

Formulation of Mathematical Model for the Investigation of Frictional Power Loss for Multi cylinder S I Engine using Dimensional Analysis

P. N. Patil¹ , Dr. D. S. Deshmukh² , Dr. V. S. Patil³ , Dr. S. P. Shekhavat⁴

¹Research Scholar, Shri Gulabrao Deokar College of Engineering, Jalgaon. (M S) .India

²Associate Professor, G H Raison College of Engineering, Hingna Road, Nagpur (M S)

India³Professor, University Institute of chemical Technology, KBC North Maharashtra

University Jalgaon⁴Professor & Vice Principal SSBT's College of Engineering & Technology, Jalgaon. (M S) India

Abstract

The aim of this study is to develop the mathematical relationships for frictional power loss in spark ignition engine for gasoline and alternative fuels used such as liquefied petroleum gas using dimensional analysis. Since the major contributor for fuel consumption are automotive sector and in this the internal combustion engine plays a major role as a power sources . Frictional power loss which affect directly on performance of any engine, defining the ability of engine to utilize the energy supplied and power developed .The correlation developed between Torque Load (Ld), Speed (N), Frictional power (Pf), Engine oil viscosity(μ_o) as a major parameter. Dimensional analysis technique is applied for reduction of variables by using Buckingham π - theorem. A probable mathematical model has formulated from obtained Pi-term with multiple regression analysis. The ANN model has also developed and tested against observed data for gasoline fuel.

Keyword- Dimensional analysis, Buckingham π - theorem , Frictional power loss, ANN.

1.0 Introduction:

In the past 20 years the automotive industry have greatly influenced by fuel efficiency and this has been achieved by improvement in engine component design with a proper lubricants or engine oils .A decrease in automotive engine friction may gives the opportunity to increase engine output and decrease the fuel consumption. It is generally accepted that the friction losses associated with the Piston ring assembly, Engine bearings, and valve train and engine auxiliaries account nearly 80% Introduction of the total mechanical losses piston assemblies are recognized to be responsible for 50% or more of the frictional power consumption[4].To find the frictional power , Some of the conventional methods are available to find the total or component engine friction such as IMEP Method from PV diagram, Morse Test Procedure, Motoring Breakdown Method, and Willians line test methods .

The use of LPG as an alternative fuel is commonly used in internal combustion engine in large quantities from a decade .Due to the high octane number, more energy content, less emission, less cost, better lubrication property ,easier transportation and storage,. Now a days the gaseous fuel as a LPG has been widely used throughout the world in S.I. Engine as a impact of green house emission losses than any other fossil fuels. A large number of studies are carried out on S.I. Engine fueled by LPG and gasoline and it is found at the use of LPG has a significant effect on engine performance and emission control . In this present investigation, Spark Ignition engine fueled by LPG has been selected for study and to formulate the field data base mathematical model for friction Analysis. Dimensional analysis (DA) is very useful for computing dimensionless parameters. This dimensional analysis can be accomplished by using Buckingham for π -theorem in which a reduction of the number of independent parameters involved in a problem. These independent parameters get expressed as dimensionless groups and these dimensionless groups are always ratios of important physical quantities involved in the problem to be considered. In experimental data base model, its main function is to reduce the amount of independent variables and to simplify the solution Therefore it can become an effective method, especially if a complete mathematical model of the investigated process is not known.

2.0 Need of Experimental Data Base Model:

To find the effects of various operating parameters of an internal combustion engine on frictional loss using LPG fuel are discussed in this paper. Engine performance is dependent on fuel consumption while frictional losses are depends on speed of engine and viscosity of engine oil. Therefore fuel consumption rate and frictional power loss is a basic dependent variable. It is an important performance parameter of an engine, varies because of any possible variable variations such as engine design (its operating conditions) and type of fuel and various viscosity multi-grade engine oils. The methodology as suggested by Schenk Hilbert Jr. 1961 has been adopted for the planning of experimentation on a model of frictional power loss by keeping objective of experimentation to formulate design data of SI Engine. In this experimentation, the independent variables are selected which have the influence on the engine performance. . This would be possible if one can have a quantitative relationship amongst various dependent and independent variables for the engine. These independent variables shall be varied over a widest possible ranges, a response data is collected and an analytical relationship is established. After developing the relationship the optimization techniques can be applied to reduce the value of independent variables at which the required responses can be minimized or maximized.

2.1 Theory of Experimentation :

The approach used for formulating the experimental model suggested by Hilbert Schenk Jr. is given in following steps.

- 1) Identification of independent, dependent and extraneous variables.
- 2) Reduction of independent variables using dimensional analysis.
- 3) Test planning comprising of determination of test envelope, Level of test points, test sequence and experimentation plan.
- 4) Physical design of an experimental set up.
- 5) Implementation of experimentation.
- 6) Purification of experimentation data.
- 7) Formulation of the mathematical model .
- 8) Optimization of Model.
- 9) Validation of the experimental data by ANN . The first six steps mentioned above constitute design of experimentation.[1]

The seventh step constitutes of model formulation where as eighth steps is for model optimization. The last step is ANN simulation of model.[5] In this analysis , Buckingham's π theorem is used to find different π terms are obtained and model get formulated .

3.0 Design of Experiments:

The engine test rig consist of afours troke four cylinder spark ignition engine, Naturally aspirated, Fiat Premier, made with K-2 series, 1.1 liter capacity with maximum output 39 KW. coupled with hydraulic dynamometer. The engine is fitted on a rigid bed and is coupled through a flexible coupling to hydraulic dynamometer which act as a loading device. The Engine is converted into bi-fuel mode for petrol and LPG fuel by installing bi-fuel conversion LPG kit. The converted engine will have flexibility of operating either on conventional petrol or LPG.

Fig. 1- Experimental Setup

In the present study, the experiments are designed on sequential classical experimental design techniques. The basic classical plan consist of holding all but one of the independent variable constant and varying this one variable over its widest range. Buckingham's π theorem is used to develop dimensionless π terms for reduction of engine

Variables .This theorem helps to better understand how the changes in the levels of any one process parameter of a π terms affects. The dependent responses for internal combustion engine operation. A response or dependant variable (Pf), as Frictional power loss defining the ability of engine to utilize the energy supplied and power developed .The correlation developed between Load (Ld), Speed(N), Frictional power (Pf), Viscosity(μ) are detailed discussed. A combination of the levels of parameters, which lead to maximum, minimum and optimum response the experiments consists of studying the relationship between seven independent process parameters with the dependent response as frictional

power loss optimization. Since simultaneous changing of these seven independent parameters was may become confusing. Hence all these independent engine parameters were reduced by dimensional analysis [6]

3.1 Formulation of Experimental data Base Model based on Dimensional Analysis:

The frictional power loss in engine cylinder is basically depends on –The frictional power (P_f) consumed during the test run can be considered as dependent upon speed (N), Load (L_d), Viscosity of engine oil (μ_o), Gravitational acceleration (g), Diameter of cylinder (D), and Density of Fuel (ρ_f).

Table 1- List of Operating Variables

S. N.	Parameter	Notation	MLT	Unit
1	Frictional power	P_f	ML 2T-3	Watt
2	Load on Engine	L_d	MLT^{-2}	N
3	Speed	N	T-1	s-1
4	Kinematic Viscosity of Engine Oil	μ_o	L^2T^{-1}	m^2/s
5	Cylinder diameter	D	L^1	m
6	Density of Fuel	ρ_f	ML^{-3}	Kg/m^3
7	Gravitational Acceleration	g	LT^{-2}	m/s^2

Step 1 : The Frictional Power Loss always affected by the various variables listed in the above table. Mathematically we have –

$$P_f = f_1 (L_d, N, D, \mu_o, \rho_f, g) \quad (I)$$

It can be written as ,

$$f_1 (P_f, L_d, N, D, \mu_o, \rho_f, g) = 0 \quad (II)$$

Therefore, total number of variables,

$n = 07$.

Number of fundamental dimensions, $m = 3$

total number of dimensionless π - terms = $7 - 3 = 4$ The four π terms are formed & hence equation (II) can be written as –

$$f_1 (\pi_1, \pi_2, \pi_3, \pi_4) = 0 \quad (III)$$

Step2: Each π term should consist of $m + 1$

variable, where $m = 3$ and also called repeating variable. Out of seven variables, three variables are to be selected as repeating variable and out of 03 repeating variable (One variable should have geometric property, second variable should have flow property and third variable should have fluid property). These three variable D, ρ_f , g are selected as repeating variables and the variable themselves should not form dimensionless term and should have fundamental dimension = m i.e. 3

Step 3 : Each π term is written according to the equation

$$\pi_1 = D^a \rho_f^b g^c P_f$$

$$\pi_2 = D^a \rho_f^b g^c L_d$$

$$\pi_4 = D^a \rho_f^b g^c \mu_o$$

$$\pi_3 = D^a \rho_f^b g^c N$$

Step 4 :The pi terms are determined by principle of dimensional homogeneity.[2]

We get the following dimensionless Pi-terms listed as following,

$$\begin{aligned}
 1) \quad & \frac{D^{7/2} \cdot \rho_f g^{3/2}}{Pf} \quad \pi_1 = \\
 2) \quad & \frac{L_d}{D^3 \rho_f g} \quad \pi_2 = \\
 & \frac{ND}{g^{1/2}} \quad 3) \pi_3 = \\
 & \frac{\rho_o}{D g} \\
 4) \quad & \pi_4 = 3/2
 \end{aligned}$$

Step 5 :Substituting the value of pi term in equation -III we get, $f_1(\pi_1, \pi_2, \pi_3, \pi_4) = 0$... (IV)

The relationship between various parameters was unknown. The dependent parameter, π_1 i.e. relating to Pf be an intricate relationship with remaining terms (ie. π_2 to π_4) evaluated on the basis of experimentation. The possible relation may be linear, log linear, polynomial with n degrees, linear with products of independent π_i terms. The raw data of the independent and dependent variables of the tested SI engine system has been collected during the experimentation. It is necessary to correlate quantitatively independent and dependent pi term involved in this engine system. This correlation is nothing but a mathematical model as a design tool for engine system analysis.. In this manner any complicated relationship can be evaluated and further investigated for error. Hence the relationship for Pf is formulated as:

$$\pi_1 = K_1 \cdot \pi_2^{a_1} \pi_3^{b_1} \pi_4^{c_1} \quad \text{. (V)}$$

There are four unknown terms in the above equation viz. constant of proportionality K_1 and indices $a_1, b_1, \text{ and } c_1$. To find the values of these unknowns, we required minimum four sets of values .

The Equation -VI can be simplify by taking log on both sides we get,

$\log(\pi_1) = \log k_1 + a_1 \log(\pi_2) + b_1 \log(\pi_3) + c_1 \log(\pi_4)$ Again simplify further and replace log terms by capital alphabet terms implies,

Let, $Z_1 = \log \pi_1$, $K_1 = \log k_1$, $A = \log(\pi_2)$,

$B = \log(\pi_3)$, $C = \log(\pi_4)$

Put the above values . equations , can be written as,

$$Z_1 = K_1 + a_1 A + b_1 B + c_1 C$$

To find the constant value of K_1 and indices a_1, b_1, c_1 The matrix method is used for solving these equations using ,matrix laboratory is given as below. Let, $A = 4 \times 4$ matrix of the multipliers of K_1, a_1, b_1, c_1

$B = 4 \times 1$ matrix of the terms on L.H.S. an

$C = 1 \times 4$ matrix of solutions or values of K_1, a_1 and b_1, c_1

Then, $C = \text{inv}(A) \cdot B$

It gives the values of K_1, a_1, b_1 and C_1 , we have determined antilog of K_1, a_1 and b_1 will be the solution . A probable mathematical form for this phenomenon could be,

$$\pi_1 = 0.9967 \cdot \pi_2^{0.84561} \pi_3^{b_0.9615} \pi_4^{-0.4490} \quad \text{. (VI)}$$

This is true linear relationship between A to C to reveal

1. Applying the theories of multiple regression analysis, the aim is to minimize the error $(E) = Y_e - Y_c$. Y_c is the computed value of π_1 by using multiple regression equation and Y_e is the value of the same term being obtained from experimental data of π_1 . An attempt has been made here using

experimental data base modelling. The value of indices indicates that how the behaviour of engine phenomenon is get affected due to interaction of various independent \square terms in the suggested model. After implementation the experimental data for suggested data base model ,following table gives the comparison of computed and predicted values for frictional power loss for gasoline fuel.

Table2-Computed values of dimensionless Pi-Terms.

Run Order	Pi-term for Response(Pf)	Pi-term for Load.	Pi-term for speed	Pi-term for Viscosity
1	254.6481	3.428	1.92	0.00018
2	248.5882	3.428	2.133	0.00023
3	254.8008	3.428	2.333	0.0003
4	441.5969	6.855	1.92	0.00023
5	467.8675	6.855	2.133	0.0003
6	471.2112	6.855	2.333	0.00018
7	555.0674	10.28	1.92	0.0003
8	652.4717	10.28	2.133	0.00018
9	700.4184	10.28	2.333	0.00023
10	764.7203	13.71	1.92	0.00018
11	800.3366	13.71	2.133	0.00023
12	835.0944	13.71	2.333	0.0003
13	897.5923	17.14	1.92	0.00023
14	997.7376	17.14	2.133	0.0003
15	1050.679	17.14	2.333	0.00018
16	1108.484	20.57	1.92	0.0003
17	1157.574	20.57	2.133	0.00018
18	1185.355	20.57	2.333	0.00023
19	1232.274	23.99	1.92	0.00018
20	1268.7	23.99	2.133	0.00023
21	1347.276	23.99	2.333	0.0003
22	1330.058	27.42	1.92	0.00023
23	1404.18	27.42	2.133	0.0003
24	1401.043	27.42	2.333	0.00018
25	1562.828	30.85	1.92	0.0003
26	1539.661	30.85	2.133	0.00018
27	1710.746	30.85	2.333	0.00023

The Experimental values and Predicted value calculated from the mathematical model are given in following table.3 which shows that both values are nearly close to each other.

Run Order	Experimental - Pf in Watt	Predicted -Pf in Watt	Error (E) = Ye-Yc.	%Error
1	616.88	617.313	-0.4332	0.00433
2	602.2	603.438	-1.2377	0.01238

3	617.25	586.745	30.5048	0.30505
4	1069.76	979.913	89.8468	0.89847
5	1133.4	967.344	166.056	1.66056
6	1141.5	1338.05	-196.55	1.96549
7	1344.64	1231.65	112.989	1.12989
8	1580.6	1729.64	-149.04	1.49038
9	1696.75	1665.35	31.3985	0.31398
10	1852.52	1993.46	-140.94	1.4094
11	1938.8	1948.65	-9.8525	0.09852
12	2023	1894.75	128.252	1.28252
13	2174.4	2126.6	47.8007	0.47801
14	2417	2099.32	317.679	3.17679
15	2545.25	2903.82	-358.57	3.58573
16	2685.28	2213.29	471.988	4.71988
17	2804.2	3108.18	-303.98	3.03979
18	2871.5	2992.66	-121.16	1.21157
19	2985.16	3199.78	-214.62	2.14622
20	3073.4	3127.86	-54.46	0.5446
21	3263.75	3041.34	222.414	2.22414
22	3222.04	3164.39	57.6534	0.57653
23	3401.6	3123.8	277.804	2.77804
24	3394	4320.9	-926.9	9.26898
25	3785.92	3118.47	667.451	6.67451
26	3729.8	4379.34	-649.54	6.49542
27	4144.25	4216.57	-72.323	0.72323

Table3-Experimental & Predicted values .

4.0 Validation by Artificial Neural Network (ANN) : ANN or Artificial Neuron Network is a computational model based on the structure and functions of biological neural networks. Information that flows through the network affects the structure of the ANN because a neural network changes or learns, in a sense - based on that input and output. ANN's have three layers that are interconnected. The first layer consist of input neurons. Those neurons send data on to the second layer, which in turn sends the output neurons to the third layer.

Fig.2 shows an artificial neural network is an interconnected group of nodes, inspired by a simplification of neurons in a brain. Here, each circular node represents an artificial neuron, an arrow represents a connection from the output of one artificial neuron to the input of another[12,14]

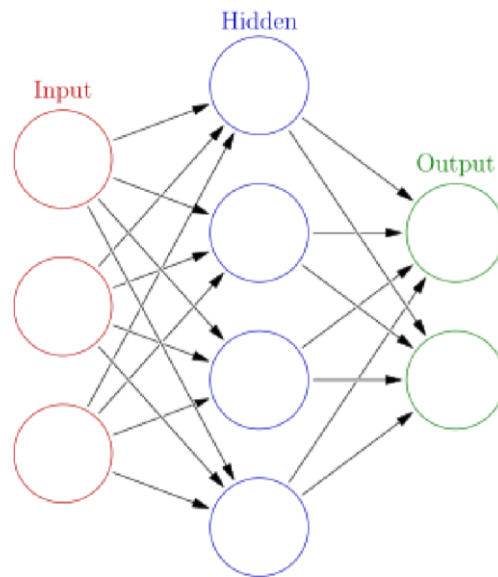


Fig.2- Artificial Neural network is an interconnected group of nodes.[13]

An ANN has several advantages but one of the most recognized of these is the fact that it can actually learn from observing data sets. In this way, ANN is used as a random function approximation tool. These types of tools help to estimate the most cost-effective and ideal methods for arriving at solutions while defining computing functions or distributions. ANN takes data samples rather than entire data sets to arrive at solutions, which saves both time and money. ANN are considered fairly simple mathematical models to enhance existing data analysis technologies. Prediction of future results are the major problems in analysis. The experimental data-based modelling achieved this through mathematical model for the dependent pi term. In such complex phenomenon involving non-linear systems it is also planned to develop using artificial neural network (ANN)[10]. The results for Training, Testing for best fit curve, and Validation by ANN are shown in fig.3 and 4.

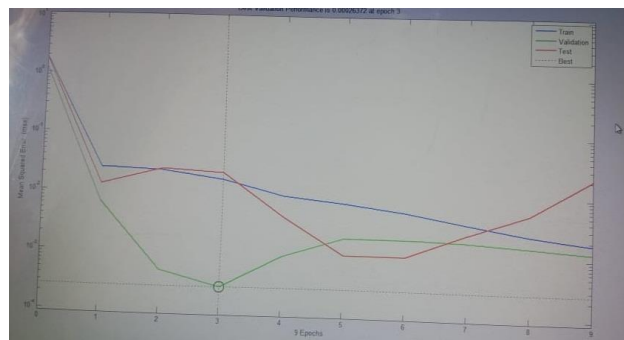


Fig.3 -Training and validation of ANN.

Output Vs Traget, R=0.93296

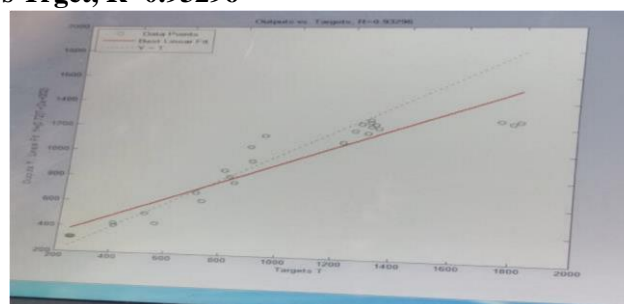


Fig.4-Testing for best fit curve

5. Conclusion

Here the Prediction model of gasoline engine for frictional power loss have been developed using artificial neural networks. From this model, we can conclude the following outcomes.

- 1). The dimensionless π terms or their group have provided the firm guidelines about combined effect of process parameters in that π terms a group helps the designer to select input variables.
- 2). The mathematical models developed with dimensional analysis for different combinations of parameters only for cutting speed, Brake Load and Lubrication oil viscosity can be effectively utilized to study the friction behavior for the engine.
- 3). In this Analysis, Standard errors of the predicted values of the response or dependent variable are found to be very low and these values are very closer averagely within 5%. The models have been formulated mathematically for the controlled environment (on stationary engine). The comparison of values of dependent term obtained from experimental data and from the mathematical model is given in Table 2. From the values of % errors, it seems that the mathematical models can be successfully used for the computation of dependent terms for a given set of independent terms.
- 4). The absolute index of π_3 highest index of π_1 viz. 0.9615. The term ' π_3 , related to engine speed, hence the engine speed parameters is the most influencing variable in this model.

From the obtained formulated mathematical model we may select a proper input as speed, load and various SAE multi-grade lubrication oils. ANN is a performance tool that saves economy and time in developing new models and methodologies for overall engine or any process management.

6.0 Future Scope:

Further research work may be performed by utilization of the above work with LPG fuel in internal combustion engine to improve the performance and to reduce the friction of engine.

Acknowledgement:

Authors are sincerely thankful to the SSBT's, College of Engineering and Technology, Bambhori, Jalgaon as well as KBC North Maharashtra University, Jalgaon for providing laboratory and central library facility. The authors also acknowledge help and cooperation from the staff and colleagues for useful support.

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