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Robust Linear State Estimation Considering PMU Measurements at Critical Buses

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Abstract

The failure of power grid system during peak and load conditions led to the development of robust state estimation method utilizing Phasor Measurement Unit (PMUs) measurements. In this paper, a novel index is proposed to determine critical buses in IEEE bus networks. A Binary Integer Programming method is utilized for optimal deployment of PMUs at critical buses using critical bus ranking. A Robust Linear State Estimation (RLSE) Algorithm is presented to obtain accurate states of the network. The conventional and PMU measurements at critical buses are integrated to form a matrix to estimate accurate states. Residual errors obtained through linear state estimation with and without PMUs are compared to show the effectiveness of the allocation. IEEE 14, 30 bus networks are computed with MATLAB programming and the results are compared with standard methods to validate the proposed method.

Keywords: *Critical bus, Binary Integer Programming method, Phasor Measurement Unit, State Estimation.*

1. Introduction

State Estimation (SE) plays a crucial role protecting the power system from blackouts. With the development of synchrophasor measurements in power system and integration with conventional measurements proved to provide accurate states with state estimation methods. Phasor Measurement Units (PMU) provides complete measurement accuracy with the time synchronization provided by Global Positioning Systems (GPS). [1-2].

SE methods that use only conventional measurements (that are provided by SCADA data) are not able to provide real time control of power systems. The first step for security analysis is to monitor current state of network. The current state of network is obtained through PMU measurements and accurate current states of network through finite SE algorithms. Many authors proposed integration of PMU measurements with conventional measurements such as to give accurate states of network. From recent blackouts occurred in India and USA, it can be observed that failure is due to sudden removal of generations or due to peak loads. For accurate estimation of states at critical buses there is in need to provide accurate SE algorithm with PMU measurements obtained at critical buses in network. Linear SE provides quick and accurate results compared to Weighted Least Square (WLS) SE. Ali Abur proposed different SE methods with PMU measurements which are nonlinear [3]. Different methods which are considered as linear are two stage methods in which first method is nonlinear and second method is linear.

In [7], the author proposed linear SE including phasor measurement technology to detect and identify bad data. In [8] the author considered to tackle missing measurements with development of hybrid algorithm with PMU measurements. integrating with conventional measurements. In [9]-[10] the authors integrated PMU measurements with remote terminal unit measurements. In [11] hybrid non- linear SE with PMU and conventional

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measurements is proposed. An Index to measure accuracy is utilized to compute the impact of PMU measurements. In [12], significance of time screw errors in conventional measurements is studied on performance of hybrid SE using PMU and conventional measurements.

In [13] the author distinguished performance of WLS and recursive Kalman Filter (KF). In [14], a method of test equations is proposed to detect gross errors in both SCADA and PMU measurements. In [15] the author proposed linear model of SE along with bad data identification.

To obtain a quick SE a new RLSE method is proposed in which PMU measurements are utilized with suitable design of matrix such that within less time the results are obtained. This is a post process method in which results are obtained in quick time. In this paper, RLSE with PMU measurements is proposed to obtain quick state estimation values in which measurements of critical buses are considered to allocate PMUs.

2. Weighted Least Square State Estimation

In this method the set of measurements are considered as Z consisting of active, reactive power injections and active and reactive power flows which are non-synchronized and non-linear data function of state vector 'y'.

$$Z = h(y) + e \tag{1}$$

Where h is non-linear function of state vector of y expressed in polar coordinates and e is measurement error vector with a covariance matrix R.

The WLS state estimation is grounded on minimization of weighted sum of squared residuals.

$$J(x) = [Z - h(y)]^T R^{-1} [Z - h(y)] \tag{2}$$

The minimization problem is elucidated using Newton Raphson method, an iterative progression in which each equation is solved.

$$G\Delta x = H^T R^{-1} \Delta Z \tag{3}$$

$G = H^T R^{-1} H$ is defined as Gain matrix, H is Jacobian matrix of size l x n, $h(y)$ is determined at a given point y^t

$$\Delta Z = Z - h(y^t) \tag{4}$$

The solution of (3) produces the vector Δy i.e., an addition to states. As a result, the updated state vector is obtained as

$$y^{t+1} = y^t + \Delta y \tag{5}$$

The convergence of iterative practice is accomplished when Δy is lesser than pre-defined tolerance value.

3. Problem Formulation

A. PMU Allocation considering Critical Bus Analysis

The problem is computed considering critical analysis for OPP. BIP method is modeled with critical constraints for OPP in network. The optimum minimization problem is formulated as

$$Min \sum_{k=1}^{N} W_k Y_k \tag{6}$$

$$\text{Subject to } AY \geq B \text{ and } A_1 Y = B_1 \tag{7}$$

Where W_k is cost weight of PMU considered at bus K, A_1 is matrix of order $N \times N$comprising of buses attained through critical analysis, which are all measured as one and for remaining buses it is zero, N is number of buses, $Y = [Y_1 \quad Y_2 \quad Y_3 \quad \ldots \quad Y_N]^T$ is a binary variable vector, B and B_1 is observability constraints which is written as$[1 \ 1 \ 1 \ 1....1]^T_{N\times 1}$, Y_k is binary variable and$A$ is incidence matrix of order $x \times y$ which is presented as

$$Y_k = \begin{cases} 1 & \text{if PMU is placed at bus } k \\ 0 & \text{otherwise} \end{cases}$$

$$A_{x,y} = \begin{cases} 1 & \text{if } x = y \text{ or connected to each other} \\ 0 & \text{otherwise} \end{cases}$$

For example consider 14-bus network, The problem is formulated as

$$Min \sum_{k=1}^{14} W_k y_k \tag{8}$$

$$Z(k) = \begin{cases} B1 = y_1 + y_2 + y_5 \geq 1 \\ B2 = y_1 + y_2 + y_3 + y_4 + y_5 \geq 1 \\ B3 = y_2 + y_3 + y_4 \geq 1 \\ B4 = y_2 + y_3 + y_4 + y_5 + y_7 + y_9 \geq 1 \\ B5 = y_1 + y_2 + y_4 + y_5 + y_6 \geq 1 \\ B6 = y_5 + y_6 + y_{11} + y_{12} + y_{13} \geq 1 \\ B7 = y_4 + y_7 + y_8 + y_9 \geq 1 \\ B8 = y_7 + y_8 \geq 1 \\ B9 = y_4 + y_7 + y_9 + y_{10} + y_{14} \geq 1 \\ B10 = y_9 + y_{10} + y_{11} \geq 1 \\ B11 = y_6 + y_{10} + y_{11} \geq 1 \\ B12 = y_6 + y_{13} + y_{12} \geq 1 \\ B13 = y_6 + y_{12} + y_{13} + y_{14} \geq 1 \\ B14 = y_9 + y_{13} + y_{14} \geq 1 \end{cases} \tag{9}$$

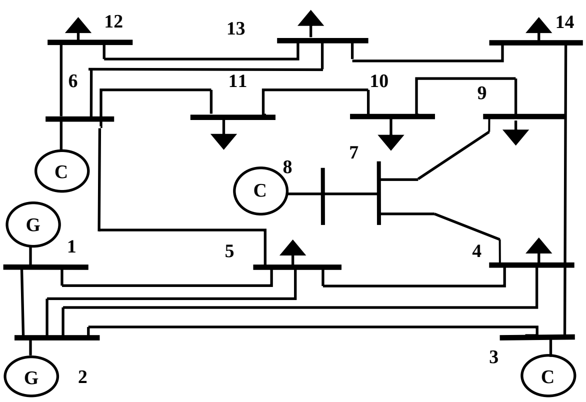

Figure.1. Schematic diagram of 14-bus network

Proper ranking of critical buses in network and OPP considering critical analysis is formulated as follows.

B. Critical bus ranking with proposed NVCI

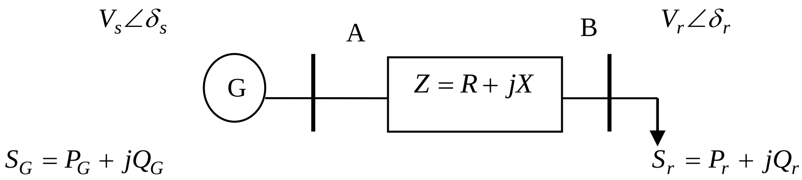

Figure.2. 2-bus network

The voltage collapse primarily depends on active and reactive power transfer through transmission lines. Voltage collapse is defined as ratio of generator voltage sensitivity and change in reactive load at each bus. The New Voltage Collapse Index (NVCI) is expressed as

$$NVCI(Q_k) = \sum_{P=1}^{M} \frac{\Delta Q_G}{\Delta Q_P} \tag{10}$$

where ΔQ_G is deviation in reactive output power of generator and is ΔQ_P deviation in reactive power output at load bus'p' With change in reactive power there is change in voltage, as reactive power and voltages are coupled to each other. The proposed collapse index is different from other existing indices. The value obtained for existing indices available in literature is between 0 and 1. For proposed NVCI, the value is always greater than one. In order to have good stability of voltage, the value of index should be closer to one. The highest value is considered as most critical bus which is vulnerable to reactive load change. The critical buses are ranked according indices of load buses. In this work only 10% of reactive power is increased at each load bus to generate NVCPI.

Steps to obtain critical buses:

1. Load flow analysis is carried out under load flow conditions
2. Increment the load bus reactive power by 10%
3. Evaluate NVCI for every load bus
4. Sort load buses in descending order
5. Rank the buses with highest index value
6. The highest index value is considered as most critical bus

The proposed algorithm to rank critical buses is computed by NVCI.

Table 1. NVCI for 14-bus system

Load Buses	NVCI	Ranking
9	1.120	1
14	1.115	2
10	1.104	3
13	1.060	4
11	1.056	5

Table 2. Critical bus ranking of 30-bus system using NVCI

Critical Buses	NVCI	Ranking
22	1.2175	1
26	1.2173	2
24	1.2089	3
21	1.2053	4
23	1.1875	5

The schematic diagram of 14- bus diagram is shown in Fig.1. Similarly for 30 bus and 57 bus system, critical buses can be computed using steps as shown above. Table I and II shows the ranking of critical buses obtained through steps for 14 bus network and 30 bus network. The main purpose to compute the critical bus ranking is to place PMUs at most critical buses to achieve complete observability of network.

4. RLSE with Optimal PMU Measurements at Critical Buses

A. RLSE with PMU measurements

The SE problem involving measurements such as power injections and power flows is nonlinear. The formulation of the measurement function with PMU measurements is linear formulated with measurements of voltage and phase angles

$$\begin{bmatrix} \begin{bmatrix} V_R \\ V_{im} \end{bmatrix}_{WLS} \\ \begin{bmatrix} V_R \\ V_{im} \end{bmatrix}_{PMU} \\ \begin{bmatrix} I_R \\ I_{im} \end{bmatrix}_{PMU} \end{bmatrix}_Z = \begin{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \\ \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \\ \begin{bmatrix} G_{ij} & -B_{ij} \\ B_{ij} & G_{ij} \end{bmatrix} \end{bmatrix}_h \begin{bmatrix} \mathrm{V_R} \\ \mathrm{V_{im}} \end{bmatrix}_x + e \qquad (11)$$

Where I is Identity matrix of dimension $n \times n$, The zero elements of the sparse matrix are replaced by 1 to form $H_{11}$and H_{22}. While H_{12} andH_{21} are represented by null matrix.

Currents in transmission network are related to voltages by series admittance $Y_{ij} = G_{ij} + jB$. Transmission lines are represented by an equivalent π -model.

Mathematically the real and imaginary parts of the injected currents can be formulated as,

$$\begin{aligned} I_R &= V_{iR}G_{ii} - V_{im}B_{ii} + \sum_{j=1, j\neq 1}^{n} (V_{jR}G_{ij} - V_{jim}B_{ij}) \\ I_{im} &= V_{iR}B_{ii} + V_{im}G_{ii} + \sum_{j=1, j\neq 1}^{n} (V_{jR}B_{ij} - V_{jim}G_{ij}) \end{aligned} \qquad (12)$$

The solution of linear model is computed directly as,

$$X = ([H]^T [R]^{-1} [H])^{-1} [H]^T [R]^{-1} [Z] \qquad (13)$$

Here R is diagonal co-variance matrix.

B. Proposed Robust Linear State Estimation Procedure

The proposed RLSE can be obtained without any iterations with proper design of matrices as shown in equations

Steps to obtain RLSE

1. The true states of the IEEE network are obtained through N_R load flow analysis.
2. State vectors obtained from WLS SE are utilized in equation.
3. Final RLSE is obtained with output of WLS and phasor measurements at critical buses.
4. Compute the errors comparing the true state from N-R load flow and states obtained through RLSE.

C. Performance of the Proposed RLSE with Root Mean Square Error (RMSE) Index

The performance and accuracy of SE process are measured by the proposed RMSE index.

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{k=1}^{N}(x_{est}(k) - x_{meas}(k))^2} \quad (14)$$

Where x_{est} is state estimated by the RLSE method,x_{meas} is the true state of the system obtained from the measurement and N is represented as a total number of buses in system. As scale of measurement is different for the two states, RMSE is calculated separately

5. Results and Analysis

The computational work is done under MATLAB/ Programming environment. MATLAB programming is carried out using Intel(R) core(TM) i3 processor at 2.20GHz with 4GB of RAM. The most critical buses 9 and 14 of 14-bus system and buses 24, 26 and 27 of 30-bus system are considered measurements of PMUs included in optimal PMU measurements for state estimation solution. Table 5. shows optimal location of PMUs considering critical buses.

Table 3. OPP considering critical bus ranking

IEEE test systems	No. of PMUs	OPP Locations
14 bus	4	2,6,9,14
30 bus	9	1,7,10,12,19,22,24,26,27

Fig.3.shows phase angle error of 14-bus network considering SE with PMU and without PMU (considering conventional measurements). Fig.4 shows voltage magnitude error of 14-bus network considering state estimation with PMU and without PMU (considering conventional measurements). The error is defined as the difference between the true value and state estimation values. Here, state estimation with and without PMUs are compared to show the effectiveness of PMU placement at critical buses. Similarly, for 30-bus system, both phase angle error and voltage magnitude error are also compared with and without PMU as shown in Fig.5 and 6. Single line diagram of 30-bus network is shown in Fig.8.

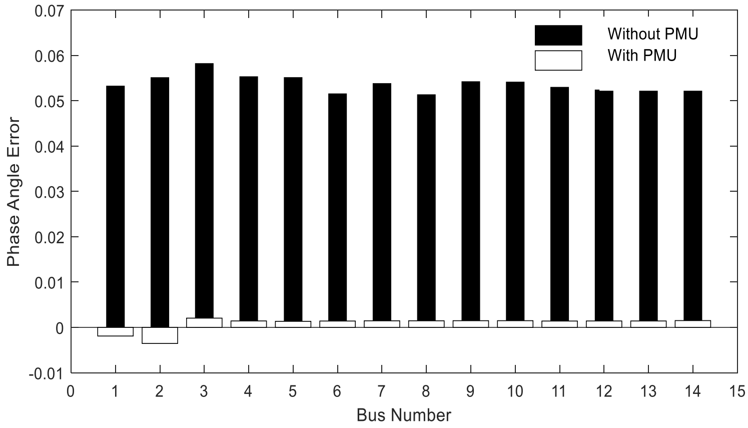

Figure.3. Phase angle of 14-bus network with and without PMU

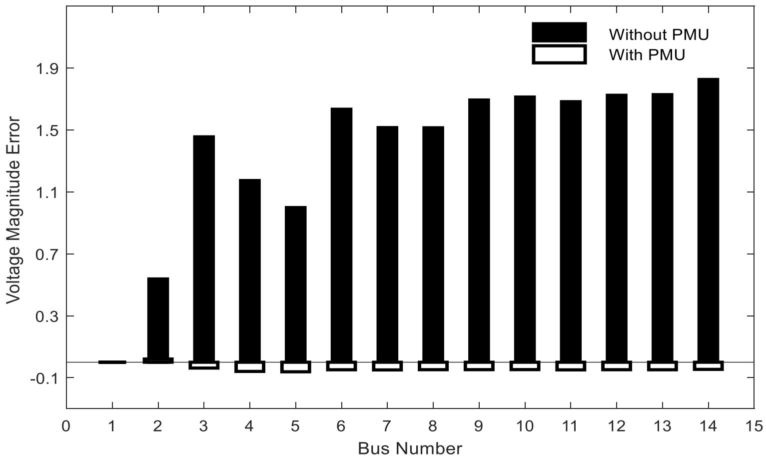

Figure. 4. Voltage Magnitude error of 14-bus network with and without PMU

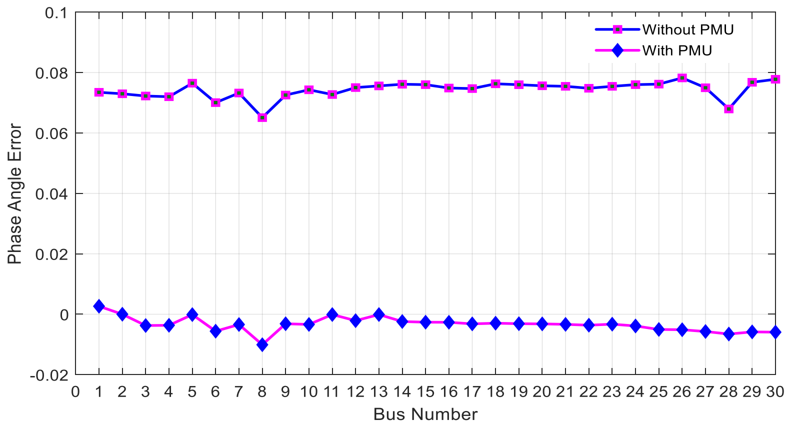

Figure.5. Phase angle error of 30-bus network with and without PMU

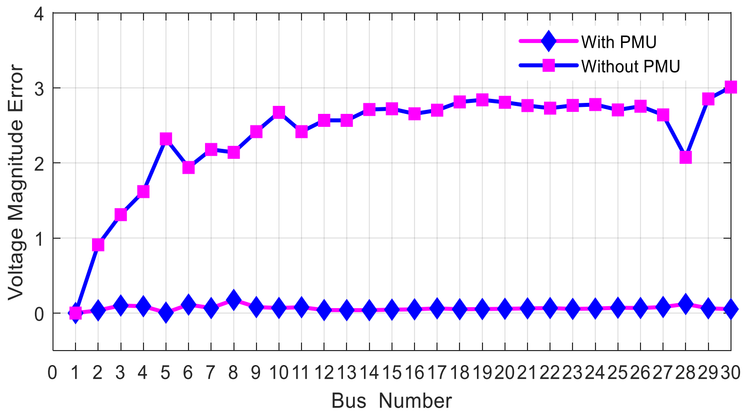

Figure.6. Voltage Magnitude of 30-bus network with and without PMU

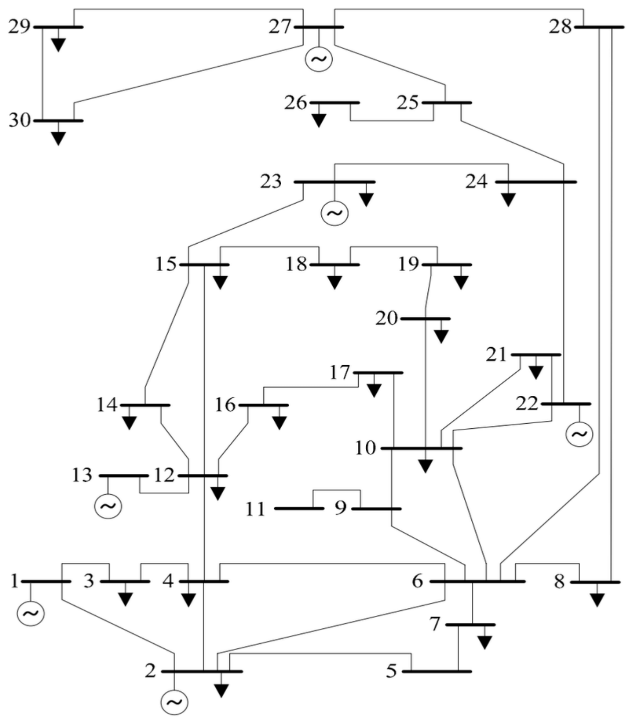

Figure.7. Schematic diagram of 30- bus network

Table 4. Comparison of Proposed RLSE Method for 30-bus system with other standard methods

Bus. No	True values N- R Method		WLS State Estimation		Proposed RLS Estimation	
	V	δ	V	δ	V	δ
1	1.060	0	0.986	0	1.057	0.000
2	1.043	-5.354	0.970	-6.263	1.043	5.390
3	1.019	-7.530	0.947	-8.842	1.023	-7.631
4	1.010	-9.284	0.938	-10.902	1.014	-9.375
5	1.010	-14.173	0.933	-16.494	1.010	-14.179
6	1.009	-11.058	0.939	-12.997	1.015	-11.170
7	1.002	-12.864	0.928	-15.044	1.005	-12.931
8	1.010	-11.819	0.944	-13.960	1.020	-11.994

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9	1.039	-14.064	0.966	-16.481	1.042	-14.144
10	1.021	-15.670	0.947	-18.344	1.024	-15.738
11	1.082	-14.064	1.009	-16.481	1.080	-14.142
12	1.049	-15.124	0.974	-17.691	1.051	-15.164
13	1.071	-15.124	0.995	-17.691	1.071	-15.163
14	1.032	-16.001	0.955	-18.713	1.034	-16.040
15	1.025	-16.008	0.949	-18.729	1.027	-16.053
16	1.030	-15.625	0.955	-18.280	1.033	-15.674
17	1.018	-15.868	0.944	-18.571	1.021	-15.931
18	1.011	-16.606	0.935	-19.419	1.014	-16.657
19	1.006	-16.765	0.930	-19.606	1.009	-16.819
20	1.009	-16.550	0.933	-19.358	1.012	-16.606
21	1.008	-16.217	0.932	-18.982	1.011	-16.280
22	1.012	-15.981	0.937	-18.711	1.015	-16.047
23	1.008	-16.229	0.933	-18.995	1.011	-16.284
24	0.999	-16.300	0.923	-19.078	1.003	-16.360
25	1.003	-16.072	0.927	-18.778	1.008	-16.142
26	0.985	-16.503	0.907	-19.259	0.990	-16.570
27	1.014	-15.655	0.939	-18.296	1.020	-15.736
28	1.007	-11.716	0.939	-13.791	1.014	-11.837
29	0.994	-16.907	0.917	-19.760	1.000	-16.971
30	0.982	-17.806	0.905	-20.817	0.988	-17.859

Table 5. RMSE Index for 14- and 30- bus systems

IEEE Test System	WLS state estimation method		GSL Estimation method with PMU measurements	
	V	δ	V	δ
14 bus	0.0146	0.4877	$4.9144\text{x}10^{-6}$	$2.4629\text{x}10^{-4}$
30 bus	0.0142	0.5496	0.0011	0.0096

RLSE method show much accurate measurements compared to other methods. Table IV shows the comparison of states, voltage and phase angles of the 30-bus system obtained for proposed RLS Estimation with WLS state estimation and true values of N-R Method. Table V shows the performance index of 14-bus network and 30-bus network and both WLS method and RSL Estimation methods are compared with respect to RMSE index.

6. Conclusion

The proposed Robust Linear State Estimation method with PMU measurements of critical buses finds accurate states of the network. The proposed novel index determines critical buses in IEEE bus networks. A Binary Integer Programming method is utilized for optimal deployment of PMUs at critical buses using critical bus ranking. Residual errors obtained through linear state estimation with and without PMUs are shows effectiveness of the placement. IEEE 14- bus network and 30-bus network are programmed in MATLAB to show the effectiveness of the proposed method.

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