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Impatient customers in an M/M/1 Queue with working vacation and Multiple Vacation

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Abstract

In this paper, using Probability generating function method, we derive an Impatient customers in an $M/M/1$ queue with working vacation and Multiple vacation. Further, we obtain the distributions of the additional queue length and the sojourn time of a customer in the stationary state. Numerical examples are given to demonstrate the system stability.

KEYWORDS: M/M/1 queue, Working vacation , Stochastic decomposition.
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1 . Introduction

In Lin et al. (2009) investigated the M/M/c queue with single working vacation. Majid and Manoharan (2017), have thought about M/M/c queue with single and different coordinated working vacation and Impatient Customers. Manoharan and Jeeva(2018), have inspected the Single Server Markovian Queuing System with Vacation Interruptions and Working Vacations with Setup time and broadened the work with waiting server and set up times.

Queueing models with clients anxiety have been concentrated by different creators, for example, Altman and Yechiali (2006, 2008), Boxma and de Waal (1994), Yechiali (2007), Baccelli et al. (1984), Daley (1965), Van Houdt et al. (2003), Yue et al. (2011,2014), where the reason for anxiety was either a considerable delay previously experienced in the line, or a significant delay foreseen by a client upon appearance. When the clock terminates, if the server neglects to restore, the clients exits from the line and never returns. Then again, if the server returns the due time, the client remains in the framework until the finishing of his administration.

In section 2 and 3 of this paper is described the model and some of the performance measures have been been calculated.

2 . Model description

In this paper, The customers arriving during the working period are served in slower rate μ_v. The regular busy period starts after the vacation time is over. The customers becoming impatient due to unavailability of server during the working vacation period due slower service rate. The impatient times are exponentially distributed at rate $n\alpha$. If

$$J(t)=\begin{cases}0, & \text{if the server is in working vacation period},\\ 1, & \text{if the server is in multiple vacation period},\\ 2, & \text{if the server is busy in normal busy period}.\end{cases}$$

Then $\{(Q(t),J(t));t\geq 0\}$, where the vacation period, and non vacation are denoted by the states

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$(K,0)K \geq 0$, $(K,1)K \geq 1$, (0,1) and $(K,2)K \geq 1$,respectively and K denotes the number of customers.

$$(\lambda+\gamma)p_{00} = \mu_b p_{12} + (\mu_v+\alpha)p_{10} \quad (1)$$
$$(\lambda+\mu_v+n\alpha+\gamma)p_{n0} = \lambda p_{(n-1)0} + (\mu_v+(n+1)\alpha)p_{(n+1)0}, n \geq 1 \quad (2)$$
$$\lambda p_{01} = \gamma p_{00} \quad (3)$$
$$(\lambda+\theta)p_{n1} = \lambda p_{(n-1)1} n \geq 1 \quad (4)$$
$$(\lambda+\mu_b)p_{12} = \theta p_{11} + \mu_b p_{22} + \gamma p_{10} \quad (5)$$
$$(\lambda+\mu_b)p_{n2} = \theta p_{n1} \lambda p_{(n-1)2} + \mu_b p_{(n+1)2} + \gamma p_{n0}, n \geq 2 \quad (6)$$

Let as define the probability generating functions

$$F_0(z) = \sum_{n=1}^{\infty} p_{n0} z^n; F_1(z) = \sum_{n=1}^{\infty} p_{n1} z^n; F_2(z) = \sum_{n=2}^{\infty} p_{n2} Z^n \quad (7)$$

with

$$F_0(1) + F_1(1) + F_2(1) = 1 \text{ and } F_{0'} = \sum_{n=1}^{\infty} nZ^{n-1} p_{n0}$$

Using (1) and (2)

$$(\lambda+\mu_v+\gamma)\sum_{n=1}^{\infty} p_{no} z^n + \alpha \sum_{n=1}^{\infty} p_{no} z^n + (\lambda+\gamma)p_{00} = \lambda z \sum_{n=1}^{\infty} p_{(n-1)0} z^n$$
$$+\frac{\mu_b}{z}\lambda z \sum_{n=1}^{\infty} p_{n+1,0} z^{n+1} + \alpha \sum_{n=1}^{\infty} (n+1)p_{n+1,0} z^n + \mu_b p_{12} + (\mu_v+\alpha)p_{10}$$

$$\alpha z(1-z)F_{0'}(z) + (\lambda z^2 - (\lambda+\mu_v+\gamma)z + \mu_v)F_0(z) + \mu_b p_{12} z - \mu_v(1-z)p_{00} = 0 \quad (8)$$

Multiplying the appropriate power of z^n in (4)

$$(\lambda+\theta)\sum_{n=1}^{\infty} p_{n1} z^n = \lambda z \sum_{n=1}^{\infty} p_{n-1,1} z^n$$
$$(\lambda+\theta-\lambda z)F_1(z) = (\lambda+\theta)p_{01} \quad (9)$$

From (5) and (6)

$$(\lambda+\mu_b)\sum_{n=2}^{\infty} p_{n2} z^n = \lambda z \sum_{n=2}^{\infty} p_{(n-1)2} z^n + \frac{\mu_b}{z}\sum_{n=2}^{\infty} p_{(n+1)2} z^{n+1} + (\lambda+\mu_b)p_{12}$$
$$+\gamma \sum_{n=2}^{\infty} p_{n0} z^n + \theta p_{11} + \mu_b p_{22} + \gamma p_{10} + \theta \sum_{n=2}^{\infty} p_{n1} z^n$$
$$(\lambda z - \mu_b)(1-z)F_2(z) = (\gamma F_0(z) + \theta F_1(z))z - (\gamma p_{00} + \theta p_{01}) \quad (10)$$
$$F_1'(z) = \frac{d}{dz}F_1(z),$$

Let we derive the solution of the differential equation (8),
If $z \neq 1$, then

$$F_{0'}(z) + \left\{\frac{-\lambda}{\alpha} + \frac{\mu_v}{\alpha z} - \frac{\gamma}{\alpha(1-z)}\right\}F_0(z) = \frac{-1}{\alpha}\left(\frac{\mu_b P_{12}}{1-z} - \frac{\mu_b P_{00}}{z}\right) \quad (11)$$

The integrating factor is

$$F_0(z) = \frac{-(\mu_b P_{12})K_1(z) + (\mu_v P_{00})K_2(z)}{\alpha e^{-\left(\frac{\lambda}{\alpha}\right)z}(1-z)^{\frac{\gamma}{\alpha}} z^{\frac{\mu_v}{\alpha}}} \quad (12)$$

where

$$K_1(z) = \int_0^z e^{-\left(\frac{\lambda}{\alpha}\right)x}(1-x)^{\frac{\gamma}{\alpha}-1} x^{\frac{\mu_v}{\alpha}} dx \quad (13)$$
$$K_2(z) = \int_0^z e^{-\left(\frac{\lambda}{\alpha}\right)x}(1-x)^{\frac{\gamma}{\alpha}} x^{\frac{\mu_v}{\alpha}-1} dx \quad (14)$$

3. Performance measures

For $j = 0,1,2$, let $P_j = F_{ij}(z) = \sum_{n=0}^{\infty} P_{jn}$, and let $F(\alpha_j) = F_{j'}(x) = \sum_{n=1}^{\infty} nP_{jn}$. Then from (5)and (6), we get

$$\gamma P_{00} + \theta P_{01} = \gamma F_0(1) + \theta F_1(1) \quad (15)$$

$$F_2(z) = \frac{(\gamma F_0(z)+\theta F_1(z))z-(\gamma F_0(1)+\theta F_1(1))z}{(\lambda z-\mu_b)(1-z)}$$

Using L'Hospital rule, we find

$$F_2(1) = \frac{\gamma F_{0'}(1)+\theta F_{1'}(1)}{\mu_b-\lambda} \tag{16}$$

where, $F_{0'}(1) = E(L_0)$, $F_2(1) = 1 - F_1(1) - F_0(1)$

Substituting the above values in (16), we get

$$1 - F_1(1) - F_0(1) = \frac{\gamma E(L_0)+\theta E(L_1)}{\mu_b-\lambda}$$

$$E(L_0) = \frac{(\mu_b-\lambda)}{\gamma}(1 - F_1(1) - F_0(1)) - \frac{\theta}{\gamma}E(L_1) \tag{17}$$

so that $E(L_0)$ can be obtained by adding (3),(4) and (6) and by rearranging the terms, we obtain

$$\lambda P_{n0} + \lambda P_{n1} + \lambda P_{n2} - [(\mu_v + (n+1)\alpha)P_{n+1,0} + \mu_b P_{n+1,2}]$$
$$= \lambda P_{n-1,0} + \lambda P_{n-1,1} + \lambda P_{n-1,2} - [(\mu_v + (n-1)\alpha)P_{n0} + \mu_b P_{n2}], n \ge 1$$

$$\lambda P_{n0} + \lambda P_{n1} + \lambda P_{n2} = [(\mu_v + (n+1)\alpha)P_{n+1,0} + \mu_b P_{n+1,2}], n \ge 0 \tag{18}$$

Adding over all possible values of n in (19), we get

$$\lambda F_0(1) + \lambda F_1(1) + \lambda F_2(1) = \mu_b F_2(1) + \nu_v(F_0(1) - P_{00}) + \alpha \sum_{n=0}^{\infty} (n+1)P_{n+1,0}, n \ge 0 \tag{19}$$

But $E(L_0) = \sum_{n=0}^{\infty} (n+1)P_{n+1,0}$ and $F_2(1) = 1 - F_0(1) - F_1(1)$

Substituting the values of $E(L_0)$ from (17) in (19) and solving we get

$$(\alpha+\gamma)(\mu_b - \lambda) = [\alpha(\mu_b - \lambda) - (\mu_v - \mu_b)\gamma]F_0(1) + (\alpha(\mu_b - \lambda) + \mu_b\gamma)F_1(1) + \mu_v\gamma P_{00} + \theta\alpha E(L_1) \tag{20}$$

Taking limit $z \to 1$ in (12), and as $0 \le F_0(1) = \sum_{n=0}^{\infty} P_{n0} \le 1$ and $\lim_{z\to 1}(1-z)^{\frac{-\gamma}{\alpha}} \to \infty$, we must have

$$P_{00} = \frac{\gamma}{\mu_v}F_0(1)\frac{K_1(1)}{K_2(1)} \tag{21}$$

Equation (9) can be simplified as

$$F_1(1) = \frac{(\lambda+\theta)\gamma}{\lambda\theta}P_{00} \tag{22}$$

$$E(L_1) = F_{1'}(1) = \frac{(\lambda+\theta)\gamma}{\theta}P_{00}$$

$$F_{2'}(z) = \frac{[\lambda(1-z)-(\lambda z-\mu_b)]}{[(\lambda z-\mu_b)(1-z)]^2}$$

$$= \frac{(\mu_b-\lambda)[(\lambda F_{0''}(1)+\theta F_{1''}(1))+\lambda(\gamma F_{0'}(1)+\theta F_{1'}(1))]}{2(\mu_b-\lambda)^2}$$

$$E(L_2) = \frac{\mu_b(\lambda F_{0''}(1)+\theta F_{1''}(1))+\lambda[(\gamma F_{0'}(1)+\theta F_{1'}(1))-(\gamma F_{0''}(1)+\theta F_{1''}(1))]}{2(\mu_b-\lambda)^2} \tag{23}$$

$$E(L) = E(L_0) + E(L_1) + E(L_2) \tag{24}$$

$$E(w) = \frac{1}{\lambda}E(L) \tag{25}$$

Which are expected queue length of the system and expected waiting time of a customer in the system respectively.

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