

On Igr^* - Continuous Functions in Intuitionistic Topological Spaces

¹P. Sathishmohan, ²U. Mehar Sudha, V. Rajendran³ and ⁴K. Rakalakshmi

^{1,3}Assistant Professor, ²Research Scholar, Department of Mathematics,
Kongunadu Arts and Science College(Autonomous),
Coimbatore-641 029.

⁴ Assistant Professor, Department of Science and Humanities,
Sri Krishna College of Engineering and Technology,
Coimbatore-641008

Email: ¹iiscsathish@yahoo.co.in, ²meharsudha1219@gmail.com,
³rajendrankasc@gmail.com, ⁴rajalakshmikandhasamy@gmail.com

Abstract

This paper focuses on Intuitionistic generalized regular star continuous functions and Intuitionistic generalized regular star irresolute functions in intuitionistic topological spaces and study some of its properties. Further, we have given appropriate examples to understand the abstract concepts clearly.

Keywords: Igr^* - continuous functions, Igr^* -irresolute functions.

1. INTRODUCTION

Norman Levine [8] initiated the idea of continuous functions in 1970. Intuitionistic fuzzy sets was introduced by Atanassov [1]. Later Coker [3] introduced intuitionistic sets and intuitionistic points in 1996. In 2000, Coker [4] developed the concept of intuitionistic topological spaces with intuitionistic sets and investigated basic properties of continuous functions and compactness topological spaces. The concept of regular continuous functions was first introduced by Arya and Gupta [2]. Later Palaniappan and Rao [9] studied the concept of regular generalized continuous functions. The concept of generalized regular star was studied by [6]. Also, the concept of generalized regular continuous functions was introduced by Mahmood [7] in general topological spaces. In this paper, the properties of intuitionistic generalized regular star continuous and functions and intuitionistic generalized regular star irresolute functions are introduced and studied.

2. PRELIMINARIES

Throughout this paper, X means an intuitionistic topological space (X, τ) and Y means an intuitionistic topological space (Y, σ) . In this section, we shall present the fundamental definitions which are useful for the sequel.

Definition 2.1. [10] A subset A of an intuitionistic topological space (X, τ) is called Intuitionistic generalized regular star closed set (briefly Igr^* -closed set) if $IrcI(A) \subseteq U$ whenever $A \subseteq U$ and U is Ig -open.

Definition 2.2. [5] A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be Intuitionistic generalized continuous if $f^{-1}(V)$ is Ig -closed in X for every intuitionistic closed subset V of Y .

- (1) Intuitionistic generalized semi continuous if $f^{-1}(V)$ is Igs-closed in X for every intuitionistic closed subset V of Y .
- (2) Intuitionistic semi generalized continuous if $f^{-1}(V)$ is Isg-closed in X for every intuitionistic closed subset V of Y .
- (3) Intuitionistic regular generalized continuous if $f^{-1}(V)$ is Irg-closed in X for every intuitionistic closed subset V of Y .
- (4) Intuitionistic generalized pre continuous if $f^{-1}(V)$ is Igp-closed in X for every intuitionistic closed subset V of Y .
- (5) Intuitionistic generalized pre semi continuous if $f^{-1}(V)$ is Igps-closed in X for every intuitionistic closed subset V of Y .
- (6) Intuitionistic generalized α continuous if $f^{-1}(V)$ is $Ig\alpha$ -closed in X for every intuitionistic closed subset V of Y .
- (7) Intuitionistic generalized pre regular continuous if $f^{-1}(V)$ is Igpr-closed in X for every intuitionistic closed subset V of Y .
- (8) Intuitionistic generalized b continuous if $f^{-1}(V)$ is Igb-closed in X for every intuitionistic closed subset V of Y .
- (9) Intuitionistic generalized semi regular continuous if $f^{-1}(V)$ is Igsr-closed in X for every intuitionistic closed subset V of Y .
- (10) Intuitionistic generalized weakly continuous if $f^{-1}(V)$ is Igw-closed in X for every intuitionistic closed subset V of Y .

Definition 2.3. [4] Let (X, τ) be an intuitionistic topological space and $A = \langle X, A_1, A_2 \rangle$ be an intuitionistic set in X . Then the interior of A is defined by

$$\text{Iint}(A) = \cup \{G : G \text{ is an IOS in } X \text{ and } G \subseteq A\}$$

It can be shown that $\text{Iint}(A)$ is an IOS in X and is an IOS in X iff $\text{Iint}(A) = A$.

3. Intuitionistic gr^* -Continuous Functions

In this section, we define and study the concept of Intuitionistic gr^* -Continuous Functions in intuitionistic topological spaces and obtain some of its properties.

Definition 3.1. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be intuitionistic generalized regular star (briefly Igr^* -continuous) if the inverse image of each intuitionistic closed set in Y is Igr^* -closed in X .

Theorem 3.2. (i) Every Igr^* -continuous function is Ig-continuous.

- (ii) Every Igr^* -continuous function is Igs-continuous.
- (iii) Every Igr^* -continuous function is Isg-continuous.
- (iv) Every Igr^* -continuous function is Irg-continuous.
- (v) Every Igr^* -continuous function is Igp-continuous.
- (vi) Every Igr^* -continuous function is Igps-continuous.
- (vii) Every Igr^* -continuous function is $Ig\alpha$ -continuous.
- (viii) Every Igr^* -continuous function is Igpr-continuous.
- (ix) Every Igr^* -continuous function is Igb-continuous.
- (x) Every Igr^* -continuous function is Igsr-continuous.

- (xi) Every Igr^* -continuous function is Iw -continuous.
- (xii) Every Igr^* -continuous function is I_p -continuous.
- (xiii) Every Igr^* -continuous function is I_b -continuous.

Proof: (i) Let $f : (X, \tau) \rightarrow (Y, \sigma)$ is Igr^* -continuous and let D be an intuitionistic closed in (Y, σ) . Since f is Igr^* -continuous function, $f^{-1}(D)$ is Igr^* -closed in (X, τ) . Since every Igr^* -closed is Ig -closed, $f^{-1}(D)$ is Ig -closed. Hence f is Ig -continuous. The proof of (ii) to (xiii) is similar to (i)

The converse of the above theorems need not be true as seen from the following examples.

Example 3.3. Let $X = Y = \{a, b, c\}$ with intuitionistic topologies $\tau = \{\emptyset, X, \langle X, \{a\}, \{b\} \rangle, \langle X, \{b\}, \{a\} \rangle, \langle X, \emptyset, \{a\} \rangle, \langle X, \{a, b\}, \emptyset \rangle, \langle X, \{a\}, \emptyset \rangle, \langle X, \emptyset, \{a, b\} \rangle\}$ and $\sigma = \{\emptyset, Y, \langle Y, \{a\}, \{c\} \rangle, \langle Y, \{c\}, \{a\} \rangle, \langle Y, \emptyset, \{a\} \rangle, \langle Y, \{a, c\}, \emptyset \rangle, \langle Y, \{a\}, \emptyset \rangle, \langle Y, \emptyset, \{a, c\} \rangle\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = b, f(b) = c, f(c) = a$ and let $D = \langle X, \{c\}, \{a\} \rangle$ then $f^{-1}(D) = \langle X, \{b\}, \{c\} \rangle$ is Ig -closed but not Igr^* -closed. Hence f is Ig -continuous but not Igr^* -continuous.

Example 3.4. Let $X = Y = \{a, b, c\}$ with intuitionistic topologies $\tau = \{\emptyset, X, \langle X, \{a\}, \{c\} \rangle, \langle X, \{c\}, \{a\} \rangle, \langle X, \{a\}, \emptyset \rangle, \langle X, \{a, c\}, \emptyset \rangle, \langle X, \emptyset, \{c, a\} \rangle, \langle X, \emptyset, \{a\} \rangle\}$ and $\sigma = \{\emptyset, Y, \langle Y, \{a\}, \{c\} \rangle, \langle Y, \{c\}, \{a\} \rangle, \langle Y, \emptyset, \{b\} \rangle, \langle Y, \{a, c\}, \emptyset \rangle, \langle Y, \{c\}, \emptyset \rangle, \langle Y, \{a\}, \emptyset \rangle, \langle Y, \emptyset, \{a, c\} \rangle, \langle Y, \emptyset, \{b, a\} \rangle, \langle Y, \emptyset, \{b, c\} \rangle\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = c, f(b) = a, f(c) = b$ and let $D = \langle X, \{a\}, \{c\} \rangle$ then $f^{-1}(D) = \langle X, \{b\}, \{a\} \rangle$ is Igs -closed but not Igr^* -closed. Hence f is Igs -continuous but not Igr^* -continuous.

Example 3.5. Let $X = Y = \{a, b, c\}$ with intuitionistic topologies $\tau = \{\emptyset, X, \langle X, \{a\}, \{c\} \rangle, \langle X, \{c\}, \{a\} \rangle, \langle X, \emptyset, \{b\} \rangle, \langle X, \{a, c\}, \emptyset \rangle, \langle X, \{c\}, \emptyset \rangle, \langle X, \{a\}, \emptyset \rangle, \langle X, \emptyset, \{a, c\} \rangle, \langle X, \emptyset, \{b, a\} \rangle, \langle X, \emptyset, \{b, c\} \rangle\}$ and $\sigma = \{\emptyset, Y, \langle Y, \{a\}, \{c\} \rangle, \langle Y, \{c\}, \{a\} \rangle, \langle Y, \{b\}, \emptyset \rangle, \langle Y, \{a, c\}, \emptyset \rangle, \langle Y, \{c, b\}, \emptyset \rangle, \langle Y, \{b, a\}, \emptyset \rangle, \langle Y, \emptyset, \{a, c\} \rangle, \langle Y, \emptyset, \{a\} \rangle, \langle Y, \emptyset, \{c\} \rangle\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = b, f(b) = a, f(c) = c$ and let $D = \langle X, \{a\}, \emptyset \rangle$ then $f^{-1}(D) = \langle X, \{b\}, \emptyset \rangle$ is Igs -closed but not Igr^* -closed. Hence f is Igs -continuous but not Igr^* -continuous.

Example 3.6. Let $X = Y = \{a, b, c, d\}$ with intuitionistic topologies $\tau = \{\emptyset, X, \langle X, \{a\}, \{b, c, d\} \rangle, \langle X, \{a, b\}, \{c, d\} \rangle, \langle X, \{c\}, \{a, b, d\} \rangle, \langle X, \{a, c\}, \{b, d\} \rangle, \langle X, \{b, c\}, \{a, d\} \rangle, \langle X, \{b, a\}, \{c, d\} \rangle, \langle X, \{a, c, d\}, \{b\} \rangle, \langle X, \emptyset, \{d\} \rangle\}$ and $\sigma = \{\emptyset, Y, \langle Y, \{a, b\}, \{c, d\} \rangle, \langle Y, \{a, b, c\}, \{d\} \rangle, \langle Y, \emptyset, \{d\} \rangle, \langle Y, \{a, d\}, \{b, c\} \rangle, \langle Y, \{c\}, \{a, b\} \rangle\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = c, f(b) = a, f(c) = b, f(d) = d$ and let $D = \langle X, \{c\}, \{a, b, d\} \rangle$ then $f^{-1}(D) = \langle X, \{d\}, \{c\} \rangle$ is Irg -closed but not Igr^* -closed. Hence f is Irg -continuous but not Igr^* -continuous.

Example 3.7. Let $X = Y = \{a, b, c\}$ with intuitionistic topologies $\tau = \{\emptyset, X, \langle X, \{a\}, \{c\} \rangle, \langle X, \{c\}, \{a\} \rangle, \langle X, \emptyset, \{c\} \rangle, \langle X, \{a, c\}, \emptyset \rangle, \langle X, \{c\}, \emptyset \rangle, \langle X, \emptyset, \{a, c\} \rangle\}$ and $\sigma = \{\emptyset, Y, \langle Y, \{a\}, \{c\} \rangle, \langle Y, \{c\}, \{a\} \rangle, \langle Y, \{c\}, \emptyset \rangle, \langle Y, \{a, c\}, \emptyset \rangle, \langle Y, \emptyset, \{c\} \rangle\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = c, f(b) = a, f(c) = b$ and let $D = \langle X, \emptyset, \{c\} \rangle$ then $f^{-1}(D) = \langle X, \emptyset, \{a\} \rangle$ is Igp -closed but not Igr^* -closed. Hence f is Igp -continuous but not Igr^* -continuous.

Example 3.8. Let $X = Y = \{a, b, c\}$ with intuitionistic topologies $\tau = \{\emptyset, X, \langle X, \{a\}, \{c\} \rangle, \langle X, \{c\}, \{a\} \rangle, \langle X, \{c\}, \emptyset \rangle, \langle X, \{a, c\}, \emptyset \rangle, \langle X, \emptyset, \{a, c\} \rangle, \langle X, \emptyset, \{c\} \rangle\}$ and $\sigma = \{\emptyset, Y, \langle Y, \{a\}, \{c\} \rangle, \langle Y, \{c\}, \{a\} \rangle, \langle Y, \{b\}, \{c\} \rangle, \langle Y, \{a, c\}, \emptyset \rangle, \langle Y, \{b, c\}, \emptyset \rangle, \langle Y, \{b, a\}, \{c\} \rangle, \langle Y, \emptyset, \{a, c\} \rangle, \langle Y, \emptyset, \{c\} \rangle\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = c, f(b) = a, f(c) = b$ and let $D = \langle X, \{a\}, \{c\} \rangle$ then $f^{-1}(D) = \langle X, \{b\}, \{a\} \rangle$ is $Igps$ -closed but not Igr^* -closed. Hence f is $Igps$ -continuous but not Igr^* -continuous.

Example 3.9. Let $X = Y = \{a, b, c, d\}$ with intuitionistic topologies $\tau = \{ \phi, X, \langle X, \{a\}, \{b, c\} \rangle, \langle X, \{b\}, \{a, c, d\} \rangle, \langle X, \{c\}, \{a, b, d\} \rangle, \langle X, \{a, c\}, \{b, d\} \rangle, \langle X, \{b, c\}, \{d\} \rangle, \langle X, \{b, a\}, \{c\} \rangle, \langle X, \{b\}, \{c, a\} \rangle, \langle X, \{d\}, \{b, c\} \rangle \}$ and $\sigma = \{ \phi, Y, \langle Y, \{a, c\}, \{b, d\} \rangle, \langle Y, \{c, d\}, \{a, b\} \rangle, \langle Y, \{a\}, \{b, c, d\} \rangle, \langle Y, \{a, c\}, \{d\} \rangle, \langle Y, \{c\}, \{a, b\} \rangle, \langle Y, \phi, \{d\} \rangle, \langle Y, \{d\}, \{a, b\} \rangle, \langle Y, \{a\}, \{c, b\} \rangle \}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = c, f(b) = a, f(c) = b, f(d) = d$ and let $D = \langle X, \{a\}, \{b, d\} \rangle$ then $f^{-1}(D) = \langle X, \{c\}, \{c, a\} \rangle$ is $Ig\alpha$ -closed but not Igr^* -closed. Hence f is $Ig\alpha$ -continuous but not Igr^* -continuous.

Example 3.10. Let $X = Y = \{a, b, c\}$ with intuitionistic topologies $\tau = \{ \phi, X, \langle X, \{a\}, \{c\} \rangle, \langle X, \{c\}, \{a\} \rangle, \langle X, \{c\}, \{b\} \rangle, \langle X, \{a, c\}, \phi \rangle, \langle X, \{c\}, \phi \rangle, \langle X, \phi, \{c, a\} \rangle, \langle X, \{c\}, \{a, b\} \rangle, \langle X, \phi, \{c, b\} \rangle$ and $\sigma = \{ \phi, Y, \langle Y, \{a\}, \{c\} \rangle, \langle Y, \{c\}, \{a\} \rangle, \langle Y, \{c\}, \{a, b\} \rangle, \langle Y, \{a, c\}, \phi \rangle, \langle Y, \phi, \{c, a\} \rangle$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = c, f(b) = a, f(c) = b$ and let $D = \langle X, \{c\}, \{a\} \rangle$ then $f^{-1}(D) = \langle X, \{a\}, \{b\} \rangle$ is $Igpr$ -closed but not Igr^* -closed. Hence f is $Igpr$ -continuous but not Igr^* -continuous.

Example 3.11. Let $X = Y = \{a, b, c\}$ with intuitionistic topologies $\tau = \{ \phi, X, \langle X, \{a\}, \{c\} \rangle, \langle X, \{c\}, \{a\} \rangle, \langle X, \{c\}, \{a, b\} \rangle, \langle X, \{a, c\}, \phi \rangle, \langle X, \phi, \{c, a\} \rangle$ and $\sigma = \{ \phi, Y, \langle Y, \{a\}, \{b\} \rangle, \langle Y, \{b\}, \{a\} \rangle, \langle Y, \phi, \{a\} \rangle, \langle Y, \{a, b\}, \phi \rangle, \langle Y, \{a\}, \phi \rangle, \langle Y, \phi, \{a, b\} \rangle$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = b, f(b) = c, f(c) = a$ and let $D = \langle X, \{b\}, \{a\} \rangle$ then $f^{-1}(D) = \langle X, \{a\}, \{c\} \rangle$ is Igb -closed but not Igr^* -closed. Hence f is Igb -continuous but not Igr^* -continuous.

Example 3.12. Let $X = Y = \{a, b, c\}$ with intuitionistic topologies $\tau = \{ \phi, X, \langle X, \{a\}, \{c\} \rangle, \langle X, \{c\}, \{a\} \rangle, \langle X, \{b\}, \phi \rangle, \langle X, \{a, c\}, \phi \rangle, \langle X, \{c, b\}, \phi \rangle, \langle X, \{b, a\}, \phi \rangle, \langle X, \phi, \{c, a\} \rangle, \langle X, \phi, \{a\} \rangle, \langle X, \phi, \{c\} \rangle$ and $\sigma = \{ \phi, Y, \langle Y, \{a\}, \{c\} \rangle, \langle Y, \{c\}, \{a\} \rangle, \langle Y, \{c\}, \phi \rangle, \langle Y, \{a, c\}, \phi \rangle, \langle Y, \phi, \{c, a\} \rangle, \langle Y, \phi, \{c\} \rangle$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = c, f(b) = a, f(c) = b$ and let $D = \langle X, \phi, \{c\} \rangle$ then $f^{-1}(D) = \langle X, \phi, \{a\} \rangle$ is $Igsr$ -closed but not Igr^* -closed. Hence f is $Igsr$ -continuous but not Igr^* -continuous.

Example 3.13. Let $X = Y = \{a, b, c, d\}$ with intuitionistic topologies $\tau = \{ \phi, X, \langle X, \{d\}, \{b, c\} \rangle, \langle X, \{b, c, d\}, \{a\} \rangle, \langle X, \{b\}, \{a, c\} \rangle, \langle X, \{a, c\}, \{d\} \rangle, \langle X, \{c, b\}, \{a\} \rangle, \langle X, \{b, a\}, \{c\} \rangle, \langle X, \phi, \{c, a\} \rangle, \langle X, \{a, b\}, \{c\} \rangle$ and $\sigma = \{ \phi, Y, \langle Y, \{a\}, \{b, d\} \rangle, \langle X, \{c\}, \{a, b\} \rangle, \langle Y, \{b, d\}, \{a\} \rangle, \langle Y, \{a, b\}, \{d\} \rangle, \langle Y, \{a\}, \{c, d\} \rangle, \langle Y, \phi, \{a, b\} \rangle$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = c, f(b) = a, f(c) = d, f(d) = b$ and let $D = \langle X, \{b, c, a\}, \{a\} \rangle$ then $f^{-1}(D) = \langle X, \{d\}, \{b, c, d\} \rangle$ is Iw -closed but not Igr^* -closed. Hence f is Iw -continuous but not Igr^* -continuous.

Example 3.14. Let $X = Y = \{a, b, c\}$ with intuitionistic topologies $\tau = \{ \phi, X, \langle X, \{a\}, \{c\} \rangle, \langle X, \{c\}, \{a\} \rangle, \langle X, \phi, \{b\} \rangle, \langle X, \{a, c\}, \phi \rangle, \langle X, \{c\}, \phi \rangle, \langle X, \{a\}, \phi \rangle, \langle X, \phi, \{c, a\} \rangle, \langle X, \phi, \{b, a\} \rangle, \langle X, \phi, \{b, c\} \rangle$ and $\sigma = \{ \phi, Y, \langle Y, \{a\}, \{c\} \rangle, \langle Y, \{c\}, \{a\} \rangle, \langle Y, \phi, \{a\} \rangle, \langle Y, \{a, c\}, \phi \rangle, \langle Y, \{a\}, \phi \rangle, \langle Y, \phi, \{a, c\} \rangle$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = b, f(b) = a, f(c) = c$ and let $D = \langle X, \{a\}, \phi \rangle$ then $f^{-1}(D) = \langle X, \{b\}, \phi \rangle$ is Ip -closed but not Igr^* -closed. Hence f is Ip -continuous but not Igr^* -continuous.

Example 3.15. Let $X = Y = \{a, b, c, d\}$ with intuitionistic topologies $\tau = \{ \phi, X, \langle X, \{a\}, \{c, d\} \rangle, \langle X, \{c\}, \{a, b\} \rangle, \langle X, \{a, b, d\}, \{c\} \rangle, \langle X, \{a, c\}, \{d\} \rangle, \langle X, \{a\}, \{b, c\} \rangle, \langle X, \phi, \{c, a\} \rangle$ and $\sigma = \{ \phi, Y, \langle Y, \{a\}, \{c, d\} \rangle, \langle Y, \{b\}, \{a, d\} \rangle, \langle Y, \{a\}, \{b, c, d\} \rangle, \langle Y, \{a, c\}, \phi \rangle, \langle Y, \phi, \{c, a\} \rangle, \langle Y, \phi, \{a, d\} \rangle$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = c, f(b) = d, f(c) = b, f(d) = a$ and let $D = \langle X, \{a, c\}, \{b\} \rangle$ then $f^{-1}(D) = \langle X, \{b, d\}, \{a, c, d\} \rangle$ is Ib -closed but not Igr^* -closed. Hence f is Ib -continuous but not Igr^* -continuous.

Theorem 3.16. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function, then the following conditions are equivalent.

- (i) f is Igr^* -continuous.
- (ii) The inverse image of intuitionistic closed set in Y is Igr^* -closed in X .

Proof: (i) $\rightarrow$ (ii) Assume f is Igr^* -continuous. Let A be an intuitionistic closed subset of Y , then $Y - A$ is intuitionistic open in Y and $f^{-1}(Y - A) = X - f^{-1}(A)$, is Igr^* -open in X which implies that $f^{-1}(A)$ is Igr^* -closed in X .

(ii) $\rightarrow$ (i) Assume the inverse of each intuitionistic closed set in Y is Igr^* -closed in X . Let D be an intuitionistic open set in Y , then $Y - D$ is an intuitionistic closed set in Y , which implies $f^{-1}(Y - D) = X - f^{-1}(D)$ is Igr^* -closed in X . Hence $f^{-1}(Y - D)$ is Igr^* -open in X , which implies that f is Igr^* -continuous.

Theorem 3.17. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function, then the following conditions are equivalent.

- (i) f is Igr^* -open.
- (ii) $f(\text{Irint}(A)) \subseteq \text{Irint}(f(A))$ for each IS A in X .
- (iii) $\text{Irint}(f^{-1}(B)) \subseteq f^{-1}(\text{Irint}(B))$ for each IS B in Y .

Proof: (i) $\rightarrow$ (ii) Let f be an Igr^* -open function. Since $f(\text{Irint}(A))$ is an Igr^* -open set contained in $f(A)$, $f(\text{Irint}(A)) \subseteq \text{Irint}(f(A))$ by definition of intuitionistic interior.

(ii) $\rightarrow$ (iii) Let B be any IS in Y . Then $f^{-1}(B)$ is an IS in X . By (ii), $f(\text{Irint}(f^{-1}(B))) \subseteq \text{Irint}(f(f^{-1}(B))) \subseteq \text{Irint}(B)$. Thus we have $\text{Irint}(f^{-1}(B)) \subseteq f^{-1}(\text{Irint}(f(f^{-1}(B)))) \subseteq f^{-1}(\text{Irint}(B))$.

(iii) $\rightarrow$ (i) Let A be any Igr^* -open set in X . Then $\text{Irint}(A) = A$ and $f(A)$ is an intuitionistic set in Y . By (iii), $A = \text{Irint}(A) \subseteq \text{Irint}(f^{-1}(f(A))) \subseteq f^{-1}(\text{Irint}(f(A)))$. Hence we have $f(A) \subseteq f(f^{-1}(\text{Irint}(f(A)))) \subseteq \text{Irint}(f(A)) \subseteq f(A)$. Thus $f(A) = \text{Irint}(f(A))$ and hence $f(A)$ is an Igr^* -open set in Y . Therefore f is an Igr^* -open.

Theorem 3.18. If $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \nu)$ is Igr^* -continuous, then their composition $\text{gof} : (X, \tau) \rightarrow (Z, \nu)$ is Igr^* -continuous function.

Proof: Let g be a intuitionistic continuous function and V be any intuitionistic open set in (Z, ν) then $f^{-1}(V)$ is open in (Y, σ) . Since f is Igr^* -continuous, $f^{-1}(g^{-1}(V)) = (\text{gof})^{-1}(V)$ is Igr^* -open in X . Hence (gof) is Igr^* -continuous.

4. Intuitionistic gr^* -Irresolute Functions

In this section, we define and study the notion of Intuitionistic gr^* -Irresolute Functions in intuitionistic topological spaces and obtain some of its properties.

Definition 4.1. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be intuitionistic generalized regular star (briefly Igr^* -irresolute) if the inverse image of each Igr^* -closed set in Y is Igr^* -closed in X .

Theorem 4.2. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be intuitionistic continuous and intuitionistic closed set. Then f is Igr^* -irresolute functions.

Proof: Let $A = \langle X, A_1, A_2 \rangle$ be any Igr^* -closed set. Then $A \subseteq \text{Ircl}(A) \subseteq U$, since f is intuitionistic continuous and intuitionistic closed it follows that $\Rightarrow f^{-1}(A) \subseteq f^{-1}(\text{Ircl}(A)) \subseteq \text{Ircl}(f^{-1}(A)) \Rightarrow f^{-1}(A) \subseteq \text{Ircl}(f^{-1}(A))$. Therefore $f^{-1}(A)$ is Igr^* -closed. Hence f is Igr^* -irresolute functions.

Theorem 4.3. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \nu)$ are Igr^* -irresolute then $\text{gof} : (X, \tau) \rightarrow (Z, \nu)$ is Igr^* -irresolute.

Proof: Let g be an Igr^* -irresolute function and V be any Igr^* -open in (Z, ν) , then $f^{-1}(V)$ is Igr^* -open in Y , since f is Igr^* -irresolute, $f^{-1}(g^{-1}(V)) = (\text{gof})^{-1}(V)$ is Igr^* -open in (X, τ) . Hence gof is Igr^* -irresolute.

Theorem 4.4. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ is Igr*-irresolute if and only if, for every Igr*-open A of Y , $f^{-1}(A)$ is Igr*-open in X .

Proof: Necessity: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is Igr*-irresolute, then for every Igr*-closed B of Y , $f^{-1}(B)$ is Igr*-closed in X . If A is any Igr*-open subset of Y , then A^c is Igr*-closed. Thus $f^{-1}(A^c)$ is Igr*-closed, but $f^{-1}(A^c) = (f^{-1}(A))^c$ so that $f^{-1}(A)$ is Igr*-open.

Sufficiency: If for all Igr*-open subsets A of Y , $f^{-1}(A)$ is Igr*-open in X , and if B is any Igr*-closed subset of Y , then B^c is Igr*-open. Also $f^{-1}(B^c) = (f^{-1}(B))^c$ is Igr*-open. Thus $f^{-1}(B)$ is Igr*-closed. Hence f is Igr*-irresolute.

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