International Journal of Future Generation Communication and Networking
Vol. 13, No. 3, (2020), pp. 2567–2577

Neuro – Spike Analysis Using Stochastic Resonance with Random Fluctuations

Sanjeev Kumar[a], Dilkeshwar Pandey[a], S.K. Bharti[b]

[a]*KIET Group of Institutions, Ghaziabad*
[b]*GCET, Greater Noida*

Abstract:

Stochastic resonance phenomenon is studied to observe the effect of small external noise on nonlinear system. Integrate and fire models are widely studied in the field of computational neuroscience. In this paper phenomenon of stochastic resonance is investigated in two models viz integrate and fire and leaky integrate and fire. Salient features of this study is that external oscillatory input is considered with random parameters i.e. phase difference, frequency and amplitude with varying noise intensity. Monte Carlo simulation method is used to generate sample paths of the underlying models which exhibit the stochastic resonance in presence of random parameters. Interestingly stochastic resonance occurs around values of noise intensity which are almost same in all cases.

Key words: - *Stochastic resonance, Integrate and fire, noise, Monte Carlo Simulation*

I. INTRODUCTION

The phenomenon of stochastic resonance (SR) has been extensively observed in various nonlinear systems (Mc Donell and Abott, 2009)[1]. Originally Benzi et al (1981) and Nicolis (1981) [2,3] coined the term ‘Stochastic Resonance’. SR effect, however, corresponds to a setting where random noise influences the dynamics of a system so as to improve system’s performance to discriminate weak information-carrying signals. The SR phenomenon is a result of understanding the amplification of a small periodic forcing by the internal noise of a system. It may be noted that SR phenomena is different from usual observation of resonance where driving frequency matches the natural frequency of oscillation/vibration of a system. In a nonlinear system, the potential well function, $U(x)$, may be approximated to be, say bimodal. Therefore , a slight perturbation considered as a weak information –carrying signal (I) , acting on a particle may role it to-and-fro at the bottom of $U(x)$. In case the particle movement is observed only when it moves from one well to other, the effect of the perturbing force goes unnoticed. The addition of noise in such a system may result in allowing the particle to occasionally exit into the neighboring well. In SR, it is observed that the exit of a particle from one well to other is not random but correlated with I.Therefore increased noise strength causes more frequent transitions between neighboring wells. Thus particle dynamics reveals the time information of the weak signal I,usually increased noise tends to blur / deteriorate the transfer and detection of information. Thus the possibility of an optimal noise strength exists for which there exists a strong correlation between the input $I$and the noise strength (McDonnell MD, et al.2015)[4] . Longtin (1993)[5] extended the study of SR effect to the excitable systems which have only one stable state but can be excited to a state which is unstable. The discovery of SR in sensory neurons when subjected to external noise (cf. Longtin et al., 1991; Bulsara et al., 1991; Chialvo and Apkarian , 1993)[6,7,8] led to further work in electrophysiological SR studies. In sensory neurons, a spike is generated as the propagating action potential reaches a threshold. On generation of a spike, a quiescent time interval occurs when no firing take place. SR phenomenon has been observed in hydrodynamically sensitive mechanoreceptor hair cells in tailfins of crayfish (Douglas et al , 1993; Wisenfeld et al , 1994; Pei et al., 1996)[9,10,11], on the cercal system of cricket (Levin et al., 1996)[12]. It is interesting to note that Pei et al.(1996) observed SR in the

ISSN: 2233-7857 IJFGCN

presence of internal noise. They varied the strength of internal noise by controlling the light intensity incident on hair cell photoreceptive area. SR equation for a generic neuronal model is given by

$$\frac{du}{dt} = f(u) + A.\sin(2\pi f t + \phi) + \sigma\xi(t) \qquad (1)$$

where u is excitation variable and $f(u)$is a general nonlinear function of the excitation variable u.The second term represents a signal carrying information and the last term represents the internal/external noise in the system, while it is considered to be Gaussian white noise process.

In SR, in the presence of some optimal noise strength σ, the amplification of input sinusoidal signal carrying the information is amplified beyond apriori threshold. The optimal noise strength σ is detected by finding the correlation between the inputs i.e., signal plus noise and the output $u(t)$.

In the following we briefly outline various neuronal models and using the Monte Carlo simulation technique, we characterize the SR process in these models.

II. INTEGRATE AND FIRE (IF) MODEL WITH RANDOM RATES AND MONTE CARLO PROCEDURE

The remarkable integrate and fire neuron model was proposed by Lapicque in 1907 [13] to understand activity of neuron in terms of dynamics of membrane potential. In the generic model, by substituting $u = V(t)$, the membrane potential, and $f(u) = \mu$, we obtain the dynamical equation for the sinusoidally excited integrate and fire model in the presence of noise as:

$$\frac{dV}{dt} = \mu + A.\sin(2\pi f t + \phi) + \sigma\xi(t) \qquad t \geq 0 \qquad (2)$$

where V is membrane potential, μ is drift parameter, σ noise intensity and $\xi(t)$is Gaussian white noise.

A. Monte Carlo Simulation of SR in IF Model

Mc Donell et all (2015)[4] studied stochastic resonance in IF neuron model with varying noise intensity . In the present study work is extended by considering phase difference, frequency and amplitude both as random parameters. In the foregoing IF model, for numerical simulation we have chosen the initial condition $V(0) = 0.01$, and $\mu = 0.5, A = 0.2$, f =1 and $\phi = 0$ (Case: 1). Fig.1 (a) shows different noise level considered at different time interval $0 \leq t \leq t_1$, its effect on the input signal and also the output response variation when it crosses the given threshold($V_{th} = 0.7$). In Fig.1 (b) the variation of the correlation coefficient between the input and the output response due to increased noise level is shown. It is clear that an optimal noise strength, say$\sigma \sim 0.25$, exist at which the observed correlation coefficient is maximum. Thus SR effect, described earlier, can be easily simulated using Monte Carlo simulation method. In order to see the effect of phase variation on SR, we considered the phase ϕ (Fig.1(c)) to be randomly varying and following uniform distribution while retaining the values of other parameters. The SR simulation shown in Fig.1 (d) shows that the optimal value of σ remains unchanged even though the phase is varying randomly.

Case 1:- With normal oscillatory input

ISSN: 2233-7857 IJFGCN

International Journal of Future Generation Communication and Networking
Vol. 13, No. 3, (2020), pp. 2567–2577

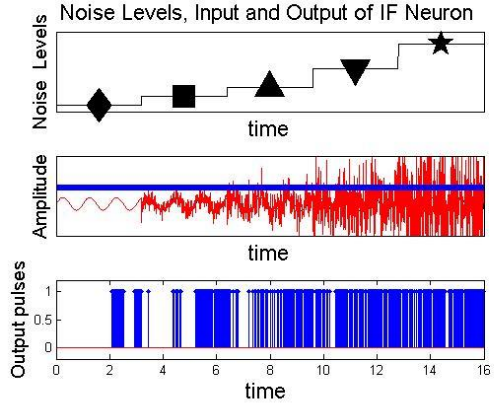


Fig. 1 (a).

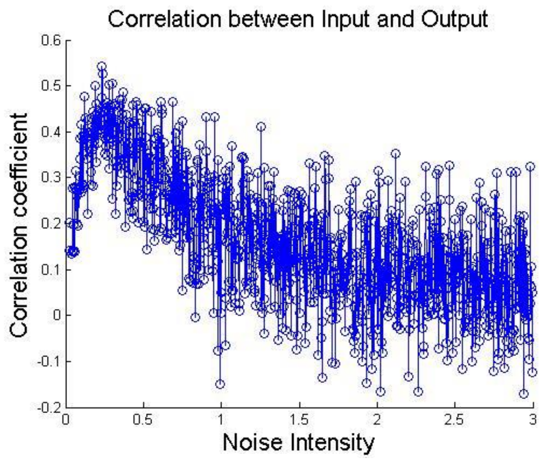


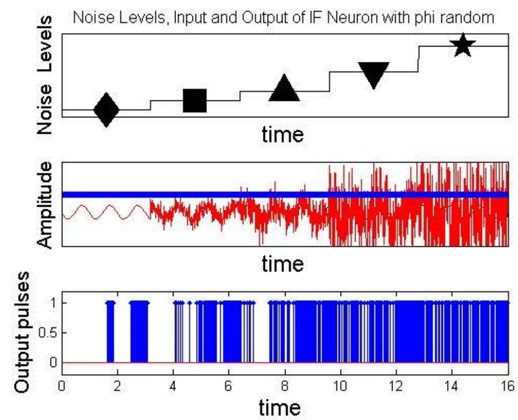
Fig. 1 (b).

It is assumed the random variable ϕ follows uniform distribution with pdf

$$p(\emptyset) = \frac{1}{2\pi} \leq \emptyset \leq 2\pi \quad (3)$$

We generate n values of ϕ from U(0,2π). For each value we draw the sample realizations.

ISSN: 2233-7857 IJFGCN



Case 2:- Oscillatory Input with random phase

Fig. 1 (c).

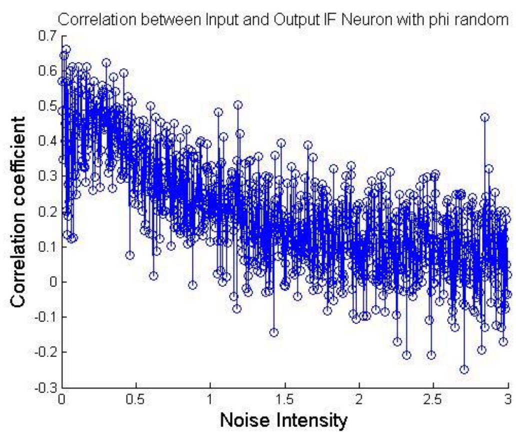


Fig. 1 (d).

Case 3:- Oscillatory Input with random frequency (f)

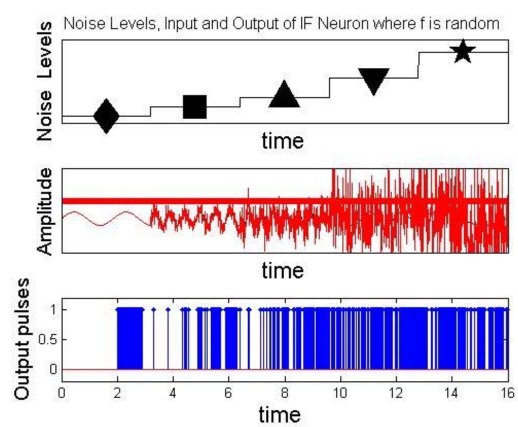


Fig. 1 (e).

ISSN: 2233-7857 IJFGCN

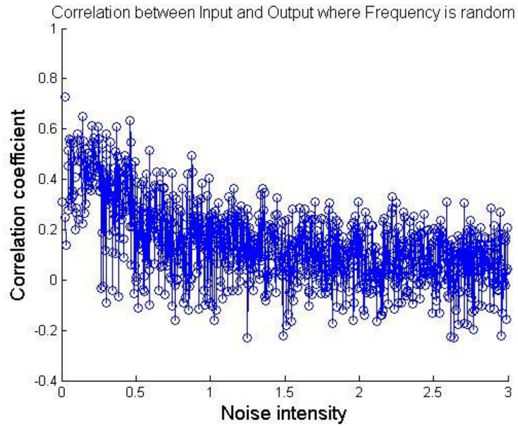


Fig. 1 (f).

The statistical distribution of frequency f is considered to be normally distributed

$f = \bar{f} + \varepsilon,$ where $\varepsilon \sim N(0,\sigma_f^2)$ (4)

Case 4:- Oscillatory Input with random Amplitude (A)

It is assumed the random variable *A* follows uniform distribution with pdf

$$p(A) = 0 \leq A \leq c * threshold, \quad c > 0 \tag{5}$$

We generate n values of *A* from U(0,c*V_{th}). For each value we draw the sample realizations.

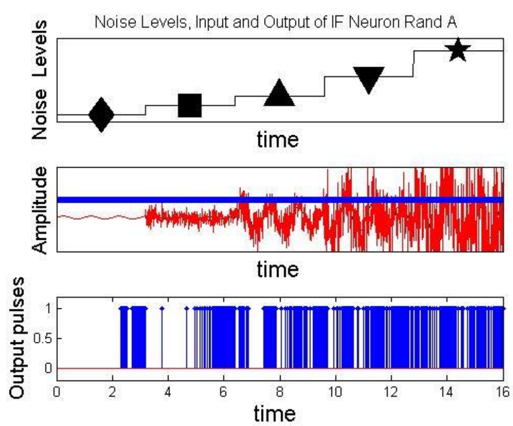


Fig. 1 (g).

ISSN: 2233-7857 IJFGCN

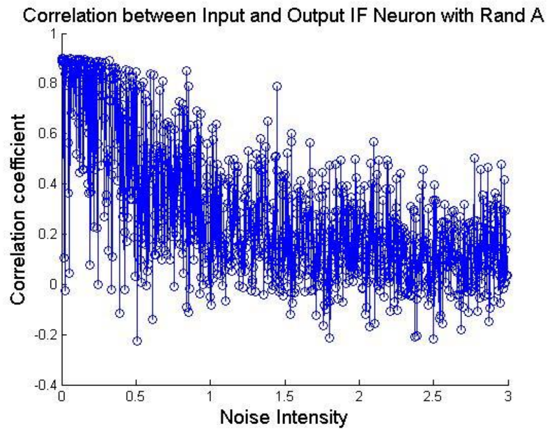


Fig. 1 (h).

Figure 1. SR simulation in IF Neuron Models

A. Leaky Integrate and Fire Neuron Model (LIF)

The IF model can be extend to a situation to consider the effect of 'leaky' or imperfect integration of input current. This model, hereafter referred to as LIF, is still widely used in neural network simulations due to its effectiveness in describing neuronal behavior. In the presence of a sinusoidal signal and noise, the equation governing the LIF model, for the study of SR effect, may be written as:

$$\frac{dV}{dt} = -\beta V + \mu + \sigma\xi(t) + A.sin(2\pi ft), t \geq 0 \tag{6}$$

where β is inverse of decay constant, μ is drift parameter.

III. LEAKY INTEGRATE AND FIRE (LIF) MODEL WITH RANDOM RATES AND MONTE CARLO PROCEDURE

In the foregoing LIF model, for numerical simulation we have chosen the initial condition $V(0) = 0.01$, and $\mu = 0.2, A = 0.2$, f =1, $\phi = 0$ and β = 1.4 (Case: 1). Fig.2 (a) shows different noise level considered at different time interval $0 \leq t \leq t_1$, its effect on the input signal and also the output response variation when it crosses the given threshold($V_{th} = 0.7$). In Fig.2 (b) the variation of the correlation coefficient between the input and the output response due to increased noise level is shown. It is clear that an optimal noise strength exist at which the observed correlation coefficient is maximum.

Case 1:- With normal oscillatory input

ISSN: 2233-7857 IJFGCN

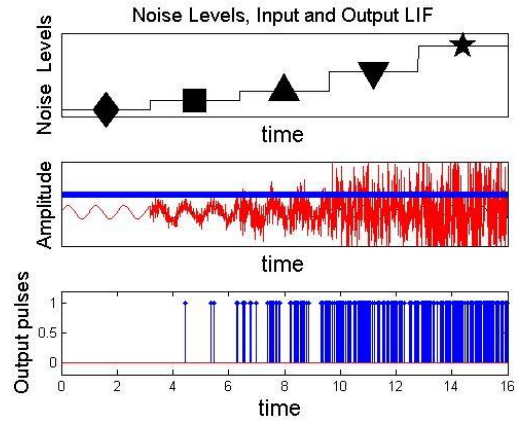


Fig. 2 (a).

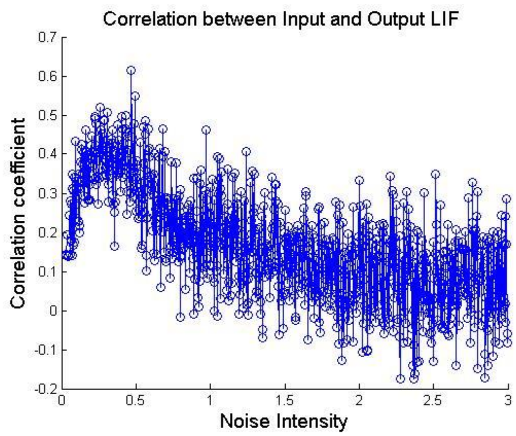


Fig. 2(b).

In order to see the effect of phase variation on SR, we considered the phase ϕ (Fig.2(c)) to be randomly varying and following uniform distribution while retaining the values of other parameters. The SR simulation shown in Fig.2 (d) shows that the optimal value of σ is slightly decreased $\sigma \sim 0.23$ when the phase is varying randomly.

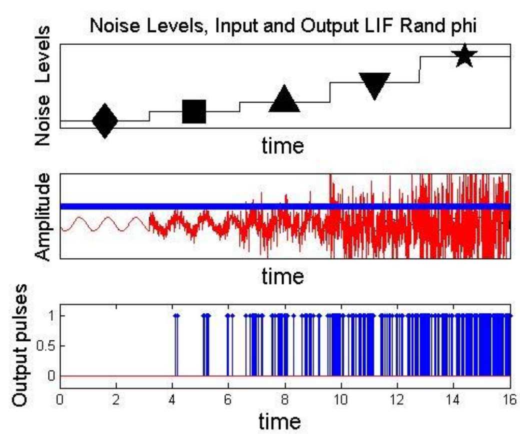


Fig. 2(c). *Case 2:- Oscillatory Input with random phase*

ISSN: 2233-7857 IJFGCN

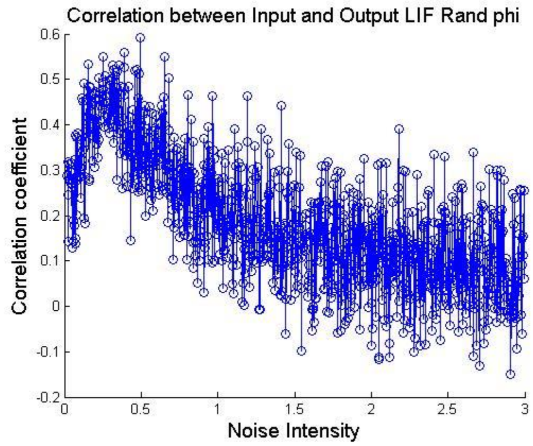


Fig. 2(d).

Further simulations show that the optimal frequency, at which the correlation coefficient shows a maximum, occur at the lower value of the noise level σ when only frequency is varying randomly and following normal distribution while other parameters have the values as in case: 1... It is shown in (Fig.2(e)) and (Fig.2(f)). Similar simulations have done by taking random amplitude following uniform distribution. The results are shown in (Fig.2 (g)) and (Fig.2 (h)).

Case 3:- Oscillatory Input with random frequency (f)

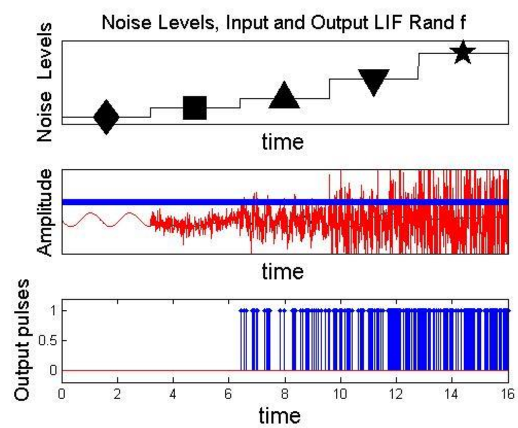


Fig. 2 (e).

ISSN: 2233-7857 IJFGCN

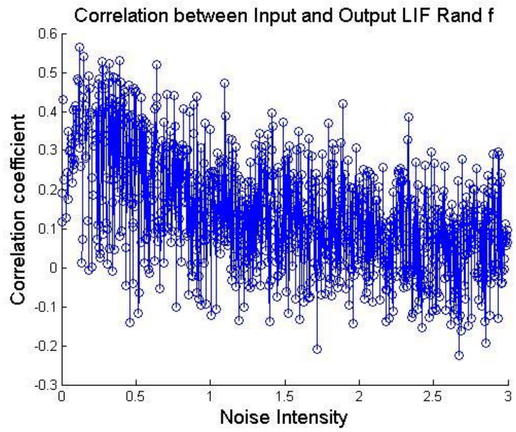


Fig. 2 (f).

Case 4:- Oscillatory Input with random Amplitude (A)

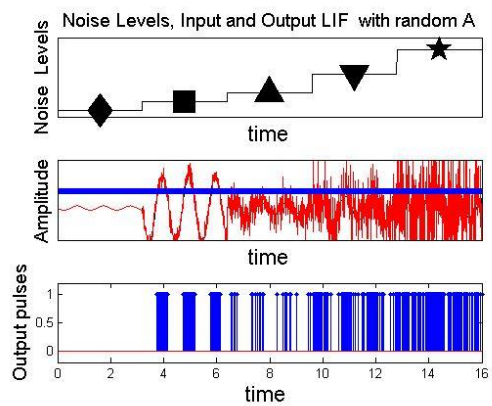


Fig. 2 (g).

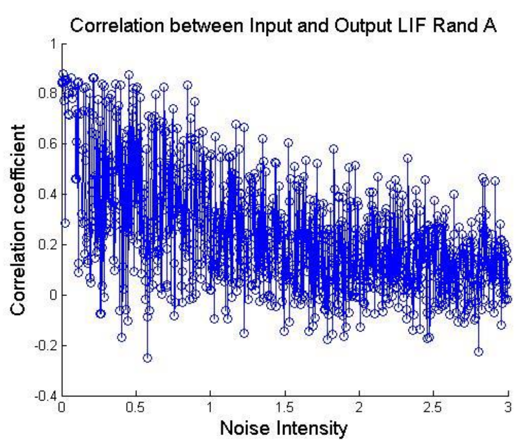


Fig. 2 (h).

ISSN: 2233-7857 IJFGCN

Figure 2. SR simulation in LIF Neuron Models

IV. CONCLUSION

Earlier work in the study of stochastic resonance is largely confined to situations where the parameters of the external stimulus remains fixed[14-17]. It is important to investigate the effect in presence of fluctuations in the parameters. In present study Monte Carlo simulation are done in presence of external oscillatory input with random fluctuations and with different noise intensities. Interesting results are noted while phase difference, frequency and amplitude of external input are taken random and maximum output of neuron is noted at optimal noise intensity $\sigma \approx 0.25$ which shows stochastic resonance. This indicates that brain uses randomness in input and noise to produce output.

ACKNOWLEDGMENT

Author are thankful to Dr. Mark D. Mcdonnell for sharing result and code to reproduce his results.

REFERENCES

[1] McDonnell MD, Abbott D. 2009. What is stochastic resonance? Definitions, misconceptions, debates, and its relevance to biology. PLoS Computational Biology 5:e1000348

[2] Benzi R, Sutera A, Vulpiani A (1981) The mechanism of stochastic resonance.

[3] Nicolis, G., Nicolis, C. & McKernan, D.Stochastic Resonance in Chaotic Dynamics, J Stat Phys (1993) 70: 125. https://doi.org/10.1007/BF01053958M.

[4] McDonnell MD, Iannella N, To MS, Tuckwell HC, Jost J, Gutkin BS, Ward LM., A review of methods for identifying stochastic resonance in simulations of single neuron models. Network: Computation in Neural Systems 2015;26(2):35-71. doi: 10.3109/0954898X.2014.990064. Epub 2015 Mar 11.

[5] Longtin A. 1993. Stochastic resonance in neuron models. Journal of Statistical Physics 70:309–327

[6] Longtin A,, Bulsara A, Moss F. 1991. Time-interval sequences in bistable systems and the noise-induced transmission of information by sensory neurons. Physical Review Letters 67:656–659.

[7] Bulsara A, Jacobs EW, Zhou T. 1991. Stochastic resonance in a single neuron model—theory and analog simulation. Journal of Theoretical Biology 152:531–555.J.-G. Lu, “Title of paper with only the first word capitalized,” *J. Name Stand. Abbrev.*, in press.

[8] DR Chialvo, AV Apkarian, Modulated noisy biological dynamics: three examples ,Journal of Statistical Physics 70 (1-2), 375-391

[9] Douglass JK, et al , Noise enhancement of information transfer in crayfish mechanoreceptors by stochastic resonance. Nature. 1993

[10] Wiesenfeld K, Pierson D, Pantazelou E, Dames C, Moss F. 1994. Stochastic resonance on a circle., Physical Review Letters 72:2125–2129.

[11] Pei, X., Wilkens, L. A., and Moss, F. ~1996!. ‘‘Light enhances hydrodynamic signaling in the multimodal caudal photoreceptor interneurons of the crayfish,’’ J. Neurophysiol. 76, 3002–3011

[12] Levin JE, Miller JP. 1996. Broadband neural encoding in the cricket cercal sensory system enhanced by stochastic resonance. Nature 380:165–168.

[13] Dayan P, Abbott LF. 2001. Theoretical neuroscience: Computational and mathematical modeling of neural systems. Cambridge, MA: The MIT Press.

[14] Takatsugu Aihara, Keiichi Kitajo, Daichi Nozaki, Yoshiharu Yamamoto, 2010, "How does stochastic resonance work within the human brain? – Psychophysics of internal and external noise, Chemical Physics, Volume 375, Issues 2–3, Pages 616-624,

[15] Krauss Patrick, Tziridis Konstantin, et al,2016, "Stochastic Resonance Controlled Upregulation of Internal Noise after Hearing Loss as a Putative Cause of Tinnitus-Related Neuronal Hyperactivity” Frontiers in Neuroscience, VOL 10

ISSN: 2233-7857 IJFGCN

[16] Marks KL, Martel DT, Wu C, et al., 2018, “Auditory-somatosensory bimodal stimulation desynchronizes brain circuitry to reduce tinnitus in guinea pigs and humans”. Sci Transl Med. 2018;10(422): eaal3175. doi:10.1126/scitranslmed. aal3175

[17] Krauss Patrick, Tziridis Konstantin, Schilling Achim, Schulze Holger, 2018, "Cross-Modal Stochastic Resonance as a Universal Principle to Enhance Sensory Processing" Frontiers in Neuroscience VOL12, PAGES=578 ,DOI=10.3389/fnins.2018.00578

ISSN: 2233-7857 IJFGCN