International Journal of Future Generation Communication and Networking
Vol. 13, No. 3, (2020), pp. 3580–3586

Control Of The Unbalanced System Of The Reversed Pendulum By Exercising The Polynomial Detain

Neeti Deswal[1], Ritesh Yadav[1], Santosh Pawar[2] and Kamal Kumar Kushwaha[3]

[1]Department of Physics, Dr. A. P. J. Abdul Kalam University, Indore (M.P.) -452016, India
[2]Department of Electronics & Communication Engineering, Dr. A. P. J. Abdul Kalam University Indore (M.P.) -452016, India
[3]Department of Physics, Government Engineering College, Jabalpur (M.P.) - 482002, India

Abstract:

This paper deals with the design of feedback control system for the unstable nonlinear system of the reversed pendulum using the polynomial approach. The resultant controllers enable stabilization of the pendulum rod in the unstable top position. Spectral factorization techniques together with the pole-placement method are utilized for the controller design and robust vs. non-robust setting of the tuning poles is presented and discussed.
Keywords--- *Unstable System, Reversed Pendulum, Feedback Control, Polynomial Approach.*

Introduction

There are a great deal of processes in the business that have unstable conduct such as different kinds of reactors, ignition systems, refining segments etc.[1],[2]. Continuous examinations with these systems without legitimate control can be a genuine danger (Stein, 2003). In such cases, process demonstrating and simulation are frequently the main safe apparatus to examine the properties of such systems. Nowadays, the job of demonstrating and simulation has risen essentially because of the expanding execution of PC innovation. There are a lot of sources gave to these science regions.[3], [4], [5].
The objective of the present work is to give one potential way to deal with the control of unstable nonlinear systems dependent on the linearization of the initially nonlinear process and resulting controller configuration dependent on the mathematical hypothesis [6], [7], [8]. The resultant controllers are determined utilizing the post position strategy with the assistance of the ghostly factorization procedure [9]. One free boundary would then be able to be utilized for tuning of the designed circle and the contrast between the vigorous and non-strong setting of this steady is demonstrated tentatively.
The paper is organized as follows: after this basic area, a depiction of the system follows along with a proposed numerical model and its boundaries. Next, linearization in a picked working point is introduced and the resultant exchange work is broke down. Further, controllers are designed for the unstable top situation of the turned around pendulum by the polynomial methodology and they are thought about by a few standards. Resultant execution and strength of the designed circle are examined and the commitment finishes up with a short rundown and some last comments.

Reversed Pendulum

The system comprises of a cart which can be moved along a metal conduct bar. An aluminum rod with a tube shaped weight is fixed to the cart by a pivot. This system is unstable and non-straight with one input and two outputs. The input signal is the control voltage of a DC motor which can change the

ISSN: 2233-7857 IJFGCN

position of the cart. The outputs are cart position and point of the pendulum rod. The two outputs are estimated by gradual encoders. Design of the system is introduced in Figure 1.

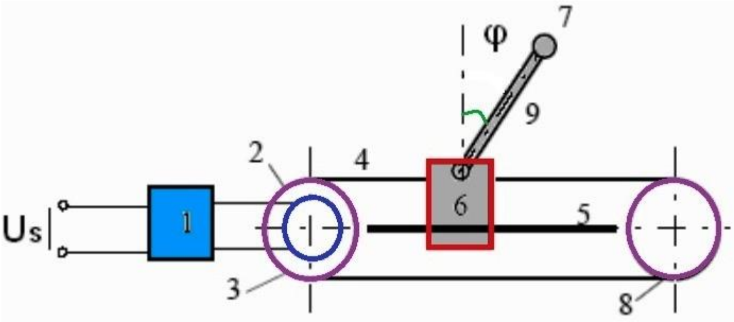

Figure 1: Design of Reversed pendulum

Where: 1 – servo amplifier, 2 – motor, 3 – drive wheel, 4 – transmission belt, 5 – metal conduct bar, 6 – cart, 7 – pendulum mass, 8 – steer roll, 9 – pendulum rod

The system can be explained by the subsequent nonlinear differential equations (Amira, 2000):

$$\varpi\gamma'' + F_\gamma\gamma' + \mu_p l\phi'' \cos\phi - \mu_p l(\phi')^2 \sin\phi = F \tag{1}$$

$$\Theta\phi'' + C\phi' - \mu_p l\xi \sin\phi + \mu_p l\gamma'' \cos\phi = 0 \tag{2}$$

Where F denotes to the input signal, which is the force generated by the DC motor. Output signals are γ cart position (γ indicates cart velocity) and ϕ pendulum angle (ϕ' signifies pendulum angular velocity). Representation ξ is the gravity acceleration constant and F_γ denotes to constant of a speed corresponding fraction of the cart. All constants and denotations are apparently characterized in Table 1. Following substitutes were utilized in the conditions (1) and (2):

$$\Theta = \Theta_\tau + \mu_p l^2 \tag{3}$$

$$\mu = \mu_c + \mu_p \tag{4}$$

Where μ_c is the cart mass μ_p is the pendulum mass l is the distance among centre of gravity of the pendulum with the centre of rotation of the pendulum moreover Θ_τ stand for the inertia moment of the pendulum rod through respect to the centre of gravity. Constant κ_α in the table indicates the gain of the servo amplifier; as shown in Figure 1.

All the employed constants were whichever taken from the manufacturer [10] or recognized by tests [11], [12] Their symbols and values are apparently characterized in the accompanying Table 1.

Parameter	Symbol	Value & Unit
cart mass	μ_c	4.00 kg
pendulum mass	μ_p	0.37 kg
total mass	μ	4.37 kg
pendulum length	l	0.425 m
inertia moment	Θ	0.08435 kg.m^2
cart grating	F_γ	6.55 kg/s
pendulum grating	C	0.00654 kg.m$^{2/s}$
velocity constant	κ_α	7.55N/V

ISSN: 2233-7857 IJFGCN

Linearization

For the function of successive controller replica, nonlinear differential equations (1) - (2) were linearized in the functioning point $\phi = 0$ (apex unstable point of the pendulum rod). Transfer function of the pendulum angle for this functioning point then:

$$\zeta_s(s) = \frac{-0.4386_s}{s^3 + 1.673_{s^2} - 18.64_s - 27.95} \tag{5}$$

Characteristic of the transfer function:

Derivative system (zero at $\psi_1 = 0$)

Unstable system (poles at $p_1 = +4.26, p_2 = -1.49, p_3 = -4.46$)

Therefore, excluding the derivative actions, the system is unstable.

Control system configuration

The conventional feedback control designexhibited in figure. 2 was used for the control system design,

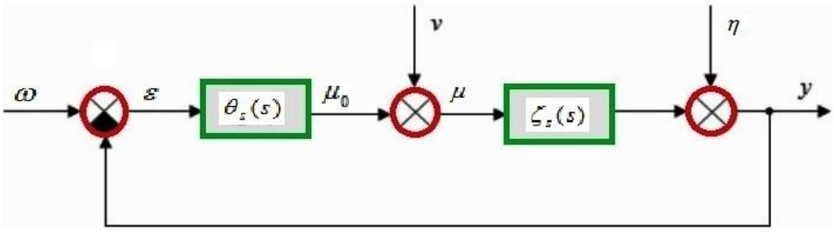

Figure 2. Control system design

where the employed variables match up to the following signals: ω — reference signal; ε - control error; μ — control input; ν, η — disturbances; y controlled output and $\theta_s(s)$, $\zeta_s(s)$ represent transfer functions of a controller and controlled system respectively; they are defined employed polynomials in the complex Laplace variable " s " as:

$$\theta_s(s) = \frac{q_s}{p_s}, \zeta_s(s) = \frac{\beta_s}{\alpha_s} \tag{6}$$

Control system requirements were created as stability, asymptotic tracking of the reference signal, disturbance reduction, and Internal appropriateness of all the utilized parts of the control system. For the reference and disturbance signal from the group of step functions and presuming the subordinate exchange capacity of the controlled system as (5), these conditions will be satisfied if the accompanying equations hold:

Stability of the control system is given by arespond of the Diophantine polynomial condition:

$$\alpha(s)p(s) + \beta(s)q(s) = \delta(s) \tag{7}$$

with $\delta(s)$ a stable attribute polynomial,

Asymptotic tracking of the reference signal as well as disturbance reduction will be guaranteed if the controller denominator comprises twice integrator:

$$p(s) = s^2 \hat{p}(s) \tag{8}$$

ISSN: 2233-7857 IJFGCN

Internal appropriateness will be satisfied if all parts of the control circle are legitimate. Utilizing this condition and considering feasibility of (7) the accompanying method for the degrees of polynomials q , $\hat{p}$ and δ must be satisfied: *deg*

$$\deg q(s) = \deg \alpha(s), \deg \hat{p}(s) = \deg \alpha(s) - 2\deg \delta(s) \geq 2\deg \alpha(s) - 1 \qquad (9)$$

Afterward, the consequential controller takes the common form:

$$\theta_s(s) = \frac{q_3 s^3 + q_2 s^2 + q_1 s + q_0}{s^2\left(\hat{p}_1 s + \hat{p}_0\right)} \qquad (10)$$

So as to calculate coefficients of this controller from (equation 7), the steady polynomial must be resolved. Here it is proposed to have it in this structure:

$$\delta(s) = \eta(s)(S + \vartheta)^2 \qquad (11)$$

where $\vartheta \succ 0$ is a correction constant and $\eta(s)$ is a steady polynomial calculate from the polynomial of the controlled system $\alpha(s)$ employing the spectral factorization technique (Grimble, 1994):

$$\tilde{\alpha}(s)\alpha(s) = \tilde{\eta}(s)\eta(s) \qquad (12)$$

(The superscripted left-right arrow signifies multifaceted conjugate polynomial $\tilde{x}(s) = x(-s)$ and the effect of the factorization is a polynomial by way of the analogouscharacteristics as the original but steady)

This decision of the properties of polynomial won't just assurance stability of the consequential control system yet additionally association with the initially method demeanor and it will leave space enough for additional conceivable correction too.

Experiments

Different experiments on the reversed pendulum were executed consecutively to verify the hypothetical ideas above. After the factorization (12) the steady properties polynomial (11) obtained this form:

$$\delta(s) = (s^3 + 10 + 19s^2 + 32s + 26 + 97)(S + \vartheta)^2 \qquad (12)$$

Then the consequential controllers are completelyresolute by the correction constant α which was selected to be 0.5; 2.5, 6.5 and 15 sequentially to contrast performance and sturdiness of the intended controllers. They are exhibitedasfollowing:

$$\text{At } (\alpha = 0.5)\ Q(s) = \frac{-103.5s^3 - 608.8s^2 - 688.2s - 15.8}{s^2(S + 9.6)}$$

$$\text{At } (\alpha = 2.5)\ Q(s) = \frac{-183.5s^3 - 1208.8s^2 - 1688.2s - 397.8}{s^2(S + 13.6)}$$

$$\text{At } (\alpha = 6.5)\ Q(s) = \frac{-634s^3 - 4132s^2 - 10335s - 7125}{s^2(S + 145)}$$

$$\text{At } (\alpha = 15)\ Q(s) = \frac{-1179s^3 - 9162s^2 - 20680s - 14340}{s^2(S + 390)}$$

Control results are introduced in the accompanying figure (Fig. 3) where both controlled yield and control input effects are demonstrated as well as, moreover in the Table 3 where examination by a few control value system is specified. During the control procedure, step disturbance of the amplitude -

ISSN: 2233-7857 IJFGCN

International Journal of Future Generation Communication and Networking
Vol. 13, No. 3, (2020), pp. 3580–3586

0.08 [rad] impacted the controlled yield in the time $t \in (15{,}15.5)$ (15; 15.5) [s] so as to test sensitivity of the intendedloop to disturbances.

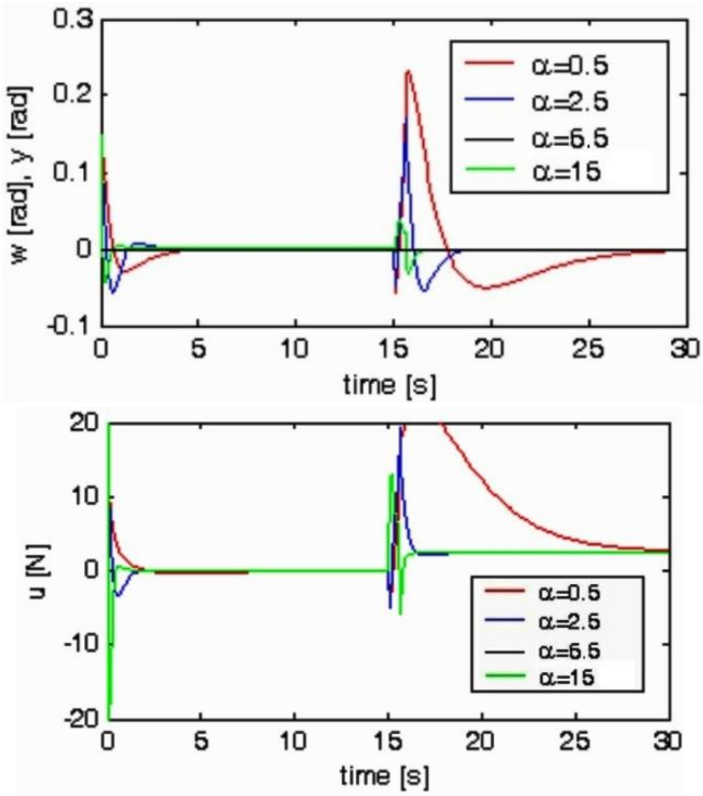

Figure 3. Control retort

The control quality was assessed with the help of following criteria:

Integral Squared Error (ISE)

$$J_e = \int_0^{tc} \varepsilon^2(t)dt$$

Integral Squared Control Input (ISCI)

$$J_u = \int_0^{tc} \mu^2(t)dt$$

Where i.e. is the overall time of control;

$\mu_{\max}[N]$ — Maximum control input for the disturbance reduction;

σ [rad] —maximum exceed of the controlled variable when satisfying the disturbance;

$t_s[s]$ —Settling time (time when the controlled output enters the area $\pm\delta$ around the reference signal and stays within; $\delta = 0.5^{\circ}$).

α	J_e	J_u	$\mu_{\max}[N]$	σ [rad]	$t_s[s]$
0.5	0.064	1935	21.8	0.24	3.0
2.5	0.017	280	19.5	0.18	1.4
6.5	0.008	420	15.2	0.09	0.9
15	0.005	842	12.8	0.04	0.5

ISSN: 2233-7857 IJFGCN

From the graphs unambiguously the control system is stable, the controlled variable tracks tangentially the reference signal, and the disturbance is reduced. It is furthermore evident that the control loop execution relies upon the correction boundary α. The table uncovers that the higher estimation of this constant the quicker the control effect (analyze settling times t_s) and better control feature communicated by the ISE measure (observe J_e in the table). Disturbance reduction is likewise influenced by the decision of this constant (observe σ, $\mu_{\max}$). So as to concentrate in detail the impact of the correction boundary an α on the power of the intended loop the affectability work is utilized, characterized as:

$$\in = \frac{\alpha(s) \times p(s)}{\delta(s)} \tag{13}$$

The climax achieve of its frequency retortspecified by the infinity norm H_∞, is a superiordeterminant of the loop sturdiness, e.g. (Skogestad& Postlethwaite, 1996). Reliance of the H_∞ standard of the sensitivity function (13) upon the limitation α is represented in the following graph, Figure 4.

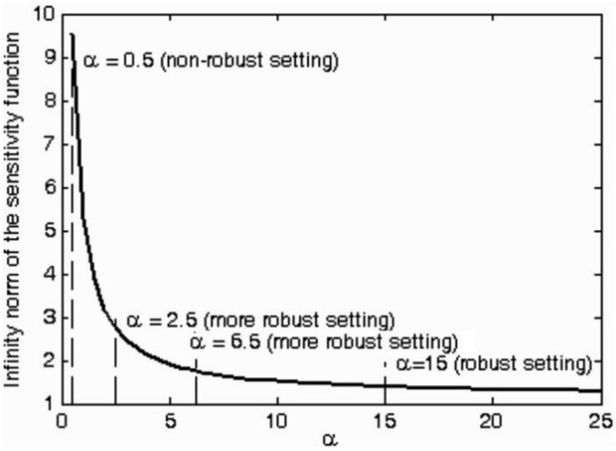

Fig. 4. H_∞ Standard of the sensitivity function with α

The figure demonstrates that the initial setting of the controller $(\alpha = 0.5)$ is non-robust with elevated sensitivity to disturbances, which is as well evident from the figure 3. Third situation, with $(\alpha = 6.5)$ also shown significant robust then the earlier one, where the disturbance is reducedrapidly but with better control effort. Last situation, with $(\alpha = 15)$finally shown most significant robust then the third one, where the disturbance is reduced extremely speedily with excellent control effort as perceived in the Table by the index J_u. As a result it is all the timesensible to seek a practicaltransaction between strength of the intended loop and realisticconstraint on the control input and its alteration, particularly when cope withunstable methods.

Conclusion

In this experiment, control of the unstable nonlinear system of the reversed pendulum was introduced. The polynomial methodology was utilized with the controllers intended utilizing the pole-placement technique and the spectral factorization procedure. A few situation of the correction constant were introduced and consider regarding utilizing various standards incorporating likewise robustness perspectives with the assistance of the sensitivity function. The proposed arrangement demonstrates to an uncomplicated mechanism for a moderately privileged method of controlling unstable processes throughadequate space left for conceivable correction of the intended loop as for its execution and

ISSN: 2233-7857 IJFGCN

robustness. Hence it tends to be used for stabilization and control of other unstable processes in a sturdy and safe manner. This will be the objective of our prospective assessment where moreover the confinement on the control data will be directed.

References

1. Chidambaram, M. (1997). Control of unstable systems: a review. Journal of Energy, Heat and Mass Transfer, 19, pp. 49-57.
2. Padma Sree, R. & Chidambaram, M. (2006). Control of unstable systems, Alpha Science Int. Ltd., ISBN 1-84265-287-7, Oxford.
3. Wellstead, P.E. (1979). Introduction to physical s ystem modelling. Academic Press Ltd, London.
4. Severance, F. L. (2001). System modeling and simulation. John Wiley & Sons, Ltd, 506pp. ISBN 0471-49694-4, Chichester.
5. Miklei, J. &Fikar, M. (2007). Process modeling, identification, and control. Springer-Verlag, 480pp. ISBN 978-3-540-71969-4, Berlin.
6. Kucera, V. (1993). Diophantine equations in control — a survey, Automatica, 29, pp. 1361-1375.
7. Hunt, K. J. (1993). Polynomial method's in optical control and filtering, Peter Peregrinus Ltd, London.
8. Anderson, B. D. 0. (1998). From Youla-Kucera to identification, adaptive and nonlinear control. Automatica, 34, pp. 1485—1506.
9. Grimble, M. J. (1994). Robu.st industrial control. Optimal design approach for polynomial systems. Prentice Hall, London.
10. Amira 2000. PS600 Laboratory Experiment Inverted Pendulum. Duisburg: Amira GmbH, 2000, 351pp.
11. Chalupa, P. &Bobal, V. (2008). Modeling and predictive control of inverted pendulum, Proceedings of 22 d European Conf. Modeling and Simulation, pp. 531-537, ISBN 978-0-9553018-5-8, Nicosia-Cyprus, June 2008, DigitaldruckPirrot GmbH Sbr.-Dudweiler.
12. Marholt, J. &Gazdoi, F. (2009). Modeling, Identification and Simulation of the Inverted Pendulum PS600. Proceedings of 10th International Carpathian Control Conference ICCC'2009, pp. 375-378, ISBN 8389772-5 1-5, Zakopane-Poland, May 2009, AGH — University of Science and Technology, Krakow.

ISSN: 2233-7857 IJFGCN