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A study of some domination parameters of undirected power graphs of Cyclic groups

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***Abstract*:**

In this paper we study some domination parameters of undirected power graph on cyclic sub-semi groups. An undirected graph $\mathcal{G}(S)$ for a semigroup S as follows.Let us denote the cyclic sub-semi group generated by $a \in S$ by$\langle a \rangle$, that is, $\langle a \rangle, = \{a^m \mid m \in N\}$, where N denotes the set of natural numbers. The graph $\mathcal{G}$ (S) is an undirected graph whose vertex set is S and two vertices $a, b \in S$ are adjacent if and only if $a \neq b$ and $a^m = b$ or $b^m = a$ (which is equivalent to say $a \neq b$ and $a^m = b$ or $b^m = a$ for some positive integer m).We obtain the domination, total domination and clique domination numbers for this graph.

Keywords: *Undirected power graph***,** *domination, total domination and clique domination numbers*

1. INTRODUCTION:

By a graph we mean, it is a pair of set (V, E), where V is the set of vertices and E is the set of edges, formed by pairs of vertices. For graph theoretic terminology we refer to the book by Chartrand and Lesniak [1]. All graphs in this paper are assumed to be non-trivial. Two vertices u and v are ***adjacent*** if they are connected by an edge, in other words, (u, v) is an edge. The ***degree of the vertex*** v, written as d(v), is the number of edges with v as an end vertex. A ***pendant*** vertex is a vertex whose degree is 1. An edge that has a pendant vertex as an end vertex is a pendant edge. An ***isolated*** vertex is a vertex whose degree is 0. The minimum degree of the vertices in a graph G is denoted δ (G) (= 0 if there is an isolated vertex in G). Similarly, we write Δ (G) as the maximum degree of vertices in G

A ***walk*** in the graph G = (V, E) is a finite sequence of the form v_{i0} , e_{j1} , v_{i1} , e_{j2} , . . . , e_{jk} , v_{ik} , which consists of alternating vertices and edges of G. A walk is a trail if any edge is traversed at most once. A ***trail*** is a path if any vertex is visited at most once except possibly the initial and terminal vertices when they are the same. A closed path is a ***circuit***. A graph is ***connected*** if all the vertices are connected to each other. A graph is ***complete*** if every pair of vertices is connected by an edge.

In recent years dominating functions in domination theory playing a key role as they have interesting applications. For the detailed study of domination see [2],[3],[4]. In this paper we study some domination parameters on undirected power graphs on semi groups.

2. POWER GRAPH:

Generating graphs from semigroups and groups are nothing new. Bosak [5] in 1964 studied certain graph over semigroups. Zelinka studied intersection graphs of nontrivial subgroups of finite Abelian groups in [6],. The study of a directed graph for the elements of a group considering as its vertex set is the Cayley digraph [7, 8, 9]. In [10] Kelarev and Quinn defined two interesting classes of directed graphs, as, divisibility graph and power graph on semigroups. Let S be a semi group.
The divisibility graph, Div(S) of a semi group S is a directed graph with vertex set S and there is an arc from u to v if and only if u≠v and u|v, i.e., the ideal generated by v contains u. On the other hand the power graph, Pow(S) of a semi group S is a directed graph in which the set of vertices is again S and for a, b ∈ S there is an edge from a to b if and only if a ≠ b and $a^m = b$ or $b^m = a$ for some positive integer m.

In [11] Ivy chakrabarty defined an undirected graph $\mathcal{G}$(S) for a semigroup S as follows.Let us denote the cyclic sub-semigroup generated by a ∈ S that is, ⟨a⟩ = {a^m | m ∈ N}, where N denotes the set of natural numbers. The graph $\mathcal{G}$ (S) is an undirected graph whose vertex set is S and two vertices a, b ∈ S are adjacent if and only if a ≠ b and $a^m = b$ or $b^m = a$ (which is equivalent to say a ≠ b and $a^m = b$ or $b^m = a$ for some positive integer m).

3. Properties of undirected power graphs.

3.1 Let S be a finite semigroup, then $\mathcal{G}$ (G) is connected if and only if S contains a single idempotent.
3.2 If G is a finite group then $\mathcal{G}$ (G) is always connected.
3.3 Let S be a semi group such that $\mathcal{G}$ (G) is connected. Then S contains at most one idempotent.

3.4 If G is a finite group then $\mathcal{G}$ (G) is always connected.
3.5 Let G be a finite group. Then $\mathcal{G}$ (G) is complete if and only if G is a cyclic group of order 1 or p^m, for some prime number p and for some m ∈ N.

3.6 The graph $\mathcal{G}$ $(U_n)$is complete if and only if n is any one of the following: n = 1, 2, 4,p, 2p, where p is a Fermat prime, i.e., p = $2^{2m} + 1$ (m ∈ Z, m ≥ 0).

3.8 Let G be a finite cyclic group of order n ≥ 3. Then $\mathcal{G}$ (G) is Hamiltonian.

4. DOMINATION OF POWER GRAPH OF FINITE CYCLIC SEMIGROUPS.

The theory of domination was introduced by C. Berge in 1958. This concept was drawn from the classical problem of covering chessboard with minimum number of chess pieces. A set D is a dominating set if for every vertex u ∈ V − D, there exists a vertex v ∈ D such that u is adjacent to v. A dominating set D in G is a minimal dominating set if no proper subset of D is a dominating set. The minimum cardinality among all the minimal dominating sets is called domination number of the graph G denoted by γ(G).

RESULTS:

Theorem 4.1: Let G be a finite group of prime order and $\mathcal{G}$ (G) be the power graph then $\gamma(\mathcal{G}\text{ (G) }) = 1$.

Proof: we know that if G is a finite group of prime order then it is cyclic. So by property 3.2 of power graph $\mathcal{G}$ (G) is a connected graph. Also by property 3.5 the graph $\mathcal{G}$ (G) will be complete graph if G is of prime order.

Also we know that in a complete graph of n vertices every vertex has n-1 as its degree.

Let us choose a set D= {{v_r}, r = 1,2,3….n}}. Then in set $V - D$ every vertex is adjacent to v_r

So every vertex dominates all other vertices.

Hence$\gamma(\mathcal{G}\text{ (G) }) = 1$.

Corollary 4.2: Let G be a group of prime order and $\mathcal{G}$ (G) be the power graph then $d(\mathcal{G}\text{ (G) }) = n$

As proved in above theorem $\gamma(\mathcal{G}\text{ (G) }) = 1$ and graph ($\mathcal{G}$ (G)) is complete graph.

This implies we get n distinct dominating set of vertices hence $d(\mathcal{G}\text{ (G) }) = n$

Corollary 4.3: $\mathcal{G}$ (G) has no independent set as the power graph $\mathcal{G}$ (G) is complete.

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5. TOTAL DOMINATION OF POWER GRAPH OF FINITE CYCLIC SEMI GROUPS:

The concept of total domination was introduced by Cockayne , Dawes and Hedetniemi in [12]. A dominating set D is a *Total dominating set [TDS]* if the induced sub graph $\langle D \rangle$ has no isolated vertices.

The minimum cardinality of TDS is denoted as $\boldsymbol{\gamma_t}(G)$.Clearly a TDS exist for any graph G without isolated vertices. A $\gamma_t(G) set$ is a minimum total dominating set.

RESULTS:

Theorem 5.1: Let G be a group of prime order and $\mathcal{g}$ (G) be the power graph then $\gamma_t(\mathcal{g}\text{ (G) }) = 2$

Proof: Since the power graph $\mathcal{g}$ (G) is a complete graph.

Let D = $\{(v_r, v_s): \deg(v_r) = n-1, \deg(v_s) = n-1\}$ then the remaining vertices in $V - D$ are adjacent to some vertex in D

$\Longrightarrow$ D is a dominating set and $\langle D \rangle$ has no isolated vertex.

So D is a total dominating set with $\gamma_t(\mathcal{g}\text{ (G) }) = 2$.

Corollary 5.2: Let G be a group of prime order and $\mathcal{g}$ (G) be the power graph then

$$d_t(\mathcal{g}\text{ (G) }) = \begin{bmatrix} \frac{n-1}{2} \; when\, n\, is\, odd \\ \frac{n}{2} \; when\, n\, is\, even \end{bmatrix.$$

Proof: Let $\mathcal{g}$ (G) is an undirected power graph of semi group. From Theorem [5.1] $\gamma_t(\mathcal{g}\text{ (G) }) = 2$ and $\mathcal{g}$ (G) is a regular graph.

So when 'n' is even we get $\frac{n}{2}$ disjoint sets of $V \in \mathcal{g}$ (G) and when 'n' is odd then we get at least one subset of V of cardinality 3.

Hence we get $\frac{n-1}{2}$ disjoint sets of $V \in \mathcal{g}$ (G).

Hence

$$d_t(\mathcal{g}\text{ (G) }) = \begin{bmatrix} \frac{n-1}{2} \; when\, n\, is\, odd \\ \frac{n}{2} \; when\, n\, is\, even \end{bmatrix.$$

6. CLIQUE DOMINATION:

Let G be a nontrivial connected graph. A dominating set D of V is a clique dominating set of $\mathcal{g}$(G) if the induced subgraph $\langle D \rangle$ of D is complete. The minimum cardinality of a clique dominating set of G, denoted by $\boldsymbol{\gamma_{cl}}(\boldsymbol{G})$, is called the clique domination number of G. A clique dominating set of G with cardinality $\boldsymbol{\gamma_{cl}}$ (G) is called a clique set of G. The concept of clique domination was first studied by Cozzens and Kelleher in [13]. Domination and other variations of domination can be found in [14] and [15].

RESULTS:

Theorem 6.1: [16][2.1] *Let $\mathcal{g}$(G) be a connected graph, then $\gamma_{cl}(\mathcal{g}\text{ (G) }) = 1$ if and only if*

$$\gamma(\mathcal{g}\,(\mathrm{G})\,) = 1$$

***Theorem* $\mathbf{6.2}$**: Let G be a group of prime order and $\mathcal{g}$(G) be the power graph then $\mathcal{g}$(G) ***has CDS with* $\mathbf{1 \leq \gamma_{cl}\,(\mathcal{g}\,(G)\,) \leq n - 1}$**.

Proof: **S**ince the power graph $\mathcal{g}$ (G) is a complete graph

Let D be a dominating set of $\mathcal{g}$ (G).

Since $\mathcal{g}$(G) is a complete graph then the induced graph $\langle D \rangle$ is also complete.

So by definition, $\langle D \rangle$ will become CDS.

Let D has one vertex.

We know that a graph with one vertex is complete .So cardinality of CDS is 1.

Now let D has 2 vertices and an edge. Then D becomes complete and hence D is a CDS with cardinality 2.

So in general when D is a complete graph with n vertices it has cardinality n-1.

Hence $1 \leq \gamma_{cl}\,(\mathcal{g}\,(\mathrm{G})\,) \leq n - 1$.

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