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New approaches to solve the Triangular Fuzzy Interval Valued Integral Equation

G. Veeramalai, K.Chitirakala
Assistant Professors, Department of Mathematics,
M.Kumarasamy College of Engineering (Autonomous), Karur

Abstract

In this article, another strategy for fuzzy function integration is considered. Another strategy for solving interval valued triangular fuzzy integral equations is proposed. The proposed technique is straightforward to apply for solving interval valued triangular fuzzy integral equations and Numerical cases are given to delineate the proposed strategy.

Keywords: *Fuzzy function, Fuzzy integral equations, Triangular valued fuzzy number, etc.*

Introduction:

One field of the integration problem assumes a noteworthy part in different regions, for example, arithmetic, material science, measurements, designing and sociologies. In all of genuine issues, not the greater part of the information can be unequivocally evaluated. At the point when data is effortlessly quantifiable or open, the data ought to be represented in fresh numbers. Fuzzy number theory makes it conceivable to fuse unquantifiable data, fragmented data and non realistic data as numerical models. The idea of fuzzy number and arithmetic task using the above numbers were first presented and examined by zadeh [1].

Numerous outcomes on interval valued fuzzy set have showed up. Essential hypothesis for interval valued is talked about by [2] and the idea of fuzzy functions integration was first presented by Dubois and Prade[25].The subjects of numerical techniques for solving fuzzy integral eqations have been quickly developing lately and have been examined. The numerical techniques for fuzzy differential equations have been contemplated by S.Abbasbandy, T.Allahviranloo,[5,6,7] and others.

This paper is structured as per the following, Section 2 starts with some fundamental definitions on fuzzy numbers and a short review of the interval valued triangular fuzzy numbers. Section 3 characterizes interval valued fuzzy triangular integral equations. Numerical cases are given in Section 4 to delineate the proposed technique. A brief conclusion is given in section 5.

Preliminaries:

We have recalled various essential definitions on fuzzy numbers in this section.

Triangular Fuzzy Number

The fuzzy system of numeration that typically utilized in applications is that the triangular (shaped) fuzzy numbers [8].

2.1 Fuzzy set: [1]

A fuzzy set L must the three axioms,

i . $\tilde{L}$ is a ordinary set.

ii. $^{\alpha}\tilde{L}$ is closed interval , for all $\alpha \in [0,1]$

iii. $\tilde{L}$, $^{0+}\tilde{L}$ is bounded.

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2.2 Triangular Fuzzy Number: [12]

A fuzzy numbers delineated with three points as: $\tilde{L} = (l_1, l_2, l_3)$

This illustration is taken as membership rule and holds the subsequent axioms

(i) Increasing function is l_1 to l_2

(ii) Decreasing function is l_2 to l_3

(iii) $l_1 \le l_2 \le l_3$

$$\mu_L(x) = \begin{cases} 0, & for\ x < l_1 \\ \dfrac{x - l_1}{l_2 - l_1} & for\ l_1 < x < l_2 \\ \dfrac{l_3 - x}{l_3 - l_1} & for\ l_2 < x < l_3 \\ 0, & for\, x > l_3 \end{cases}$$

2. 3. A Triangular fuzzy number is positive is defined as $\tilde{L} = (l_1, l_2, l_3)$, here $l_i > 0, \ \forall i$

2.4 A Triangular fuzzy number is negative is defined as $\tilde{L} = (l_1, l_2, l_3)$, here $l_i < 0, \ \forall i$

2.5. Two triangular fuzzy numbers $\tilde{L}$ and $\tilde{M}$ are identically equal, that is $\tilde{L} = \tilde{M}$, if and only if $l_1 = m_1$, $l_2 = m_2$ and $l_3 = m_3$

2.6 Interval arithmetic

Let, $\tilde{k} = [k_1, k_2]$, $\tilde{l} = [l_1, l_2]$

(I). Addition

$$\tilde{k} + \tilde{l} = [k_1 + l_1, k_2 + l_2]$$

(II). Subtraction

$$\tilde{k} - \tilde{l} = [k_1 - l_2, k_2 - l_1]$$

(III). Multiplication

$$\tilde{k}.\tilde{l} = [\min(k_1 l_1, k_1 l_2, k_2 l_1, k_2 l_2), \max(k_1 l_1, k_1 l_2, k_2 l_1, k_2 l_2)]$$

(IV). Division

$$\frac{[p, q]}{[r, s]} = [p, q].[\frac{1}{r}, \frac{1}{s}]$$ If $0 \notin [r, s]$

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(V). $\mu\tilde{k} = [\mu k_1, \mu k_2]\, for \mu \geq 0$

$[\mu k_2, \mu k_1]\, for \mu < 0$

(VI). Inverse

$$[k_1, k_2]^{-1} = [\frac{1}{k_2}, \frac{1}{k_1}],\ for 0 \notin [k_1, k_2]$$

(VII). $[k_1, k_2]^n = [k_1{}^n, k_2{}^n], if k_1 \geq 0$

$= [k_2{}^n, k_1{}^n], if k_2 < 0$

$= [0, \max\{k_1{}^n, k_2{}^n\}], otherwise$

2.2.7. Interval valued Triangular Fuzzy number:

If $\tilde{A} = [A^-, A^+]$, $\tilde{B} = [B^-, B^+]$ are two interval valued triangular fuzzy number and $\tilde{A}^- = (a_1^-, a_2^-, a_3^-)$, $\tilde{A}^+ = (a_1^+, a_2^+, a_3^+)$, $\tilde{B}^- = (b_1^-, b_2^-, b_3^-)$ and $\tilde{B}^+ = (b_1^+, b_2^+, b_3^+)$ then for $\forall k \in R^+$

$$\tilde{A} + \tilde{B} = [A^- + B^-,\ A^+ + B^+]$$

$$\tilde{A} - \tilde{B} = [A^- - B^-,\ A^+ - B^+]$$

$$k\tilde{A} = [kA^-,\ kA^+]$$

Where $A^- \pm B^- = (a_1^- \pm b_1^-,\ a_2^- \pm b_2^-, a_3^- \pm b_3^-)$

$A^+ \pm B^+ = (a_1^+ \pm b_1^+,\ a_2^+ \pm b_2^+, a_3^+ \pm b_3^+)$

$kA^- = (ka_1^-,\ ka_2^-, ka_3^-)$ and $kA^+ = (ka_1^+,\ ka_2^+, ka_3^+)$

3. Proposed Method

We define the one dimensional interval valued triangular fuzzy integral equation $\int_{[a,b]} F(t)dt$. As F is continuous, it follows that two continuous real valued functions $\underline{F}$ and $\overline{F}$ are there in such a way that when t is real, $F(t) = [\underline{F}(t),\ \overline{F}(t)]$

Also, we define the integral by $\int_{[a,b]} F(t)dt = \left[\int_{[a,b]} \underline{F}(t)dt,\ \int_{[a,b]} \overline{F}(t)dt \right]$

Suppose $0 < a < b$, and $\int_{[a,b]} F(t)dt = \int_{[a,b]} P(t)dt$, here $P(t)$ interval valued triangular fuzzy number

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$$\int_{[a,b]} F(t)dt = \int_{[a,b]} \left(P_1(t), P_2(t), P_3(t)\right)dt$$

We have

$$\int_{[a,b]} F(t)dt = \left(\left[\int_{[a,b]} \underline{P_1}(t)dt, \int_{[a,b]} \overline{P_1}(t)dt\right], \left[\int_{[a,b]} \underline{P_2}(t)dt, \int_{[a,b]} \overline{P_2}(t)dt\right], \left[\int_{[a,b]} \underline{P_3}(t)dt, \int_{[a,b]} \overline{P_3}(t)dt\right]\right)$$

$$= \left(\left[\underline{P_1}(b-a), \overline{P_1}(b-a)\right], \left[\underline{P_2}(b-a), \overline{P_2}(b-a)\right], \left[\underline{P_3}(b-a), \overline{P_3}(b-a)\right]\right)$$

$\int_{[a,b]} F(t)dt$ shows that it should be more concenterated when power type functions having interval coefficients are integrated. Here we will summarize the results for an interval polynomial of the form

$P(t) = A_0 + A_1 t + A_2 t^2 + ... + A_k t^k$, All A_i's are interval valued triangular fuzzy number and t is real.

Two cases are there

Case (i): Consider a and b with same sign, then

$$\int_{[a,b]} P(t)dt = A_0\left(b-a\right) + A_1(b-a) + ... + A_k \left(b^{k+1} - a^{k+1}\right)\Big/ k+1$$

Cases (ii): Consider $a < 0 < b$

then $$\int_{[a,b]} P(t)dt = T_0 + T_1 + T_2 + ... + T_k$$

$$T_i = \begin{cases} A_i\left(\left(b^{i+1} - a^{i+1}\right)\Big/ i+1\right), & i \text{ is even}, \\ \left[\left(\left(\underline{A_i}\, b^{i+1} - \overline{A_i}\, a^{i+1}\right)\Big/ i+1\right), \left(\left(\overline{A_i}\, b^{i+1} - \underline{A_i}\, a^{i+1}\right)\Big/ i+1\right)\right], & i \text{ is odd} \end{cases}$$

Where

4. Numerical Example:

Consider the following interval valued triangular fuzzy integral equations and by proposed method.

$$\int_{[a,b]} F(t)\,dt = \int_{[0,1]} \left(\frac{2r}{1+x^2}, \frac{4-2r}{1+x^2}\right)dx, \quad r \in (0,1)$$

We have the interval valued triangular fuzzy integral equation

$$\int_{[a,b]} F(t)\,dt = \int_{[a,b]} \left(P_1(t), P_2(t), P_3(t)\right)dt = \int_{[0,1]} \left(\left[0, \frac{4}{1+x^2}\right], \left[\frac{1}{1+x^2}, \frac{3}{1+x^2}\right], \left[\frac{2}{1+x^2}, \frac{2}{1+x^2}\right]\right)dx$$

Triangular fuzzy interval valued integral equation, we obtained

$$\int_{[0,1]} \left(\left[0, \frac{4}{1+x^2}\right], \left[\frac{1}{1+x^2}, \frac{3}{1+x^2}\right], \left[\frac{2}{1+x^2}, \frac{2}{1+x^2}\right]\right)dx = \left([0, 3.1348], [0.7837, 2.3511], [1.5674, 1.5674]\right)$$

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(i.e.) the solution is Midpoint of interval **is 3.1348** and the exact solution is

$$\int_{[a,b]} F(t)\,dt = \int_{[0,1]} \left(\frac{2r}{1+x^2}, \frac{4-2r}{1+x^2} \right) dx = 3.1348$$

Conclusion:

In this Work, another strategy is used to solve the interval valued triangular fuzzy integral equations. Here interval valued triangular fuzzy equation is utilized to build another strategy for solving interval valued integral equations and the legitimacy of the suggested technique is inspected with numerical cases. The developed technique is productive to resolve the interval valued triangular fuzzy integral equations happening in the real life situations.

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