c }, { b, d }} and X= { a, b } then $\tau_R(X) = \{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$. In Example 3.2, the sets { b }, { d }, { a, b } and { a, d} are nano spg - closed set but not a nano semi c(s) generalized closed set in $(U, \tau_R(X))$.
Let A = { a, d } be a Ngsp - closed set and also A be a Nc(s)- set in $(U, \tau_R(X))$.
Now $Nscl(A) = A \cup N\text{int}(Ncl(A)) = \{a,d\} \cup U = U \not\subseteq \{a,d\} = A.$ (ie) $Nscl(A) \not\subseteq A.$
So A is not a Nsc(s)g-closed set in $(U, \tau_R(X))$.

Remark 3.38:
The diagram given below represents none of the implications can be reversed.



Nano semi c(s) generalized closed set

Remark 3.39: The union of two Nsc(s)g – closed set need not be Nsc(s)g – closed set as seen from the following example.

Example 3.40: In example 3.2, the sets { a } and { b, d } are Nsc(s)g – closed but their union { a, b, d } is not a Nsc(s)g – closed set in $(U, \tau_R(X))$.

Let A = { a, b, d } be a Nc(s)-set in $(U, \tau_R(X))$.

$Nscl(A) = A \cup N\text{int}(Ncl(A)) = \{a, b, d\} \cup U = U \not\subseteq \{a, b, d\} = A.$

(ie) $Nscl(A) \not\subseteq A.$ So A is not a Nsc(s)g-closed set in $(U, \tau_R(X))$.

Remark 3.41: The intersection of two Nsc(s)g – closed set need not be Nsc(s)g – closed set as seen from the following example.

Example 3.42: In example 3.2, the sets { b, c } and { b, d } are Nsc(s)g – closed but their intersection { b } is not a Nsc(s)g – closed set. Let A = { b } be a Nc(s)-set in $(U, \tau_R(X))$. $Nscl(A) = A \cup N\text{int}(Ncl(A)) = \{b\} \cup \{b, d\} = \{b, d\} \not\subseteq \{b\} = A.$ (ie) $Nscl(A) \not\subseteq A.$ So A is not a Nsc(s)g-closed set in $(U, \tau_R(X))$.

Remark 3.43: Ngp – closed sets and Nsc(s)g – closed sets are independent as seen from the following example.

Example 3.44: In example 3.2, The sets { b }, { d } are Ngp – closed sets but not Nsc(s)g – closed sets in $(U, \tau_R(X))$.Let A = { d } be a Ngp closed set and also A be a Nc(s)-set in $(U, \tau_R(X))$.
Now $Nscl(A) = A \cup N\text{int}(Ncl(A)) = \{d\} \cup \{b, d\} = \{b, d\} \not\subseteq \{d\} = A.$

(ie) $Nscl(A) \not\subseteq A.$ So A is not a Nsc(s)g-closed set in $(U, \tau_R(X))$.

In example 3.2, The sets { a }, { b, d } are Nsc(s)g – closed sets but not Ngp –closed sets in $(U, \tau_R(X))$.
Also from example 3.2, Let A = { b, d } be a Nsc(s)g-closed set and also A be a nano open set in $(U, \tau_R(X))$. $Npcl(A) = A \cup Ncl(N\text{int}(A)) = \{b, d\} \cup \{b, c, d\} = \{b, c, d\} \not\subseteq \{b, d\} = A.$

(ie) $Npcl(A) \not\subseteq A.$ So A is not a Ngp closed set in $(U, \tau_R(X))$.

Remark 3.45: Nrg – closed sets and Nsc(s)g – closed sets are independent as seen from the following example. .

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Example 3.46: In example 3.2, the sets { a, b }, { a, d },{ a, b, d } are Nrg – closed sets but not Nsc(s)g – closed sets. sLet A = { a, b } be a Nrg closed set and also A be a Nc(s)-set in $(U,\tau_R(X))$. Now $Nscl(A) = A \cup N\text{int}(Ncl(A)) = \{a,b\} \cup U = U \not\subseteq \{a,b\} = A.$

$(ie)\ Nscl(A) \not\subseteq A.$ So A is not a Nsc(s)g-closed set in $(U,\tau_R(X))$. Also from example 3.2, the sets { a }, { b, d } are Nsc(s)g – closed sets but not Nrg – closed sets. Let A = { a } be a Nsc(s)g-closed set and also A be a nano regular open set in $(U,\tau_R(X))$. Ncl(A) = { a, c } $\not\subseteq$ { a } = A. Ncl(A) $\not\subseteq$ A. So A is not a Nrg-closed set in $(U,\tau_R(X))$.

Remark 3.47: Ngpr – closed sets and Nsc(s)g – closed sets are independent as seen from the following example.

Example 3.48: In example 3.2, the sets { b }, { d },{ a, b }, { a, d }, { a, b, d } are Ngpr – closed sets but not Nsc(s)g –closed sets in $(U,\tau_R(X))$. Let A = { a, d } be a Ngpr closed set and also A be a Nc(s)-set in $(U,\tau_R(X))$. Now $Nscl(A) = A \cup N\text{int}(Ncl(A)) = \{a,d\} \cup U = U \not\subseteq \{a,b\} = A.$

$(ie)\ Nscl(A) \not\subseteq A.$ So A is not a Nsc(s)g-closed set in $(U,\tau_R(X))$. Also from example 3.2, the sets { a }, { b, d } are Nsc(s)g – closed sets but not Ngpr – closed sets in $(U,\tau_R(X))$. Let A = { b, d } be a Nsc(s)g-closed set and also A be a nano regular open set in $(U,\tau_R(X))$. $Npcl(A) = A \cup Ncl(N\text{int}(A)) = \{b,d\} \cup \{b,c,d\} = \{b,c,d\} \not\subseteq \{b,d\} = A.$ $(ie)\ Npcl(A) \not\subseteq A.$ So A is not a Ngpr-closed set in $(U,\tau_R(X))$.

Remark 3.49: Strongly Ng*–closed sets and Nsc(s)g – closed sets are independent as seen from the following example.

Example 3.50: In example 3.2, the sets { b },{ d } are strongly Ng* -closed sets but not Nsc(s)g – closed sets in $(U,\tau_R(X))$.

Let A = { b } be a strongly Ng* closed set and also A be a Nc(s)-set in $(U,\tau_R(X))$.But $Nscl(A) = A \cup N\text{int}(Ncl(A)) = \{b\} \cup \{b,d\} = \{b,d\} \not\subseteq \{b\} = A.$ $(ie)\ Nscl(A) \not\subseteq A.$ So A is not a Nsc(s)g-closed set in $(U,\tau_R(X))$. Also from example 3.2, the sets { a }, { b, d } are Nsc(s)g – closed sets but not strongly Ng* - closed sets in $(U,\tau_R(X))$.Let A = { a } be a Nsc(s)g-closed set and also A be a Ng- open set in $(U,\tau_R(X))$. But N*cl*(N*int*(A)) = { a, c } $\not\subseteq$ { a } = A. (ie) N*cl*(N*int*(A)) $\not\subseteq$ A. So A in not a strongly Ng* - closed sets in $(U,\tau_R(X))$.

Remark 3.51: Npg – closed sets and Nsc(s)g – closed sets are independent as seen from the following example.

Example 3.52: In example 3.2, The sets { b }, { d } are Npg – closed sets but not Nsc(s)g – closed sets in $(U,\tau_R(X))$.Let A = { d } be a Npg closed set and also A be a Nc(s)-set in $(U,\tau_R(X))$. Now $Nscl(A) = A \cup N\text{int}(Ncl(A)) = \{d\} \cup \{b,d\} = \{b,d\} \not\subseteq \{d\} = A.$ $(ie)\ Nscl(A) \not\subseteq A.$ So A is not a Nsc(s)g-closed set in $(U,\tau_R(X))$.Also from example 3.2, The sets { a }, { b, d } are Nsc(s)g – closed sets but not Npg –closed sets in $(U,\tau_R(X))$.Let A = { b, d } be a Nsc(s)g-closed set and also A be a nano pre open set in $(U,\tau_R(X))$. $Npcl(A) = A \cup Ncl(N\text{int}(A)) = \{b,d\} \cup \{b,c,d\} = \{b,c,d\} \not\subseteq \{b,d\} = A.$ $(ie)\ Npcl(A) \not\subseteq A.$ So A is not a Npg closed set in $(U,\tau_R(X))$.

Remark 3.53: Nwg – closed sets and Nsc(s)g – closed sets are independent as seen from the following example.

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Example 3.54: In example 3.2, The sets { b }, { d } are Nwg – closed sets but not Nsc(s)g – closed sets in $(U,\tau_R(X))$. Let A = { d } be a Nwg closed set and also A be a Nc(s)-set in $(U,\tau_R(X))$. Now $Nscl(A)= A\cup N\text{int}(Ncl(A)) = \{d\}\cup\{b,d\} = \{b,d\} \not\subseteq \{d\} = A.$ (ie) $Nscl(A) \not\subseteq A.$ So A is not a Nsc(s)g-closed set in $(U,\tau_R(X))$. Also from example 3.2, The set { b, d } is Nsc(s)g – closed sets but not Nwg –closed set in $(U,\tau_R(X))$. Let A = { b, d } be a Nsc(s)g-closed set and also A be a nano open set in $(U,\tau_R(X))$. $Ncl(N\text{int}(A) = \{b,c,d\} \not\subseteq \{b,d\} = A.$ (ie) $Ncl(N\text{int}\, A) \not\subseteq A.$ So A is not a Nwg closed set in $(U,\tau_R(X))$.

Remark 3.55: Ng*p – closed sets and Nsc(s)g – closed sets are independent as seen from the following example.

Example 3.56: In example 3.2, The sets { b }, { d } and {a, b, d} are Ng*p – closed sets but not Nsc(s)g – closed sets in $(U,\tau_R(X))$.Let A = { d } be a Ng*p closed set and also A be a Nc(s)-set in $(U,\tau_R(X))$. Now $Nscl(A)= A\cup N\text{int}(Ncl(A)) = \{d\}\cup\{b,d\} = \{b,d\} \not\subseteq \{d\} = A.$ (ie) $Nscl(A) \not\subseteq A.$ So A is not a Nsc(s)g-closed set in $(U,\tau_R(X))$.Also from example 3.2, The set { b, d } is Nsc(s)g - closed sets but not Ng*p –closed set in $(U,\tau_R(X))$.Let A = { a } and { a, b, c } be a Nsc(s)g-closed set and also A be a nano g open set in $(U,\tau_R(X))$. $Npcl(A)= A\cup Ncl(N\text{int}(A)) = \{a\}\cup\{a,c\} = \{a,c\} \not\subseteq \{a\} = A.$ (ie) $Npcl(A) \not\subseteq A.$ So A is not a Ng*p closed set in $(U,\tau_R(X))$.

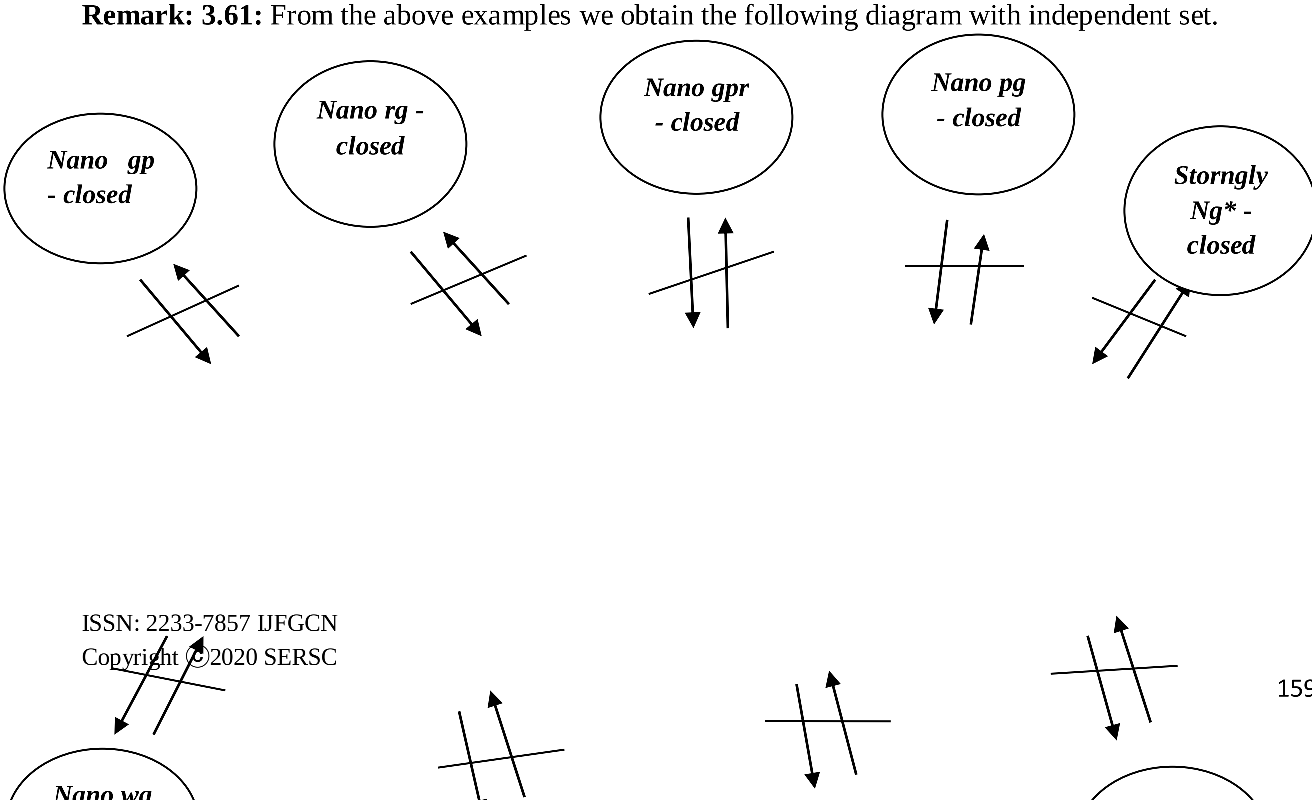
Remark 3.57: Ng*s – closed sets and Nsc(s)g – closed sets are independent as seen from the following example.

Example 3.58: In example 3.2, The set {a, b, d} is Ng*s – closed set but not Nsc(s)g – closed set in $(U,\tau_R(X))$ and the set { a, b, c } is Nsc(s)g – closed set but not Ng*s –closed set in $(U,\tau_R(X))$.

Remark 3.59: Nano pre – closed sets and Nsc(s)g – closed sets are independent as seen from the following example.

Example 3.60: In example 3.2, The set { d }, {a, d} and {a, b, d} is Nano pre – closed sets but not Nsc(s)g – closed sets in $(U,\tau_R(X))$ and the sets {a, c}, {b, c} and { a, b, c } are Nsc(s)g – closed sets but not Nano pre – closed set in $(U,\tau_R(X))$

Remark: 3.61: From the above examples we obtain the following diagram with independent set.



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Nano semi c(s) generalized closed set

References.

[1] Ahmad Al – Omari and Mohd. Salmi Md. Noorani(2009). On Generalized b – closed sets, Bulletin of the Malaysian Mathematical Sciences Society (2) 32 (1) 19-30.
[2] D.Andrijevic(1996). On b – open sets. Mathematicki venik 48 59-64.
[3] Bhattachraya and B . K.Lahiri., Semi generalized closed sets in topology, Indian J.Math, 29(3)375 – 382(1987).
[4] K.Bhuvaneshwari and K.Ezhilarasi, On Nano semi generalized and Nano generalized semi closed sets, IJMCAR, 4(3)(2014), 117-124.
[5] K.Bhuvaneshwari and K.Mythili Gnanapriya, Nano generalized closed sets, International journal of scientific and Research Publications 4(5)(2014), 1-3.
[6] K.Bhuvaneshwari and K.Mythili Gnanapriya, On Nano generalized pre closed sets and Nano pre generalized closed sets in Nano Topological Spaces, International journal of Innovative Research in Science, Engineering and Technology, 3(10)(2014), 16825-16829.
[7] A.Dhanis Arul Mary and I.Arockiarani (2015) On Nano gb – closed sets in Nano Topological Spaces. International Journal ematical Archive (6), 54-58.
[8] A.Dhanis Arul Mary and I.Arockiarani (2015) On Characterizations of Nano rgb – closed sets in Nano Topological Spaces. International Journal of modern Eigineering Research (5), 68-75.
[9] Y.Gnanmbal, On Nano β open sets,Int Jr. of Engineering, 1(2) (2015), 1-6.
[10] E.Hatir, T. Noiri , and S.Yuksel, A decomposition of continuity, ActaMath. Hungar 70(1996), 145 -150.
[11] M.Lellis Thivagar, Carmel Richard, On Nano Forms of Weakly open sets, International Journal of Mathematics and Statistics Invention, 1 (2013), 31-37.
[12] N. Levine, Generalized closed sets in Topology, Rend. Circ. Mat. Palermo,19(1970), 89-96.
[13] N. Levine, Semi – open sets and semi continuity in Topological spaces,Amer.Math. Monthly 70(1963), 36-41.

ISSN: 2233-7857 IJFGCN

[14] A.S.Mashhour, M.E. Abd EI- Monsef and S.N. EI-Deep, On pre – topological spaces, Bull.Math.de la soc. R.S. de Roumanie 28(76)(1984), 39-45.
[15] Miguel caldas, A note on some applications of α – open sets, IJMMS, 2 (2003), 125-130
O. Njastad, On some classes of nearly open sets, Pacific J.Math. 15 (1965), 961-970.
[16] R,Nithyakala, and A.Pushpalatha, Semi c(s)-Generalized Closed Sets in Topological Spaces, Journal of Informatics and Mathematical Sciences, Vol. 9, No. 3, pp. 521–528, 2017,ISSN 0975-5748 (online); 0974-875X (print), Published by RGN Publications.
[17] N.Nagaveni and M.Bhuvaneswari, On Nano Weakly Generalized closed sets, Internationa journal of Pure and Applied Mathematics, Volume 106, 7(2016), 129-137.
[18] C.R.Parvathy and S.Praveena, On Nano generalized pre regular closed sets in Nano Topological Spaces, IOSR Journal of Mathematics (IOSR – JM), (13)(2)(2017), 56-60.
[19] M.Rajamani, Studies on decomposition of generalized continuous maps in topological spaces, Ph.D thesis, Bharathiar university, Coimbatore ,(2001).
[20] M.Rameshpandi, Strongly Nano g^*- closed sets, International Journal of Current Research in Science and Technology, Vol -3, Issue 11 (2017), 1-6.
[21] V. Rajendran, P.Sathishmohan and K.Indirani, On Nano Generalized Star Closed Sets in Nano Topological Spaces, International journal of Applied Research. 1(9)(2015), 04-07.
[22] V. Rajendran, P.Sathishmohan and N.Suresh, On Nano Generalized Star Pre - Closed Sets in Nano Topological Spaces, International journal of Applied Research. 1(9)(2015), 04-07.
[23] V. Rajendran, B.Anand and S.Sharmila banu, On Nano Generalized Star semi-Closed Sets in Nano Topological Spaces, International journal of Applied Research.1(9)(2015), 142-144.
[24] S.B.Shalini and K.Indtrani, On Nano generalized semi pre closed sets and Nano Semi pre generalized closed sets in Nano Topological Spaces, International journal of Innovative Research in Science, Engineering and Technology, 5(3)(2015), 27-31.
[25] P.Sulochana Devi andK. Bhuvaneswari., On Nano Regular Generalized and Nano Generalized Regular closed sets, IJETT, 8(13)(2014), 386-390.
[26] R.Thanga Nachiyar and K.Bhuvaneswari, On Nano generalized A-closed sets and Nano A-generalized closed sets in Nano Topological Spaces, International journal of Engineering Technology(IJETT), Volume 13,(6)(2014), 257-260, ISSN 2231-5381.
[27] J.Tong, On decomposition of continuity in topological spaces, Acta.Math. Hunga. 54(1-2)(1989), 51-55.

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