# Edge Litact Domination in graphs

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### *Abstract:*

*In the manuscript the Edge litact domination number of the standard graphs like Cycle graph C_p wheel graph W_p ,Complete graph K_p, Bi-partite graph K_(m,n), Trees, and so-on are calculated and tried to bring up the relationship between the edge  litact domination number of G with different specifications of G and also with other domination specifications of G. Further the Nordhaus- Gaddum types of results are also obtained.*

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## 1.Introduction :

The vast applications of domination theory in diverse fields motivated many authors to study. The domination concept of  graph theory was originated from a chess board problem in 1850 which was mathematically described by *C. F. de Jaenisch* in the year 1862. More applications of domination concept can be found in many famous problems like *facility location , electrical networks, land surveying* etc., For several domination parameters defined on a graph, readers are referred to *T.W.Haynes, S. T.Hedetniemi, P. J.Slater* [11] & [12] and *V.R.Kulli* [13]. The current article concentrates on edge litact domination of graphs .

In a graph, the domination of edges was initiated by *Mitchell and Hedetniemi* [7] which is applicable in the analysis of social relations in which the edge between any two members denotes a good relation between them and no edge represents that they hate each other. We are interested in finding the edge domination in a litact graph which was introduced and described by *Kulli and M H Muddebihal* [5]. Many authors explored the variant in their research. *Vaidya and Pandit* [8] discussed the *edge domination in path and cycle related graphs* while the *end edge domination in graphs* is discussed by *M.H.Muddebihal and A.R.Sedsmkar* [3]. The *complimentary edge semi total block domination in graphs* is studied by *M.H.Muddebihal and Y.A.Bagewadi*[4] where as *M.H.Mudebihal and Milind S Chippalkatti* [5] has given many  bounds for the the *edge total block domination in graphs* .A brief study on *edge lict domination in graphs* can be found in [1]. A brief study on *complementary edge domination in graphs* is given by *Kulli and Soner* [15] and a discussion on *Fractional edge domination in graphs* is given by *S. Arumugam and S. Jerry*[9].The *edge domination sets* in graphs is further studied by *M.Yannakakis and F.Gavril*[6] and *S. Arumugam and S. Velammal* [10].
The aim of  investigation is to determine edge domination number of litact graph of graphs and  families of graph and also to obtain few bounds with various graph parameters.

## 2.Preliminaries :

All the notations and their corresponding definitions can be found in *F.Harary[*2] and *V.R.Kulli*[13]. Simple, finite, non-trivial, undirected and connected graphs are used to depict the notations.
Here we will recall some useful definitions for the present investigation.
**Definition 2.1: Induced Subgraph:** A sub graph traced by a subset of vertices in $G$ having end points as vertices together with edges of a graph.It is denoted by $\langle x \rangle$.
**Defintion 2. 2: Cut vertex :** The removal of a vertex which leaves the graph disconnected is a cut vertex .
**Definition 2.3: Litact Graph*:*** A litact graph $m(G)$ , in which the vertices are edges and cut vertices of G and edges are obtained by connecting the edges and cut vertices when they are adjacent and incident in G.
**Definition 2.4: Litact Dominating number*:*** A vertex set $D$ dominating in $m(G)$ is litact dominating, if every vertex of $V(G) - D$ is adjacent with atleast one vertex in $D$. Litact domination number is, $\gamma_m(G) = \min|D|$.
**Definition 2.5: Edge Litact Dominating number:** A set, $D \subseteq E(m(G))$ dominating in $m(G)$ is Edge litact dominating, if every edge not in $D$ is adjacent to at least one edge in $D$. Edge Litact domination number is, $\gamma'_m(G) = min|D|$.

## 3. Results

For more results we require the succeeding theorems.

**Theorem A** [13]: For every graph $G$, $\gamma(G) \leq p - \Delta(G)$.
**Theorem B** [13]: For each graph $G$, $\frac{q}{\Delta'(G)+1} \leq \gamma'(G)$
**Theorem C** [13]: For each graph $G$, $p - q \leq \gamma(G)$ only if each component of G is a star.
**Theorem D** [13]: For every graph $G$, $\gamma_t(G) \geq \frac{diam(G)+1}{2}$
**Theorem E** [13]: For each graph $G$, $\gamma(G) \leq \beta_0[G]$ .
**Theorem F** [13]**:** In every graph $G$, $\frac{p}{\Delta(G)+1} \leq \gamma_c(G) \leq 2q - p$
**Theorem G** [13]: If $G$ has no isolated vertices, $\gamma[G] \leq (p + 2 - \delta(G))/2$
**Theorem H** [13]**:** In any graph $G$, $\gamma[G] \leq \frac{\left(p+1-(\delta(G)-1)\frac{\Delta(G)}{\delta(G)}\right)}{2}$

## 4. Theorems:

The Edge litact domination number for few standard graphs is given in the theorem below.
**Theorem 4.1:**
(i) For each Cycle graph $C_p$ with a minimum of three vertices, $\boldsymbol{\gamma'_m(C_P)} = \left\lceil \frac{p}{3} \right\rceil$

(ii) For each Wheel graph $W_p$ with at least four vertices , $\boldsymbol{\gamma'_m(W_p)} = \boldsymbol{p - 2}$.

iii) For each Path $P_p$ with at least four vertices, $\boldsymbol{\gamma'_m(P_p)} = \left\lfloor \frac{p}{2} \right\rfloor$

(vi) For each Star graph $K_{1,p}$ with a minimum of three vertices, $\boldsymbol{\gamma'_m(K_{1,p})} = \boldsymbol{2}$

We relate $\gamma'_m(G)$ and vertex set in $G$.
**Theorem 4. 2: In every graph $G$ having more than two vertices, $\boldsymbol{\gamma'_m(G) < 2p}$ .**
**Proof**: Let edge set be $E[G]$ and cut vertex set be $C[G]$ in $G$. Then $E[G] \cup C[G] = V[m(G)]$. Let $D_1 \subseteq E[m(G)]$ be a minimal set which is dominating in $m(G)$. Then, we have
$$|D_1| + |E[m[G]] - D_1| = E[m[G]] < 4p \text{ -------(1)}$$
Also,  $|D_1| \leq |E[m[G]] - D_1|$
$$\Rightarrow |D_1| - |E[m[G]] - D_1| \leq 0 \text{ --------(2)}.$$

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Adding (1) and (2) we get
$$|D_1| < 2p \Rightarrow \gamma'_m(G) < 2p.$$
We relate edge litact domination number of graph $G$ with its number of vertices and maximum degree in the next corollary.

**Corollary 4.1: For each graph $G$, $\gamma'_m(G) + \gamma(G) < 3p - \Delta(G)$.**
**Proof:** From Theorem A,
$$\gamma(G) \leq p - \Delta(G) \ \text{.......} \ (1)$$
and Theorem 4.2 we have
$$\gamma'_m(G) < 2p \ \text{............} \ (2)$$
Adding (1) and (2) we get the result .
The next corollary gives a relation between $\gamma'_m(G)$, $\gamma(G)$ and the $q$.

**Corollary 4.2: In each graph $G$, $\gamma'_m(G) < 2(\gamma(G) + q)$.**
**Proof** : From Theorem C we have
$$p - q \leq \gamma(G)$$
$$\Rightarrow p \leq q + \gamma(G) \text{.......} (1)$$
and Theorem 4.2 we have
$$\gamma'_m(G) < 2p \ \text{.......} \ (2)$$
we get the corollary from (1) and (2).
We obtain a bound of $\gamma'_m(G)$ with $q$ and $\Delta'(G)$ in the next theorem.

**Theorem 4.3: In each graph $G$, $\frac{q}{\Delta'[G]+1} \leq \gamma'_m(\text{G})$ and when $G \cong C_3$ & $P_3$, $\frac{q}{\Delta'(G)+1} = \gamma'_m(G)$.**

**Proof**: Let $q = |E[G]|$, $m(G)$ be the Litact graph and $\Delta'(G)$ is the edge maximum degree. And let $D$ be the minimal set which is edge dominating in $m[G]$ and so $|D| = \gamma'_m[G]$.
Then
$$|D|\Delta'[G] < 2 |E[m[G]]| - |D|$$
$$\Rightarrow |D|\Delta'[G] + |D| < 2|E[m[G]]|$$
$$\Rightarrow |D|(\Delta'[G] + 1) < 2|E[m[G]]| \ \text{...........} \ (1).$$
Since $V[m(G)] = E[G] \cup C[G]$, we have $2|E[m(G)]| > |E[G]|$ .
Thus from (1), we have
$$2|E[m[G]]| > |D|(\Delta'[G] + 1) \geq |E[G]| = q$$
and this implies
$$\frac{q}{\Delta'(G)+1} \leq |D| = \gamma'_m(G)$$
$$\Rightarrow \frac{q}{\Delta'(G)+1} \leq \gamma'_m(G).$$
The corollary given below relates the $\gamma'_m(G)$ & $\gamma'(G)$.

**Corollary 4.4: In a graph $G$, $\gamma'_m(G) \geq \gamma'(G)$**
**Proof** : From Theorem B
$$\frac{q}{\Delta'(G)+1} \leq \gamma'(G) \ \text{............} \ (1)$$
and Theorem 4.3
$$\frac{q}{\Delta'(G)+1} \leq \gamma'_m(G) \ \text{.........} \ (2)$$
Subtracting (1) from (2) we get, $\gamma'_m(G) \geq \gamma'(G)$.

Here we obtain a relation between diameter and edge litact domination number

**Theorem 4.4: In a graph $G$, $\left\lceil \frac{diam(G)-1}{2} \right\rceil < \gamma'_m(G)$.**
**Proof:** Let $F = \{e_1, e_2, \ldots, e_i\} \subseteq E(G)$ be an edge set with longest path in $G$ of length $|F| = diam[G]$. Let $D$ be $\gamma'_m$ set in $G$ which gives $|D| = \gamma'_m(G)$ .Since $D$ is $\gamma'_m$ -set and the path of the diameter includes at least $\gamma'_m(G) + 1$ edges combining the neighbourhoods of the edges in $D$.
Hence
$$diam(G) \leq \gamma'_m(G) + \gamma'_m(G) + 1 = 2\gamma'_m(G) + 1$$
$$\Rightarrow \left\lceil \frac{diam(G)-1}{2} \right\rceil < \gamma'_m(G).$$
The next result gives a relation for $\gamma'_m(G)$ & $\gamma_t(G)$

**Theorem 4.5: For each $G$ with a minimum of two vertices, $\gamma'_m(G) \geq \gamma_t(G) - 1$**

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**Proof** : From Theorem 4.4 we have,

$$\left\lceil \frac{diam(G)-1}{2} \right\rceil < \gamma'_m(G) \text{ which gives } \frac{diam(G)-1}{2} < \gamma'_m(G) \quad \ldots\ldots (1)$$

Subtracting Theorem D from (1) we get,

$$-1 \leq \gamma'_m(G) - \gamma(G)$$

which gives $\quad\quad\quad \gamma'_m(G) \geq \gamma_t(G) - 1.$

We obtain a relation for $\gamma'_m(G)$ with $\gamma'(G)$ & $\gamma_t(G)$ .

**Theorem4.6: In a graph $G$, $2\gamma'_m(G) \geq \gamma'(G) + \gamma_t(G) - 1$** .

**Proof** : Adding Corollary 4.4 and Theorem 4.5  we get the required result.

We obtain a relation for $\gamma'_m(G)$ with diameter and $\alpha_0(G)$

**Theorem 4.7: In each $G$, $\gamma'_m(G) < \alpha_0(G) + diam(G)$** .

**Proof** : Let a minimal vertex set covering the edges in G be V .Then $|V| = \alpha_0(G)$ . Let $S = \{e_1, e_2, \ldots, e_i\}$ be the edge set in $G$ so that $|S| = diam(G)$. Let $D = \{e_1, e_2, \ldots, e_k\}$  be a set of edges in $G$ and $|D| = \gamma'_m(G)$. Therefore clearly $|D| < |S| + |V|$ which implies  $\gamma'_m(G) < \alpha_0(G) + diam(G)$.

The following theorem gives a relation of $\gamma'_m(G)$ with the diameter, $|V(G)|$ and $\gamma(G)$ .

**Theorem 4.8: In each graph $G$, $\gamma'_m(G) + diam(G) \leq p + \gamma(G)$.**

**Proof** : Let $uv$ be the maximum length between any two vertices in $G$, then we have   $dist(u,v) = diam(G)$. Let

$\gamma$-set be  $V \subseteq V(m(G))$ in $G$. Then $|V| = \gamma[G]$ . And let  $D = \{e_1, e_2, \ldots, e_j\}$ be the edge set in $m(G)$ in which at least one edge of D is adjacent with every edge in $E[m(G)] - D$. Then $|D| = \gamma'_m(G)$.

Hence we have,     $|D| + dist(u,v) \leq p + |V|$

$\quad\quad \Rightarrow \gamma'_m(G) + diam(G) \leq p + \gamma(G).$

The following corollary is a relation between  $\gamma'_m(G), \alpha_0(G)$ , $\beta_0(G)$ & $\gamma(G)$.

**Corollary 4.5: In each $G$, $2\gamma'_m(G) < 2\alpha_0(G) + \beta_0(G) + \gamma(G)$** .

**Proof**: Adding Theorem 4. 7 and Theorem 4.8 we get the result.

The next theorem is relation for $\gamma'_m(G)$ with $\gamma(G)$

**Theorem 4.9: In a graph $G$, $\gamma'_m(G) \leq 4\gamma(G)$** .

**Proof**: Let the edge set be  $D$ in $m(G)$ so that every edge in $E[m(G)] - D$ is adjacent with any edge in $D$. Then $|D| = \gamma'_m(G)$ . Let $D' = \{v_1, v_2, \ldots, v_i\} \subseteq V(G)$  be the set with minimum vertices and is dominating in $G$ so that $|D'| = \gamma(G)$. Since $D$ contains at most $4\gamma(G)$ edges, we have clearly $|D| \leq 4|D'|$ and this gives the result.

 The corollary given below relates edge litact domination number with $\beta_0(G)$

**Corollary 4.6 : In graph $G$, $\gamma'_m(G) \leq 4\beta_0(G)$** .

**Proof :** From Theorem E

$$\gamma(G) \leq \beta_0[G] \quad \ldots\ldots (1)$$

and Theorem 4.9

$$\gamma'_m(G) \leq 4\gamma(G) \quad \ldots\ldots (2)$$

 The result is from (1) and (2).

The following corollary is a relation between edge litact domination number , p and minimum degree of the graph.

**Corollary 4.7: For each graph $G$, $\gamma'_m(G) \leq 2(p + 2 - \delta(G))$**

**Proof** : It is clear from Theorem 4.9 & Theorem G

We get a relation for edge litact domination number of $G$ with $p, \Delta(G)$ & $\delta(G)$

**Corollary 4. 8: For a graph $G$, $\gamma'_m(G) \leq 2\left(p + 1 - (\delta(G) - 1)\frac{\Delta(G)}{\delta(G)}\right)$**

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**Proof:** From Theorem H and Theorem 4. 9, the proof is clear .

In the succeeding theorem we establish a relationship between $\gamma'_m(G)$ & $q$

**Theorem 4.10: In each graph $G$, $\gamma'_m(G) \leq q - 1$.**

**Proof** : Let $D \subseteq V(m(G))$ be the set of edges dominating in $m(G)$. This gives $|D| = \gamma'_m(G)$ . Since the set $D$ contains at most $q - 1$ edges, we have $\gamma'_m(G) \leq q - 1$.

The following result is a relation of Edge litact domination number in $G$ with $\gamma_c(G)$

**Theorem 4. 11: In a graph $G$, $2\,\gamma'_m(G) > \gamma_c(G)$ .**

**Proof**: Let $D = \{e_1, e_2, \dots, e_j\}$ be $\gamma'_m$ -set in $G$. This implies $|D| = \gamma'_m(G)$. Let $P$ be the minimal set which is dominating in $G$. Consider a set $P_1 = N(P) - P$ and $P_2 \subseteq P_1$ so that the subgraph traced by $P_1 \cup P_2$ is connected. Then $P_1 \cup P_2$ is a connected dominating set which gives $|P_1 \cup P_2| = \gamma_c(G)$. Then clearly it follows that

$|D| > \frac{1}{2}|P_1 \cup P_2|$ which implies that $2\,\gamma'_m(G) > \gamma_c(G)$ .

In the following corollary we get a relation for $\gamma'_m(G)$ in terms of $p$ & $\Delta(G)$

**Corollary 4.9: In a graph $G$, $\frac{p}{\Delta(G)+1} < 2\,\gamma'_m(G)$**

**Proof**: We get the result using Theorem F and Theorem 4.11.

The following theorem is a relation between $\gamma'_m(G)$ and $\gamma(G), q, p$

**Theorem 4.12 : For every graph $G$, $\gamma'_m(G) + \gamma(G) < 2q + p$**

**Proof**: We get the result by adding Theorem F and Theorem 4.11.

**Theorem 4.13(Nordhaus-Gaddum type outcomes)**:

**In a graph $G$,**

(i) $\gamma'_m(G) + \gamma'_m(\bar{G}) \leq p$  (ii) $\gamma'_m(G) \cdot \gamma'_m(\bar{G}) \leq p\,q$

**Conclusion remarks :**

In this paper the accurate values of edge litact dominating set of a graph is calculated for different graphs and tried to obtain some bounds and relations for the parameter with different variants of the graph.

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