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Energy Efficiency Analysis of a single-cell Massive MIMO Systems

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Abstract

We analyze EE for the multi-user massive MIMO systems in the single-cell. Primarily the circuit model of power consumption is elaborated, which describes the process of influence the circuit power consumption by the antenna number and user number in detail. Then it demonstrates the closed-form expression of EE, which depicts the relationship between the influence factors and obtained downlink EE. The factors include the number of users, the transmission power and the number of BS antennas. The simulation provides information where it is shown that with medium numbers of antennas and users optimal EE is acquired.

Keywords: *Massive MIMO, Energy Efficiency, Zero Forcing Pre-coding, Lambert Function*

1. Introduction

The rapid development of the ICT industry has brought about a rapid increase in energy consumption. Today mobile operators have become an important component of energy consumption. It is reported that 18% of the total expenditure of European operators is energy consumption, and Indian operators have consumed 32% of the total cost in energy consumption. It can be seen that for operators, improving energy efficiency will help increase their economic benefits. From the user's perspective, the battery durability of various terminal devices is a significant reference for standardizing the performance of the device. Therefore, both from the supplier's perspective and from the user's perspective, energy-efficient wireless communication systems are essential.

Many international projects are currently investigating how to make wireless communication systems energy efficient. The Green Radio project provides an accurate energy consumption model; optimizes hardware equipment by updating digital processing technology and increasing the reuse rate of users and base stations [1] and adopts an energy-efficient optimized architecture and resource management. OPERA-Net project is based on business load and through cell scaling, network.

Structural adjustments achieve energy savings to achieve energy-efficient optimized network access; use scalable MIMO detection [2], amplitude modulation, and fountain codes at the link level; design energy-aware terminals study efficient power amplifier and energy regeneration technologies. The eWin project uses a novel compromise relationship to design a low-consumption architecture optimizes the architecture according to local conditions adopts flexible and variable spectrum resource management technology and also manages wireless resources based on heterogeneous relationships of competition and collaboration. The EARTH project analyzes the life cycle of energy consumption of telecommunications products, formulates system-level optimization guidelines, and uses technologies such as dynamic load and transmission mode adaptation. The research content of these projects is carried out from the aspects of energy efficiency measurement criteria, energy efficiency information theory [3], and energy efficiency optimization methods.

Demonstrated through the statistics, the energy consumption of mobile networks is a reason out of around 0.2% of overall carbon emissions and the electricity bill reflects roughly 18-40% of operator expenditures in various countries [4]. Green communications

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have been placed onward to change better state EE assuring that other parameters of communication systems, like SE and reliability, are not deteriorated, etc. (OpEx) [4]. Recent global societal and economic issues including the power usage of the communication technology industry and the associated carbon emissions [5]. It has stirred researchers and industries to exquisite practice in the brand-new field of green cellular networks research [6], lately sparked on by the study of the SMART 2020 report [7] and the Green Touch consortium [8].

To ensure the sustainability of development, EE has turn out to be an essential criterion for the design of 5G cellular networks. As one of 5G's key technology, massive MIMO ensures the spectrum efficiency and energy efficiency of the network and is an important driving force for 5G. Therefore, the EE analysis of massive MIMO systems is of great significance to the development of wireless communications. Many factors such as the number of BS antennas, the number of users per cell, and the pilot multiplexing factor, the base station density, and the transmission power affect the EE of the system. This study will analyze the impact of the three influence factors of transmit power, the number of BS antennas, and the number of cell users on the downlink energy efficiency[9], and give closed-form expression results. The EE of a communication system is usually measured in units of bits / Joule, which is numerically equal to the ratio of the average reachable rate (bits/symbol) to the total average power consumption (Joule/symbol), where the average power consumption model is the focus of research.

2. System Model

Considering the downlink transmission of a single-cell system as shown in figure 1, to communicate with N single-antenna users the BS uses M antennas. It is assumed that the channel matrix h_n between the BS and the Nth user obeys the Rayleigh block fading$CN(0,\zeta_N I_N)$, that is, it is satisfied that the channel remains unchanged for T signal times. Users are selected in a polling manner from users who move in a given area, and the user base changes over time. $\zeta_1,\zeta_2,\cdots\zeta_N$ Are the influencing factors of N users related to the user distribution in a given area and the path loss model that is sensitive to location information.

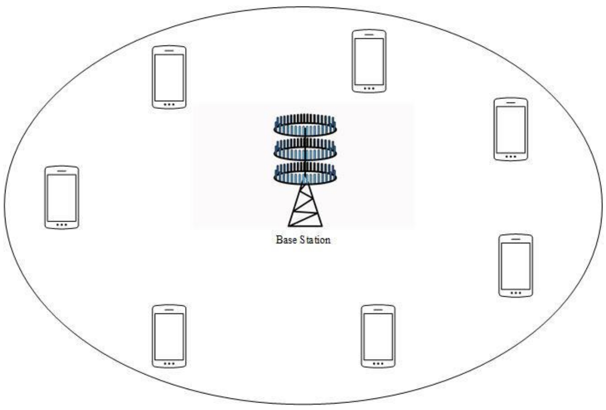

Figure 1. Single-cell Multi-user Massive MIMO System

To obtain immediate channel state information (CSI), consider using an orthogonal uplink pilot sequence TDD model. By using the channel's reciprocity characteristics and Gaussian codebook, and treating interference between users as noise, the average reachable downlink rate of the Nth user can be obtained as,

$$G_N = \left(1 - \frac{N}{T}\right) E\left\{log_2\left(1 + \frac{|h_n^H u_n|^2}{\sum_{j \neq n}|h_n^H u_j|^2 + \sigma^2}\right)\right\} \quad (1)$$

Whereas, the preamble factor (1-N/T) represents the required pilot overhead, and $\sigma^2$is the noise variance. The pre-coding matrix is represented by u_n, which can be calculated from the real-time channel state information at the base station. The expected value in the above formula is closely related to $\{h_n\}$, $\{u_n\}$, and $\{\zeta_n\}$.

In traditional systems, there is a large difference between peak and average rate, and the system here strives to ensure that each selected user has the same rate value, which is $G_n = G\ (\geq 0)$. Specifically, the purpose of this study is to find the optimal number of BS antennas, the number of users and rate value G to maximize the energy efficiency of the system.

3. Downlink Power Consumption Model

The traditional system usually models the circuit power consumption in the power consumption model as a constant that is independent of the number of BS antennas M and the number of cell users N. At this time, the total power consumption of the BS will be approximately equivalent to the transmitting power, but this hypothesis is not always correct. Making this assumption in massive MIMO will make people mistakenly believe that energy efficiency can increase infinitely when $M \to \infty$ [10]. To sum up, massive MIMO systems need a more reasonable energy consumption model.

In addition to the power dissipated by the amplifier, the total power consumption includes circuit power from analog filters and digital signal processing. Evolving based on the power consumption model in [1, 11, 12], an improved model can be obtained, which specifically describes the power consumption P_{sum} relation with the number of base station antennas M and the number of cell users N. Total downlink power consumption (Joule/symbol) is

$$P_{sum} = \sum_{n=1}^{N} \frac{E\{\|u_n\|^2\}}{\eta} + \sum_{i=0}^{3} B_{i,0} N^i + \sum_{i=0}^{2} B_{i,1} N^i M \quad (2)$$

Where, η is the efficiency of the power amplifier, η϶(0,1). Where $B_{0,0}$is a fixed hardware power consumption greater than zero, which does not change with M and N, and The rest of the parts related to the higher-order terms of (M, N) will be described in detail below. Combining the above formula we can acquire the average EE expression,

$$EE = \frac{\sum_{n=1}^{N} G_n}{P_{sum}} \quad (3)$$

The following specifically describes the higher-order terms related to (M, N) in Equation (2). The main sources are the following four aspects:

1. Transceiver links: The typical MIMO transceiver link power consumption is $P_{TX} + NP_{RX} + P_{OSC}$ (Joule/symbol). P_{TX} It is the antenna component of the BS, including the power consumption of converters, mixers, and filters. There is a separate P_{OSC} oscillator, which works for all base station antennas. Considering a single antenna user here, P_{RX} is the power consumption of all receiving components including amplifiers, mixers, oscillators, and filters.

2. Encoding and decoding: The base station encodes and modulates the N signal sequences, and then each user uses some sub-optimal algorithms to decode the signals it receives. Therefore, the total power consumption is N $(P_{enc} + P_{dec})$ calculated in Joule/symbol, where P_{enc} and P_{dec} are the power consumption of encoding and decoding, respectively.

3. Channel estimation and pre-coding: Let the computational efficiency of the system be J flops / Watt, then the uplink CSI estimation will receive M signals from each user and then scale each one up proportionally. The scale factor is related to the estimator. The system estimates each coherent time, so the energy it consumes is $\frac{MN}{JT}$ (Joule/symbol); the pre-coding is also calculated in each coherent time slot, We consider ZF pre-coding here, and its power consumption is $\frac{3N^2M+2NM}{JT}+\frac{2N^3}{3JT}$ (Joule/symbol) (produced by matrix inversion). During signal transmission, multiplying the pre-coding matrix by the signal vector requires $\frac{MN}{J}\left(1-\frac{N}{T}\right)$ (Joule/symbol) of power consumption.
4. Firmware consumption: The system architecture will have a fixed power consumptionP_C, which does not change with M or N, including fixed consumption of control signals and independent load consumption of the baseband processor.

After sorting and summing up the above four assumptions, we can get the specific expression of the total downlink power consumption under ZF pre-coding.

$$P_{sum}=\sum_{n=1}^{N}\frac{E\{\|u_n\|^2\}}{\eta}+P_C+P_{OSC}+(P_{RX}+P_{enc}+P_{dec})N+P_{TX}M+\frac{3+T}{JT}MN+\frac{2}{JT}MN^2+\frac{2}{3JT}N^3 \tag{4}$$

The coefficients of the higher-order terms are as follows:

$$B_{0,0}=P_C+P_{OSC},B_{1,0}=P_{enc}+P_{dec}+P_{RX},B_{2,0}=0,B_{3,0}=\frac{2}{3JT},B_{0,1}=P_{TX},B_{1,1}=\frac{3+T}{JT},B_{2,1}=\frac{2}{JT}$$

4. Optimal Energy Efficiency Analysis

Here we discuss the case where the number of BS antennas M is greater than the number of users N and assuming that the pilot signal provides the ideal CSI for the BS, ZF pre-coding is considered here. The channel matrix$H=[h_1,h_2\cdots h_n]$. It is assumed that ZF pre-coding enables each user to obtain a statistics of$\left(1-\frac{N}{T}\right)\log_2\left(1+p(M-N)\right)$, whereas p is the normalized transmission power. The total transmit power at the base station is [13],

$$\sum_{n=1}^{N}E\{\|u_n\|^2\}=pNZ_\zeta \tag{5}$$

Whereas, $Z_\zeta=\int_0^\infty\frac{\sigma^2}{s}f_\zeta(s)ds$ is determined by the transmission environment. Combining equations (2) and (3) to get the final expression of energy efficiency as

$$EE=\frac{N\left(1-\frac{N}{T}\right)log_2\left(1+p(M-N)\right)}{\frac{pNZ_\zeta}{\eta}+\sum_{i=0}^{3}B_{i,0}N^i+\sum_{i=0}^{2}B_{i,1}N^iM} \tag{6}$$

Use the above formula to optimize M, N, and p, respectively. When users are evenly distributed in a cell, the maximum and minimum radius of the cell is represented respectively by r-max and r-min. Where G is the fixed channel fading, r is the distance among the user and the BS, and n is the path loss factor, then we can get

Z_ζ where $Z_\zeta = \frac{\sigma^2\left(r_{max}^{\eta+2}-r_{min}^{\eta+2}\right)}{G\left(1+\frac{\eta}{2}\right)\left(r_{max}^2-r_{min}^2\right)}$. The following section will introduce how to set the number of BS antennas so that the transmission power and the number of users can make the EE optimum.

4.1. Optimum Number of BS Antennas

First, the influence of the number of BS antennas on the optimal energy efficiency is analyzed. The control variable method is used here, that is, when analyzing the optimal number of antennas, the other two influence factors are regarded as constant. The optimization problem for solving the optimal number of antennas is expressed as

$$\max_{M\geq N} \frac{N\left(1-\frac{N}{T}\right)log_2\left(1+p(M-N)\right)}{\frac{pNZ_\zeta}{\eta}+\sum_{i=0}^{3} B_{i,0}N^i+\sum_{i=0}^{2} B_{i,1}N^i M} \quad (7)$$

The Lambert function and related optimization knowledge can be used to get the optimal solution to the problem of optimization.

$$M^{optimal} = \frac{e^{q\left(\frac{\frac{p^2NZ_\zeta}{\eta}+p\sum_{i=0}^{3}B_{i,0}N^i}{e\sum_{i=0}^{2}B_{i,1}N^i}+\frac{Np-1}{e}\right)+1}+Np-1}{p} \quad (8)$$

Where Q (·) represents the Lambert function, and the optimal number of BS antennas for maximizing EE is given by this. Using the properties of the Lambert function, we can see that when the normalized transmit power p is large, the optimal M almost linearly increases with p, the optimal M is proportional to $B_{i,0}$, and inversely proportional to $B_{i,1}$ & Z_ζ grows linearly, and from the previous analysis, we can see that in a circular cell, Z_ζ is proportional to r_{max}^{η}, where r_{max} is the cell radius and κ is the path loss factor.

It is worth noting that M obtained by equation (7) is a real number, but it must be an integer, and the number of BS antennas of the actual cell BS must be an integer, therefore, the final feasible optimal solution should be one of the two nearest integers obtained by the above formula as the final solution.

4.2. Optimal Transmit Power

As mentioned in the above analysis, p is the normalized transmission power, and the total transmit power is pNZ_ζ. The optimization problem to find the optimal transmit power is expressed as follows,

$$\max_{p\geq 0} \frac{N\left(1-\frac{N}{T}\right)log_2\left(1+p(M-N)\right)}{\frac{pNZ_\zeta}{\eta}+\sum_{i=0}^{3} B_{i,0}N^i+\sum_{i=0}^{2} B_{i,1}N^i M} \quad (9)$$

The solution of the above formula is

$$p^{optimal} = \frac{e^{q\left(\frac{(M-N)\eta(\sum_{i=0}^{3}B_{i,0}N^i+\sum_{i=0}^{2}B_{i,1}N^iM)}{NZ_\zeta e}+\frac{1}{e}\right)+1}-1}{M-N} \quad (10)$$

The above formula gives the transmission power with maximum energy efficiency. According to the nature of the Lambert function, the optimal transmission power increases with the increase of the circuit power coefficient $B_{i,j}$. This is reasonable, if the power consumption of a static circuit is large, you can increase the transmission power more so that it has less effect on the total power consumption.

As mentioned earlier, the transmission power in a large-scale MIMO system is inverse proportional to M and ($\sqrt{M}$in the case of non-ideal CSI). This is a well-known conclusion, but it can be found from the above formula that this conclusion does not hold for energy efficiency. To achieve a better performance of energy efficiency which is almost completely the opposite, that is, as the number of BS antennas M escalates the transmission power increase. The reason for this is as follows: If the power utilization of the circuit increases with the increase of M, then the value of the transmission power can be increased so that it does not hinder the increase of energy efficiency. In the special case where the power consumption of the circuit is independent of M ($B_{i,1} = 0, \forall i$), the transmit power should decrease within (M), and the reduction speed at this time is much slower than with M. In any case, this is only implemented in a scenario (there is no circuit power consumption) that does not match the actual situation.

4.3. Optimal Number of Users

The problem of the optimal number of users can be described by the following optimization problem. For the convenience of description, let the transmit power satisfy p^T= Np and the number of antennas per user is $\alpha = M/N$. The optimization problem is expressed as

$$\max_{N\geq 0} \frac{N\left(1-\frac{N}{T}\right)log_2\left(1+p^{tot}(\alpha-1)\right)}{\frac{p^T Z_\zeta}{\eta}+\sum_{i=0}^{3} B_{i,0}N^i+\sum_{i=0}^{2} B_{i,1}\alpha N^{i+1}} \tag{11}$$

The optimization problem is quasi-concave and can be solved using the roots of the fourth-order polynomial,

$$xy_3N^4 = 2z_3xN^3 - (wx_2 + xy_1)N^2 - 2xy_0N + y_0w \tag{12}$$

Where,

$w = \log_2\left(1 + p^{tot}(\alpha - 1)\right), x = \frac{w}{T}, y_0 = B_{0,0} + \frac{p^{tot}Z_\zeta}{\eta}, y_1 = B_{1,0} + \alpha B_{0,1}, y_2 = B_{2,0} + \alpha B_{1,1}, y_3 = B_{3,0}\alpha B_{2,1}$

In the special case of $y_3 = 0$, the optimum solution is

$$N^{optimal} = \sqrt{\left(\frac{xy_0}{wy_2+xy_1}\right)^2 + \frac{y_0w}{wy_2+xy_1}} + \frac{xy_0}{wy_2+xy_1} \tag{13}$$

The fourth-order polynomial in Eq. (11) generally has 4 roots, here only one is given as an example. The specific calculation method of the fourth-degree polynomial root is presented in [14]. Similar to the previous analysis of the optimal number of BS antennas, the number of users with the highest energy efficiency calculated by the formula (12) is not necessarily an integer. According to the quasi-concave characteristic of the above formula, the number of optimal users is between the two nearest integers.

To discuss this issue further, the highest order terms in equation (2) are ignored here, that is, let $B_{3,0}$ and $B_{2,1}$ equal to zero, respectively, and $\alpha = M/N$ is fixed. The optimum number of users $N^{optimal}$ is a declining function of $B_{1,0}$, $B_{2,0}$, $B_{0,1}$, $B_{1,1}$, these coefficients are related to M or N, however, wherein the terms independent of M and N dominate the power consumption, the number of users gradually increases, which means that $N^{optimal}$ escalates with the fixed hardware power consumption B0, 0 and the channel environmental parameter Z_{ζ}.

5. Simulations and Analysis

This segment simulates how the optimal EE p changes with M, analyzes the relationship among the EE, M, and N, and also details how to jointly design the number of BS antennas, transmit power, and the number of users to make EE better as in section 3. Users are theoretically distributed uniformly in a circular cell with a radius of 30 to 200 meters, a classic path loss model is used, and the transmission parameter Z_{ζ} as described in Section 4. The simulation parameters are shown in the following table.

Table 1. Simulation Parameters

S/No	Parameters	Value
1	T (time)	5760 s.Hz
2	J (flops)	10^{9} flops/Watt
3	η	0.3
4	R	$10^{-3.5}$
5	r_{min}	30 m
6	r_{max}	200 m
7	σ^2	10^{-20} J/Symbol

6. Conclusion

The performance of single-cell large-scale MIMO systems from the perspective of energy efficiency has been analyzed. Firstly, a more practical energy consumption model is analyzed, and the EE expression is written. Then, optimal energy efficiency is analyzed under a typical path loss model. The fixed variable method is used to analyze the optimal number of BS antennas, the optimal transmit power and the optimum number of users. The closed-form expressions are given for the Joint optimal iterative algorithm. It can be known from the simulation that the optimum transmission power escalates almost linearly with the number of BS antennas. The joint optimization of the three influence factors can be obtained by an iterative algorithm, and the EE reaches the maximum when the number of BS antennas is 150 and it communicates with 80 users simultaneously.

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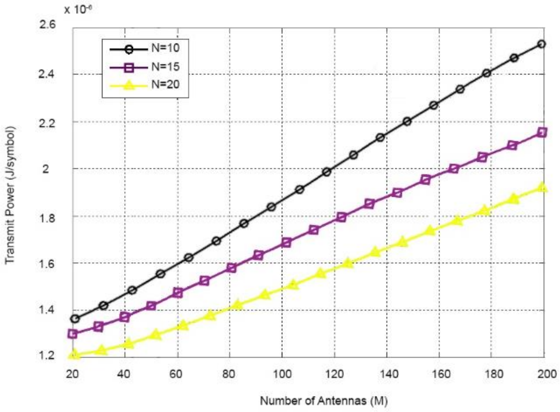

Figure 2. Transmit Power Relationship with Number of BS Antennas

Figure 2 exemplifies the relationship amongst the optimal transmit power p and the number of BS antennas M. The black curve is relatively steep and is obtained with 10 users per cell, which is reasonable, because when the number of antennas is fixed, the more users, the more energy-efficient the transmission power will naturally be. It can be known from some classic literature that when the number of BS antennas M approaches to infinity and the system has ideal channel state information, the user's the transmit power is inversely proportional to the BS antennas (in the case of non-ideal CSI, it is inversely proportional to the square root of the number of antennas), although this conclusion has been known widely, but not applicable in the analysis here. As we can observe from the figure 2, when the value of M is large, the optimal transmission power almost linearly increases with the M BS antennas.

The previously mentioned Lambert function $Q(s)$ satisfies $Q(0) = 0$. When s is greater than 0, the function is an increasing function which satisfies the following inequality.

$$\frac{se}{\log(s)} \leq e^{Q(s)+1} \leq \frac{s}{\log(s)}(1+e), \quad s \geq e \tag{14}$$

Using this inequality, when M is large, the optimal power in (9) satisfies

$$p^{optimal} \geq \frac{(\tilde{B}_0+\tilde{B}_1 M)-\frac{ln\left((M-N)(B_0+B_1 M)-1\right)}{M-N}}{\ln\left((M-N)(\tilde{B}_0+\tilde{B}_1 M)-1\right)-1} = \begin{cases} O\left(\frac{M}{lnM}\right), & \tilde{B}_1 > 0 \\ O\left(\frac{1}{lnM}\right), & \tilde{B}_1 = 0 \end{cases} \tag{15}$$

Where, $\tilde{B}_0 = \frac{\eta \sum_{i=0}^{3} B_{i,0} N^i}{N Z_\zeta}$ $\tilde{B}_1 = \frac{\eta \sum_{i=0}^{2} B_{i,0} N^i}{N Z_\zeta}$ the above expression uses the representation method of the symbol O. When M increases, the denominator increase is negligible compared to the numerator. Therefore, the maximum emission power of EE increases almost linearly with M. which is fully proved by the figure above.

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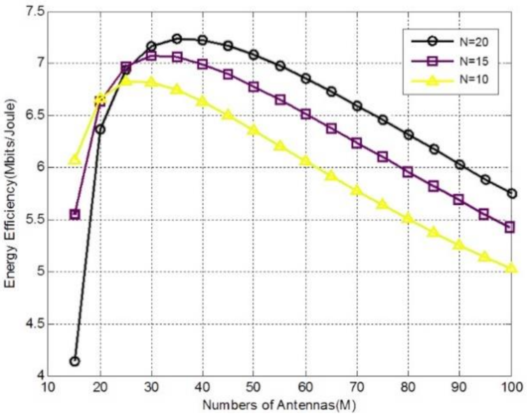

Figure 3. Energy Efficiency Analysis with Number of Antennas

Figure 3 describes the influence of the base station antennas on EE for the different number of users, and the transmission power is optimal. The overall trend of the curve is to increase first and then decrease. The maximum EE is obtained when the number of BS antennas is moderate. This has a strong guiding significance for the actual communication system. If you want to obtain better energy efficiency performance in the practical system, the more antennas the better EE will be. The three curves in the figure from high to low are respectively the number of users equal to 20, 15, and 10. In the leftmost part of the curve, the more the number of users is, the lower the EE is. When the number of BS antennas is increased to a certain value, the more the number of users is, the greater the energy efficiency will be. According to formula (6), the number of users N has an impact on the sum-rate of the numerator and the power consumption has on the denominator, but the growth rate of the numerator and denominator is related to the specific value of N, which is not absolute. This is also reflected in figure 4, where the top black curve has a downward trend when N is 100.

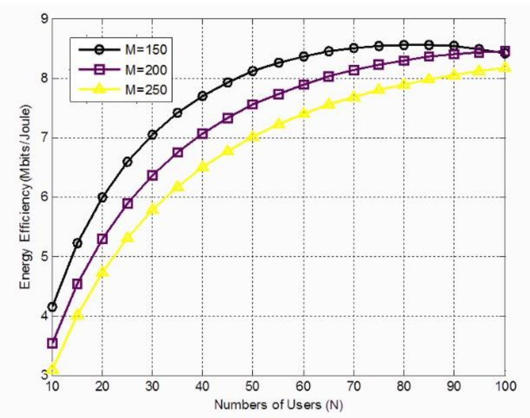

Figure 4. Energy Efficiency Analysis over Number of Users

Figure 4 shows the change in energy efficiency with the number of users under different antenna numbers, and the transmission power is optimal. Figure 3 and figure 4 are corresponding, and the conclusions obtained from both figures can also confirm each other. For example, when N is fixed at a larger value (150 to 200 or 250), the energy efficiency value will decrease, when M is fixed (which is a larger value), N increases at a smaller value (10 to 15 or 20), and the energy efficiency value will increase.

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