# Dominating Critical In Hesitancy Fuzzy Graphs

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***ABSTRACT:***

*In this paper, we introduce the concept of dominating critical in hesitancy fuzzy graph. Further investigate the properties of the dominating critical.*

***KEYWORDS:*** *Hesitancy fuzzy graph, dominating critical.*

## 1. INTRODUCTION:

Fuzzy set theory was introduced by Zadeh LA. Most of the real world problems are enormously complex and contain vague data. In order to measure the lack of certainty, extra development to Fuzzy sets was introduced by Torra V and he named it as Hesitant Fuzzy Sets(HFSs). HFSs are encouraged to handle the common trouble that appears in fixing the membership degree of an element from some potential values. This circumstances is rather common in decision making problems too while an professional is asked to assign different degrees of membership to a set of elements {x, y, z,…} in a set A. Frequently problems arise due to uncertain issues and situations hence one is faced with hesitant moments. The investigator had to find ways and means to take the problems and arrive at a solution. Therefore investigators have taken up the learning and application of HFS. HFSs have been extended Xu Z. and Zhu B, from different perspectives such as, both quantitative and qualitative.

In this paper, we introduce the concept of dominating critical in hesitancy fuzzy graph. Further investigate the properties and bounds of the dominating critical.

## 2. PRELIMINARIES :

A Hesitancy fuzzy graph $G(\mathrm{V},\mathrm{E})$, where the vertex set V is a triplet fuzzy functions it is defined by $\mu_1 : V \to [0,1]$, $\nu_1 : V \to [0,1]$ and $\beta_1 : V \to [0,1]$,these functions are called as membership, non-membership and hesitancy of the vertex $v_i \in V$ respectively and $\mu_1(v_i)+\nu_1(v_i)+\beta_1(v_i)=1$, $\beta_1(v_i)=1-[\mu_1(v_i)+\nu_1(v_i)]$. The edge set of $G(\mathrm{V},\mathrm{E})$ is a triplet fuzzy functions it is defined by $\mu_2 : V\times V \to [0,1]$, $\nu_2 : V\times V \to [0,1]$ and $\beta_2 : V\times V \to [0,1]$, such that

$$\mu_2(\mathrm{uv}) \le \mu_1(\mathrm{u}) \wedge \mu_1(\mathrm{v}),$$

$$\nu_2(\mathrm{uv}) \le \nu_1(\mathrm{u}) \vee \nu_1(\mathrm{v})$$

$$\beta_2(\mathrm{uv}) \le \beta_1(\mathrm{u}) \wedge \beta_1(\mathrm{v})$$

and $0 \le \mu_2(\mathrm{uv})+\nu_2(\mathrm{uv})+\beta_2(\mathrm{uv}) \le 1$ for every $\mathrm{uv} \in \mathrm{E}$.

ISSN: 2233-7857 IJFGCN

In a hesitancy fuzzy graph $G(V,E)$ there is a strong edge between every pair of vertices, then $G(V,E)$ is said to be as Complete hesitancy fuzzy graph.

The cardinality of the vertex $v \in V$ in the hesitancy fuzzy graph $G(\mathrm{V},\mathrm{E})$ is defined by $|v| = \left[\dfrac{1+\mu_1(v)+\beta_1(v)-\gamma_1(v)}{3}\right]$.

An edge $uv \in E$ in a hesitancy fuzzy graph $G(\mathrm{V},\mathrm{E})$, is said to be an strong edge such that $\mu_2(\mathrm{uv}) = \mu_1(\mathrm{u}) \wedge \mu_1(\mathrm{v})$, $\gamma_2(\mathrm{uv}) = \gamma_1(\mathrm{u}) \vee \gamma_1(\mathrm{v})$, $\beta_2(\mathrm{uv}) = \beta_1(\mathrm{u}) \wedge \beta_1(\mathrm{v})$. The vertex $u$ *and* $v$ are said to be adjacent vertices and neighbourhood vertices. The neighbourhood set $N(u)$ is set all vertices that are adjacent to the vertex $u$.

The neighbourhood degree and effective neighbourhood degree of the vertex $u \in V$ in the hesitancy fuzzy graph $G(\mathrm{V},\mathrm{E})$ is defined by $d_N(u) = \sum_{v \in N(u)} |v|$ and $d_E(u) = \sum_{v \in N(u)} \left[\dfrac{1+\mu_2(uv)+\beta_2(uv)-\gamma_2(uv)}{3}\right]$.

The order of the vertex $u \in V$ in the hesitancy fuzzy graph $G(\mathrm{V},\mathrm{E})$ is defined by $O(G) = \sum_{v \in V} |v|$ .

A set D of V is said to be dominating set of a hesitancy fuzzy graph $G(A,B)$ if every $v \in V - D$ there exits $u \in D$ such that u dominates v. A dominating set D of a hesitancy fuzzy graph $G(A,B)$ is called minimal dominating set of G, if every node $v \in D$, $D-\{v\}$ is not a dominating set. The dominating number $\gamma_{hf}(G)$ of the hesitancy fuzzy graph $G(A,B)$ is the minimum cardinality taken over all minimal dominating set of G.

## 3. DOMINATING CRITICAL IN HESITANCY FUZZY GRAPHS

**Definition 3.1.** Let $G(V,E)$ be a hesitancy fuzzy graphs. The set of all vertices $V^0 = \{u | \gamma(G-u) = \gamma(G)\}$ is called a null dominating critical in $G(A,B)$.The set of all vertices $V^+ = \{u | \gamma(G-u) > \gamma(G)\}$ is called a positive dominating critical in $G(A,B)$.The set of all vertices $V^- = \{ u \mid \gamma(G-u) < \gamma(G)\}$ is called a negative dominating critical in $G(A,B)$.

**Remarks:** In hesitancy fuzzy graphs $G(V,E)$ the vertex set $V = V^0 \cup V^+ \cup V^-$

**Theorem 3.1:** Let $G(V,E)$ be a complete hesitancy fuzzy graph hesitancy fuzzy graphs. D is a dominating set of $G(V,E)$. Then

(i) $D = V^+$

(ii) $V - D = V^0$

**Proof: (i).** Let $G(V,E)$ be a complete hesitancy fuzzy graph hesitancy fuzzy graphs. D is a dominating set of $G(V,E)$. Therefore $D = \{v\}$, v is the vertex having the minimum cardinality in

$G(V,E)$. This implies we get $<G-v>$is also a complete hesitancy fuzzy graph hesitancy fuzzy graphs and the cardinality of all the vertices $u \in V-\{v\}$ is greater than cardinality of the vertex $v \in D$. Since D is a minimal dominating set of $G(V,E)$. Therefore $\gamma(G-v) > \gamma(G)$. This implies we get $D = V^{+}$.

**(ii).** Let $u \notin D$,the sub graph $<G-u>$ is also a complete hesitancy fuzzy graph. Therefore $D=\{v\}$, v is the vertex having the minimum cardinality in $G(V,E)$. This implies $\gamma(G-u)=\gamma(G)$, every vertex $u \notin D$is an element of $V^{0}$. This implies $V-D=V^{0}$.

**Example 3.1**

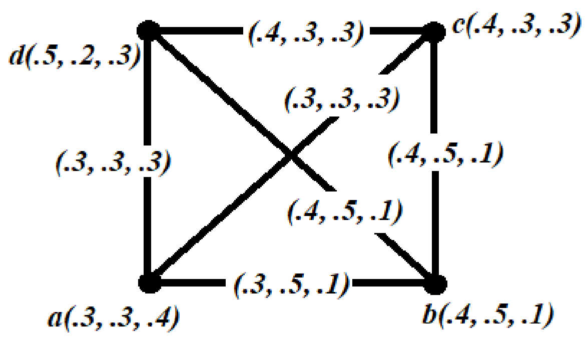


**Figure 3.1:** Complete hesitancy fuzzy graph $G(A,B)$

In the Complete hesitancy fuzzy graph $G(A,B)$, degree of the vertices are $|a|=0.47, |b|=0.33, |c|=0.47 \; and \; |d|=0.53$. The null dominating critical $V^{0}=\{a,c,d\}$ and positive dominating critical $V^{+}=\{b\}$.

**Theorem 3.2:** Let $G(V,E)$ be a hesitancy fuzzy graph and $d_N(v)=\Delta_N(G)$. Then $v \in V^{+}$.

**Proof:** Let $G(V,E)$ be a hesitancy fuzzy graph and $d_N(v)=\Delta_N(G)$. This implies $v \in D$, D is a minimal dominating set of $G(V,E)$. Note that $D-v$is not a dominating set of $G(V,E)$. There are some vertices $u_i \in N(v), i=1,2,...n$ not dominated by the set $D-v$. Therefore $(D-v)\bigcup\{u_i\}, i=1,2...n$is a dominating set of $<G-v>$. This implies $\gamma(G-v) > \gamma(G)$, we get $v \in V^{+}$.

**Theorem 3.3:** Let $G(V,E)$ be a hesitancy fuzzy graph and D is a minimal dominating set of $G(V,E)$. Then $D \subseteq V^{+}$.

**Proof:** Let $G(V,E)$ be a hesitancy fuzzy graph and D is a minimal dominating set of $G(V,E)$. This implies $D-\{v\}$ is not a dominating set of $G(V,E)$. There exist some vertex u in hesitancy sub graph $<G-v>$is not dominated by the set $D-\{v\}$. There is a vertex $w \in V$ is adjacent to u such that

ISSN: 2233-7857 IJFGCN

$(D-\{v\})\cup\{w\}$ is a minimal dominating set of $<G-v>$. Clearly $\gamma(G-v)>\gamma(G)$ this implies every vertex $v\in D$ belongs to $V^{+}$.

**Theorem 3.4:** Let $v\in V$ is an isolated vertex in a hesitancy fuzzy graph $G(V,E)$. Then $v\in V^{-}$.

**Proof:** Let $v\in V$ is an isolated vertex in a hesitancy fuzzy graph $G(V,E)$. Therefore $v\in V$ is a vertex dominating itself such that $v\in D$, D is a minimal dominating set of $G(V,E)$. This implies $D-\{v\}$ is dominating set of a sub graph $<G-\{v\}>$. Clearly we get $\gamma(G-u)<\gamma(G)$ . Therefore the vertex $v\in V^{-}$.

**Example.3.2**

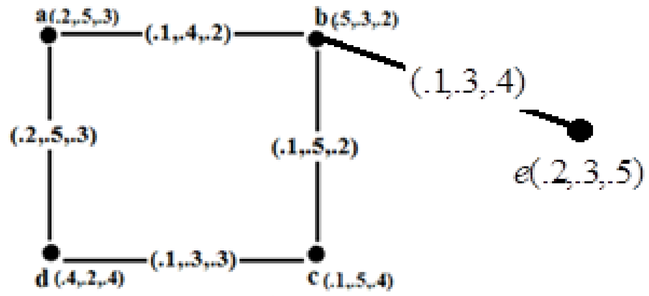


**Figure 3.2:** Hesitancy fuzzy graph $G(A,B)$

In the hesitancy fuzzy graph $G(A,B)$, neighbourhood degree of the vertices are $d_N(a)=0.53, d_N(b)=0.33, d_N(c)=0.47, d_N(d)=0.33, d_N(e)=0$. The minimal dominating set $D=\{a,c,e\}$. The null dominating critical $V^{0}=\{b,d\}$, positive dominating critical $V^{+}=\{a,c\}$ negative dominating critical $V^{+}=\{e\}$.

**Definition 3.2:** Let $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ are two hesitancy fuzzy graphs. The join of $G_1$ and $G_2$ is defined by

$$(\mu_{11}+\mu_{12})(u)=\begin{cases}\mu_{11}(u) & \text{if } u\in V_1\\ \mu_{12}(u) & \text{if } u\in V_2\end{cases}\quad (\eta_{11}+\eta_{12})(u)=\begin{cases}\eta_{11}(u) & \text{if } u\in V_1\\ \eta_{12}(u) & \text{if } u\in V_2\end{cases}$$

$$(\beta_{11}+\beta_{12})(u)=\begin{cases}\beta_{11}(u) & \text{if } u\in V_1\\ \beta_{12}(u) & \text{if } u\in V_2\end{cases}$$

and Edge set E is defined by

$$(\mu_{12}+\mu_{22})(uv)=\begin{cases}\mu_{12}(uv)\ , & \text{if } uv\in E_1\\ \mu_{22}(uv)\ , & \text{if } uv\in E_2\\ \mu_{11}(u)\wedge\mu_{21}(v) & \text{otherwise}\end{cases}\quad (\eta_{12}+\eta_{22})(uv)=\begin{cases}\eta_{12}(uv)\ , & \text{if } uv\in E_1\\ \eta_{22}(uv)\ , & \text{if } uv\in E_2\\ \eta_{11}(u)\vee\upsilon_{21}(v) & \text{otherwise}\end{cases}$$

ISSN: 2233-7857 IJFGCN

$$(\beta_{12}+\beta_{22})(uv)=\begin{cases}\beta_{12}(uv)\ , & if\ \ uv\in E_1\\ \beta_{22}(uv)\ , & if\ \ uv\in E_2\\ \beta_{11}(u)\wedge\beta_{21}(v) & otherwise\end{cases}$$

**Theorem 3.5:** Let $(G_1+G_2)$ is a join of two hesitancy fuzzy graphs $G_1(V_1,E_1)$ *and* $G_2(V_2,E_2)$. The sets $D_1$ *and* $D_2$ are dominating of sets of $G_1(V_1,E_1)$ *and* $G_2(V_2,E_2)$.

i) If $|D_1|<|D_2|$, $D_1\in V^+, D_2\in V^0$

ii) If $|D_2|<|D_1|$, $D_2\in V^+$, $D_1\in V^0$

**Proof:** Let $(G_1+G_2)$ is a join of two hesitancy fuzzy graphs $G_1(V_1,E_1)$ *and* $G_2(V_2,E_2)$. The sets $D_1$ *and* $D_2$ are dominating of sets of $G_1(V_1,E_1)$ *and* $G_2(V_2,E_2)$.

i). Assume $|D_1|<|D_2|$. Every vertex $v\in V_1$ such that there exist a vertex $u\in D_1$ such that u dominates v. since $D_1$ is a minimal dominating set of $G_1(V_1,E_1)$. By the definition of $(G_1+G_2)$, there is a strong edge between vertices in $D_1$ and $V_2$. clearly $D_1$ is the minimal dominating set of $(G_1+G_2)$ since $|D_1|<|D_2|$. In the hesitancy fuzzy sub graph, $<G-u>$, here $u\in D_1$. Therefore there is some vertex $v\in V_1$ is not dominated by $D_1$. This implies $(D_1-u)\cup\{v\}$ is a dominating set of $(G_1+G_2)$. Hence we get $\gamma(G-u)<\gamma(G)$ here $G=(G_1+G_2)$, $D_1\in V^+$. In the hesitancy fuzzy sub graph, $<G-u>$, here $u\in D_2$. Note that every vertex is dominated by $D_1$. This implies $\gamma(G-u)=\gamma(G)$, hence we get $D_2\in V^0$.

ii). Assume $|D_2|<|D_1|$. Every vertex $v\in V_2$ such that there exist a vertex $u\in D_2$ such that u dominates v. since $D_2$ is a minimal dominating set of $G_1(V_1,E_1)$. By the definition of $(G_1+G_2)$, there is a strong edge between vertices in $D_2$ and $V_1$. clearly $D_2$ is the minimal dominating set of $(G_1+G_2)$ since $|D_2|<|D_1|$. In the hesitancy fuzzy sub graph, $<G-u>$, here $u\in D_2$. Therefore there is some vertex $v\in V_2$ is not dominated by $D_2$. This implies $(D_2-u)\cup\{v\}$ is a dominating set of $(G_1+G_2)$. In $G=(G_1+G_2)$ we get $\gamma(G-u)<\gamma(G)$, therefore $D_2\in V^+$. In the hesitancy fuzzy sub graph, $<G-u>$, here $u\in D_1$. Note that every vertex in $<G-u>$ is dominated by $D_2$. This implies $\gamma(G-u)=\gamma(G)$, hence we get $D_1\in V^0$.

**Example 3.3.**

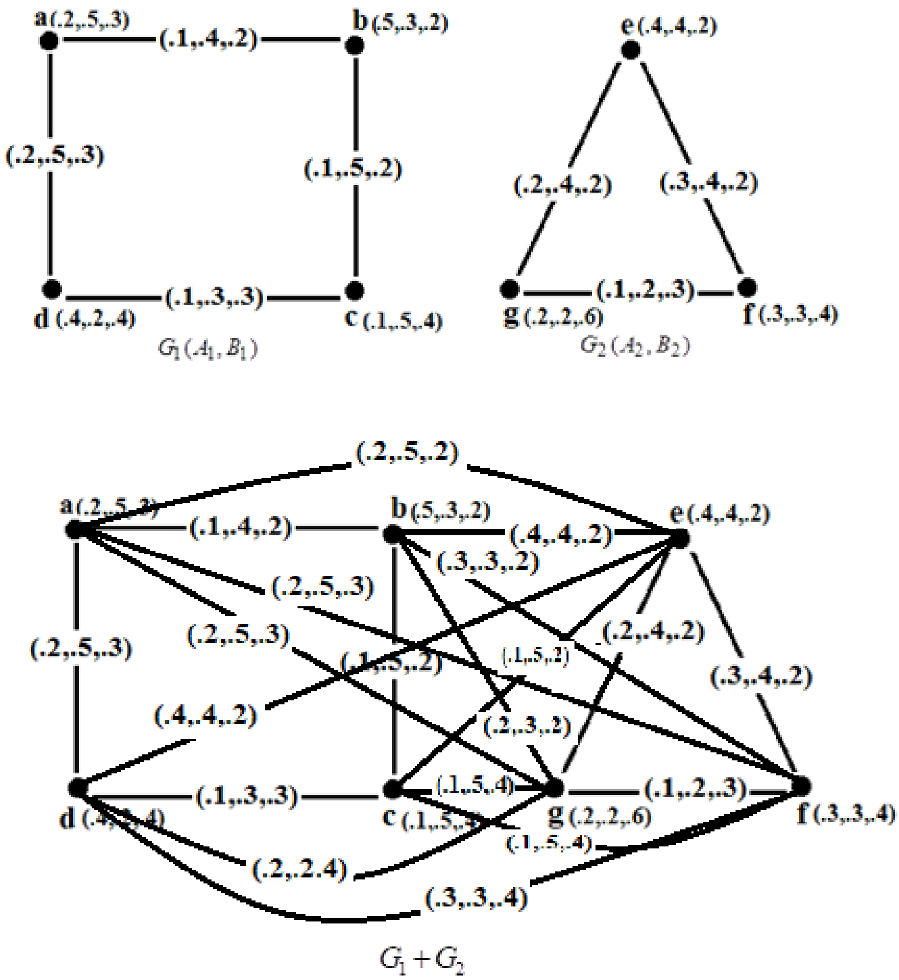


**Figure 3.3**

In Figure 3.3, the dominating sets of the hesitancy fuzzy graphs $G_1(A_1,B_1)$ *and* $G_2(A_2,B_2)$ are $D_1=\{a,c\}$ *and* $D_2=\{e\}$.The domination number of $G_1(A_1,B_1)$ *and* $G_2(A_2,B_2)$ are $\gamma(G_1)=0.66$, $\gamma_T(G_2)=0.4$ . In $G_1+G_2$, $V^+=\{e\}$ and $\{a,c\}\in V^0$ .

**Conclusion:**

In this work, we introduce the concept of dominating critical in hesitancy fuzzy graph. Further investigate the properties and bounds of the dominating critical. In future we will define the different types of dominating critical in hesitancy fuzzy graph.

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ISSN: 2233-7857 IJFGCN

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