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Pilot Decontamination Approach in Massive MIMO Systems

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Abstract

We considered the massive MIMO-OFDM system to minimize the impact that can be reduced on pilot pollution in the uplink channel estimation. A proposal of a pilot scheme for three interfering cells by injecting zero pilot symbols in the training sequences is presented. In that proposal, the ML channel estimation is executed merging with Cramer Rao Bound with the OFDM sub-carriers. The simulations demonstrate the methodology, which can help to decrease the influence of pilot pollution and also it is easier to implement the pilot multiplexing.

Keywords: *Massive MIMO, SU-MIMO, MU-MIMO, OFDM, Pilot Contamination, Cramer Rao Bound*

1. Introduction

Massive multiple-input multiple outputs (MIMO) have aroused as the leading technology in the last decade for the future wireless communication system, which can achieve spectral efficiency and high data rates [1-3]. In comparison with single-user MIMO, the multi-user MIMO with a large antenna array is more desirable. From recent experiments, it has been observed that, if the BS is equipped with a large number of antenna arrays (i.e. tens, hundreds or thousands), the communication system can attain energy efficiency, huge gain, security, and throughput. Massive MIMO is a bright source of research problems. Because it has the capability of providing EE, coverage, improved link reliability, high data rates it has got the attraction of both industry and academia. Despite the benefits we can get from massive MIMO, there are still a lot of problems, which need to be tackled[4]. Massive MIMO is a bright source of research problems. Because it has the capability of providing EE, coverage, improved link reliability, high data rates it has got the attraction of both industry and academia. Despite the benefits we can get from massive MIMO, there are still a lot of problems, which need to be tackled. If OFDM is combined with the massive MIMO, the existing system's performance will increase further [3]. In a massive MIMO cellular system, each BS is equipped with a large number of transceiver antennas, thus greater diversity and multiplexing gains can be attained in the uplink and downlink, in comparison to the conventional MIMO equipped with small arrays of antennas. Hence fast fading and additive white Gaussian noise (AWGN) effects are reduced substantially.

Also because of inadequate orthogonal pilot sequences and dense network organization, the performance of the channel estimation will greatly be affected by the re-use of the pilot patterns in the desired and neighboring cells [3, 5]. The users re-use the pilot sequence in multi-cells and transmit towards the desired BS which cannot extricate efficiently, thus it will cause pilot contamination [6].

The effect of pilot contamination can be minimized by the non-linear algorithm presented in [7]. [8] Studied the sum rate lower bound in the presence of pilot contamination. Usually, the channel can be estimated by two methods, the training based algorithms, and blind channel estimation algorithms. The performance of training based algorithms is better than the blind algorithms [9].

The pilot patterns, in general, can be scattered pilot pattern, block pilot pattern, diamond pilot pattern and comb type pilot pattern. Each pilot pattern has an altered effect on the estimation of the channel [10]. To mitigate pilot contamination in a massive MIMO system, it is necessary to carefully design the pilot pattern scheme. A semi-time shifted comb types pilot pattern [11]has been investigated for the multi-cell massive MIMO-OFDM system with less number of sub-carriers[12, 13].

The pilot contamination can be greatly reduced if an appropriate pilot scheme is designed. In this work, we aim to appraise the circumstances in which the pilot contamination is a substantial problem in practical systems and find solutions to alleviate this problem. Motivated from this, we proposed a time-shifted comb-type pilot scheme in a multi-cell massive MIMO-OFDM system. An unbiased ML estimator is employed to estimate the channel, also Cramer Rao Bound[14] is used to minimize the variance of the ML estimator by the Fisher Information matrix.

2. System Model

An uplink multi-cell system has been considered including the interfering cells and the desired cell as shown in figure 24. Whereas, in each cell, the BS is equipped with large antenna arrays, which can be determined to receive antennas and transmit antennas. Moreover, it is assumed that the BS in each cell will serve the single-antenna users. We also satisfied the condition of a large-scale MIMO system. To communicate with the BS, each user will use OFDM pilot symbols in the uplink to estimate the channel. The length of orthogonal pilot sequences must be equal or less than the length of OFDM symbols, which allow demonstrating the pilot matrix's inverse non-singular. It is anticipated, that the orthogonal pilot sequences are assigned to users in the desired cell. Though, the neighboring cell will re-use the same pilot sequence which will result in pilot pollution. Subsequently, the users in the desired cell will encounter interference from the users of neighboring cells, sharing the same orthogonal pilot sequence.

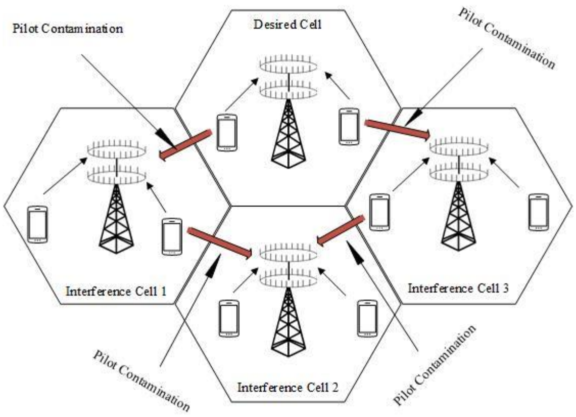

Figure 1. Pilot Contamination Scenario in Multi-cell Massive MIMO System

3. Signal Model

Let $x_1, x_2 \cdots x_S$ be pilot/training symbols, and $y_1, y_2 \cdots, y_S$ be observed in output pilot symbols. So, the pilot outputs will be

$$\begin{bmatrix} y_{(1)} \\ y_{(2)} \\ \vdots \\ y_{(S)} \end{bmatrix} = \begin{bmatrix} h \\ \mathrm{h} \\ \vdots \\ \mathrm{h} \end{bmatrix} + \begin{bmatrix} q_{(1)} \\ q_{(2)} \\ \vdots \\ q_{(S)} \end{bmatrix}$$

We will get

$$y_S = h + q_S \tag{1}$$

Where y_S is Gaussian output $(h; \sigma^2)$ and q_S is Gaussian noise $CN(0, \sigma^2)$, $q_1, q_2, \cdots, q_S$ are i.i.d (Independent Identically Distributed). Since $q_1, q_2, \cdots, q_S$ are independent, so the output $y_1, y_2, \cdots, y_S$ are also independent.

4. ML Estimate

The modular conceptualization is to measure and conclude statistical parameters. In estimating MLE has various key properties: sufficiency; consistency; efficiency; and parameterization in variance. Additionally, most of the statistical inference techniques are formulated based on MLE. For example, the chi-square test, the G-square test, Bayesian methods, inference with lost data, random effect modeling, MLE is a prerequisite, as are other requirements for model selection, such as the Akaike knowledge criterion [15] and the Bayesian information criteria [16].

In R.A. 1920s. Fisher introduced the basics of maximum probability estimation (MLE), that describes the optimal distribution of probability that makes the observed data "most likely”, which means the numerical quantity of the parameter vector that maximizes the probability function is ideally desired. The MLE estimate is commonly regarded as the resultant vector parameter, which is seek-after by looking for the multi-dimensional space parameter. To reiterate, estimating maximum likelihood is a process of searching for the most presumptive distribution of probability which produces the observed data.

Joint PDF of observations is given as:

$$F_{Y(\mathrm{s})}{}^{(\mathrm{y(s)})} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{1}{2\sigma^2}(y_{(\mathrm{s})}-h)^2} \tag{2}$$

Now Product of individuals PDFs of $y_1, y_2, \cdots, y_S$

$$F_{Y(1)}{}^{(y(1))} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(y_{(1)}-h)^2}$$

$\times$

$$F_{Y(2)}{}^{(y(2))} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(y_{(2)}-h)^2} \quad (3)$$

$\times$

$$F_{Y(S)}{}^{(y(s))} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(y_{(n)}-h)^2}$$

We will get

$$F_{Y(S)}{}^{(y(s))} = (\frac{1}{\sqrt{2\pi\sigma^2}})^{\frac{n}{2}} e^{-\frac{1}{2\sigma^2}\sum_{s=1}^{N}(y_{(s)}-h)^2} \quad (4)$$

This is the Joint PDF of observations, which are also called the likelihood function of channel coefficient 'h'.

We can define $F_{(y(1),y(2),\dots,y(S))}{}^{y(1)y(2)\dots y(S)} = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^S e^{-\frac{1}{2\sigma^2}\sum_{s=1}^{S}(y_s-h)^2}$ as $L(\bar{y};h)$

$$\bar{y} = \begin{pmatrix} y(1) \\ y(2) \\ \vdots \\ y(n) \end{pmatrix} \quad (5)$$

Now we will maximize this $L(\bar{y};h)$ likelihood function to find the maximum likelihood estimate which is known as ML.

Taking Natural log of likelihood function $L(\bar{y};h)$

$$L(\bar{y};h) = \log(\bar{y};h) \ or \ \ln(\bar{y};h) \quad (6)$$

When the function is maximum, its log is also maximum.

$$\ln L(\bar{y};h) = \ln((\frac{1}{2\pi\sigma^2})^{S/2} e^{-\frac{1}{2\sigma^2}\sum_{s=1}^{S}(y(s)-h)^2}) \quad (7)$$

$$\ln L(\bar{y};h) = -\frac{S}{2}\ln 2\pi\sigma^2 - \frac{1}{2\sigma^2}\sum_{s=1}^{S}(y_{(s)}-h)^2 \quad (8)$$

Now we will minimize $\sum_{s=1}^{S}(y(s)-h)^2$ this part. Now we will differentiate and set equate to zero.

$$\frac{\partial}{\partial h}\sum_{s=1}^{S}(y_s-h)^2 = \sum_{s=1}^{S}2(y_s-h) = 0 \quad (9)$$

$$\sum_{s=1}^{N} y(s) = Sh \quad (10)$$

$$\widetilde{h_{ML}} = \frac{1}{S}\sum_{s=1}^{S} y_S \tag{11}$$

4.1. Cramer Rao Lower Bound

$\mathrm{L}(\bar{y};h)$

Where 'h' is an unknown parameter and y is a vector of observation

$$\bar{y} = \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(S) \end{bmatrix}$$

$\mathrm{L}(\bar{y};h)$ Is PDF w.r.t observation vector.

Hence the integral of PDF is 1, this PDF must integrate to 1.

$$\int_{-\infty}^{\infty} \mathrm{L}(\bar{y};h)d\bar{y} = 1 \tag{12}$$

Now differentiate w.r.t 'h'

$$\frac{\partial}{\partial h}\int_{-\infty}^{\infty} \mathrm{L}(\bar{y};h)d\bar{y} = \frac{\partial}{\partial h}1 = 0 \tag{13}$$

$$\int_{-\infty}^{\infty} \frac{\partial L(\bar{y};h)}{\partial h} d\bar{y} = 0 \tag{14}$$

Multiplying & Dividing by $\mathrm{L}(\bar{y};h)$

$$\int_{-\infty}^{\infty} \frac{1}{\mathrm{L}(\bar{y};h)} \frac{\partial L(\bar{y};h)}{\partial h} d\bar{y} = L(\bar{y};h)d\bar{y} = 0 \tag{15}$$

$$\frac{\partial}{\partial h}\ln \mathrm{L}(\bar{y};h) \tag{16}$$

$$\int_{-\infty}^{\infty} \frac{\partial}{\partial h}\ln \mathrm{L}(\bar{y};h)\,\mathrm{L}(\bar{y};h)d\bar{y} = 0 \tag{17}$$

Multiplying by 'h' on both sides

$$h \times \int_{-\infty}^{\infty} \frac{\partial}{\partial h}\ln \mathrm{L}(\bar{y};h)\,\mathrm{L}(\bar{y};h)d\bar{y} = h \times 0 = 0 \tag{18}$$

$$\int_{-\infty}^{\infty} h\frac{\partial}{\partial h}\ln \mathrm{L}(\bar{y};h)\,\mathrm{L}(\bar{y};h)d\bar{y} = 0 \tag{19}$$

Considering now an unbiased estimator

$E\{\hat{h}\} = h$

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$$\int_{-\infty}^{\infty} \hat{h}\,\mathrm{L}(\bar{y};h)\,d\bar{y} = h \text{ Average value of } \hat{h}$$

Now differentiate both sides with 'h', we have

$$\frac{\partial}{\partial h}\int_{-\infty}^{\infty} \hat{h}\,\mathrm{L}(\bar{y};h)\,d\bar{y} = \frac{\partial}{\partial h} h = 1 \tag{20}$$

$$\frac{\partial}{\partial h}\int_{-\infty}^{\infty} \hat{h}\,\mathrm{L}(\bar{y};h)\,d\bar{y} = 1 \tag{21}$$

Multiplying & Dividing by $\mathrm{L}(\bar{y};h)$

$$\int_{-\infty}^{\infty} \hat{h}\frac{1}{\mathrm{L}(\bar{y};h)}\frac{\partial}{\partial h}\mathrm{L}(\bar{y};h)\,\mathrm{L}(\bar{y};h)\,d\bar{y} = 1 \tag{22}$$

$$\int_{-\infty}^{\infty} \hat{h}\frac{\partial}{\partial h}\ln\mathrm{L}(\bar{y};h)\,\mathrm{L}(\bar{y};h)\,d\bar{y} = 1 \tag{23}$$

Subtracting Equation 19 from equation 23. We will obtain,

$$\int_{-\infty}^{\infty} \hat{h}\frac{\partial}{\partial h}\ln\mathrm{L}(\bar{y};h)\,\mathrm{L}(\bar{y};h)\,d\bar{y} = 1 \tag{24}$$

$$\int_{-\infty}^{\infty} h\frac{\partial}{\partial h}\ln\mathrm{L}(\bar{y};h)\,\mathrm{L}(\bar{y};h)\,d\bar{y} = 0 \tag{25}$$

We will get

$$\int_{-\infty}^{\infty} (\hat{h}-h)\frac{\partial}{\partial h}\ln\mathrm{L}(\bar{y};h)\,\mathrm{L}(\bar{y};h)\,d\bar{y} = 1 \tag{26}$$

Where $(\hat{h}-h)$ is estimation error and $\frac{\partial}{\partial h}\ln\mathrm{L}(\bar{y};h)$ is derivative of the log-likelihood function.

$$E\{(\hat{h}-h)\partial\ln\mathrm{L}(\bar{y};h)\} = 1 \tag{27}$$

For two random variables x,y

$$E\left\{x^2\right\}E\left\{y^2\right\} \geq E^2\left\{xy\right\} \tag{28}$$

Cauchy Schwarz Inequality for random variables

$$E\{(\hat{h}-h)\}\,\mathrm{E}\{\frac{\partial}{\partial h}\ln\mathrm{L}(\bar{y};h)\} \geq E^2\left\{(\hat{h}-h)\frac{\partial}{\partial h}\ln\mathrm{L}(\bar{y};h)\right\} = (1)^2 = 1 \tag{29}$$

$$E\{(\hat{h}-h)\}\,\mathrm{E}\{(\frac{\partial}{\partial h}\ln\mathrm{L}(\bar{y};h))^2\} \geq 1 \tag{30}$$

$$E\{(\hat{h}-h)\} \geq \frac{1}{\mathrm{E}\{(\frac{\partial}{\partial h}\ln\mathrm{L}(\bar{y};h))^2\}} \tag{31}$$

Where $E\{(\hat{h}-h)\}$ is the variance of estimator & $\frac{1}{E\{(\frac{\partial}{\partial h}\ln L(\bar{y};h))^2\}}$ is Cramer Rao Bound

$$E\{(\frac{\partial}{\partial h}\ln L(\bar{y};h))^2\} = I(h) \quad (32)$$

Whereas, $I(h)$ is Fisher information of the parameter.

$$E\{(\hat{h}-h)\} \geq \frac{1}{I(h)} = \frac{1}{E\{(\frac{\partial}{\partial h}\ln L(\bar{y};h))^2\}} \quad (33)$$

4.2. Pilot Schemes

For the uplink, a comb-type pilot pattern is proposed in figure 25. A multi-cell system is considered for the desired cell and 3 interfering cells. Each line in Figure 25 represents the OFDM symbol. The first line represents the desired cell, where the second, third and fourth lines represent the second, third and fourth interfering cells respectively.

From Figure 25, we can see that a zero pilot symbol is inserted between the neighbor training symbol X_{i1} and X_{i2} (i=1, 2, 3) in the transmitter of the uplink. After inserting a zero pilot symbol, the neighboring subcarrier spacing between training symbols will become 2τ. $Num_{pilot} = [^{N_{sc}}/_{2\tau}]$ represents the number of non-zero training symbols, whereas [.] corresponds to the smallest integer. The number of a subcarrier for data transmission is reduced to Num_pilot after we insert zero pilot symbol. The same training sequence is used in the desired cell along with interfering cell 1, with different (time-shifted) pilot positions. The interfering cell 1, 2 and 3 employs the same pilot positions with different orthogonal training sequences. Therefore the desired cell does not encounter pilot interference from interfering cell 1 and 2, this by employing this proposed pilot pattern pilot contamination is reduced.

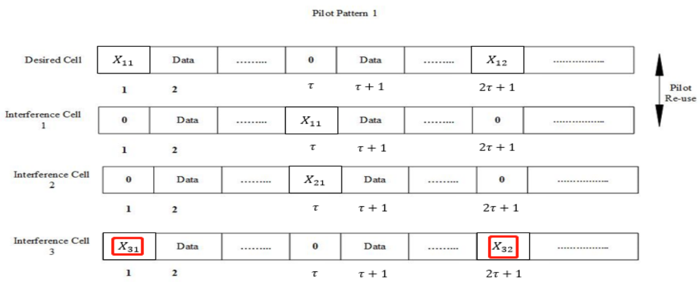

Figure 2. Pilot Scheme with Zero Symbols

Figure 3. Pilot Scheme 2 with all Cells Pilots Overlapping

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5. Simulation Scenario

With a desirable cell as well as three interfering cells, the simulation assumption is compared. Each cell includes an uplink user. The number of OFDM subcarriers is N_{sc} , the cyclic prefix length is N_g. QPSK modulation is utilized for data symbols in each OFDM symbol. The taps of the multi-path channel adapt the exponential function model is DS, where each path of the channel adapts Rayleigh distribution. Simulation parameters are provided in table 2.

Table 1. Simulation Parameters for Massive MIMO-OFDM

S/No	Parameters	Values
1	M_r	200
2	M_t	4
3	N_{sc}	128
4	N_g	16
5	DS	6
6	Num_pilot	4

5.1. Results

In the uplink the receiver first receives the OFDM training sequences to calculate the channel matrix of the desired cell and estimate the channel by the ML-CRB methods. In this study we calculated the MSE and BER to assess the performance of the system.

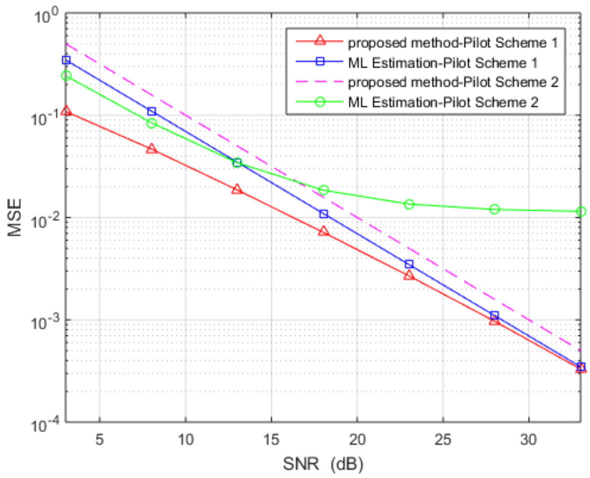

Figure 4. MSE Comparison of Pilot Schemes

In figure 4 MSE is simulated with $M_t = 4$,and $M_r = 200$ for the ML and ML-CRB for pilot scheme 1 and pilot scheme 2. Zero pilot symbols are inserted so that the interference can be avoided among desired cell and interfering cell 1. From the figure we can observe that the pilot scheme 1 gives better performance if compared with pilot scheme 2 with ML method and the performance is improved more if ML-CRB with scheme 1 is simulated.

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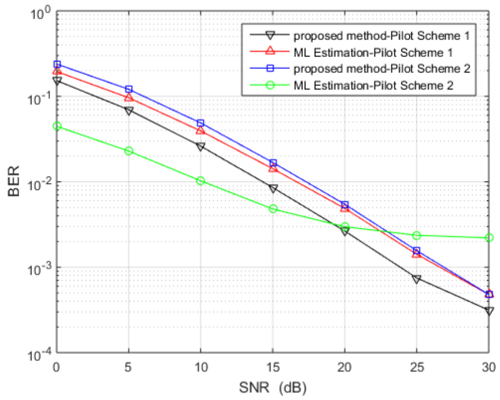

Figure 5. BER Comparison of Pilot Schemes

In figure 5 we calculated the BER performance among pilot scheme 1 and pilot scheme 2 by ML and ML-CRB methods. From figure we can observe that the pilot scheme 1 performs better than the pilot scheme 2.

6. Conclusion

A pilot scheme is proposed with zero pilot symbols for three interfering cells. An ML channel estimation is performed and merged with Cramer Rao bound for the multi-cell massive MIMO-OFDM system. The simulations demonstrate the proposed approach, which is helpful to reduce the effect of pilot contamination and easy to implement as well. Future research will consider the configurations with more number of transmit antennas and more uplink transmitters. It will also work on the usage of MIMO channel statistics reducing the pilot contamination.

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