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# Difference Perfect Square Cordial Labeling of Subdivision of Snake Graphs

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***Abstract***

*A graph $G=(p,q)$ with $p$ vertices and $q$ edges is said to have a Difference Perfect Square Cordial labeling if there exists a bijection $f:V(G)\to\{1,2,3,\ldots,p\}$ such that for each edge $e=uv$ the induced map $f^*:E(G)\to\{0,1\}$ is defined by,*

$$f^*(uv)=1 \quad \text{if } u^2-2uv+v^2=1$$
$$=0 \quad \text{otherwise}$$

*and $|e_f(0)-e_f(1)|\le 1$ where , $e_f(0)$ =number of edges with zero label and $e_f(1)$ =number of edges with one label.*

*In this paper we obtain Difference perfect square cordial labeling of $S(T_n)$, $S(Q_n)$, $S(DT_n)$, $S(DQ_n)$, $S(AT_n)$, $S(AQ_n)$, $S(DAT_n)$, $S(DAQ_n)$ graphs.*

***Keywords---*** *Difference perfect square cordial labeling, $S(T_n)$, $S(Q_n)$, $S(DT_n)$, $S(DQ_n)$ graphs.*

## I. INTRODUCTION

All the graphs in this paper are finite and undirected. The symbols $V(G)$ & $E(G)$ denotes the vertex set and edge set of a graph $G$. An excellence reference on this subject is the survey by J. A. Gallian [1].

U. V. Vaghela and D. B. Parmar [3] has define a concept of new labeling which is Difference perfect square cordial labeling. The definitions which are useful for the present investigation are below. We refer Gross and Yellen [2], for all kinds of definitions and notations.

A graph $G=(p,q)$ with $p$ vertices and $q$ edges is said to admit Difference perfect square cordial labeling if there exists a bijection $f:V(G)\to\{1,2,3,\ldots,p\}$ such that for each edge $e=uv$ the induced map $f^*:E(G)\to\{0,1\}$ is defined by,

$$f^*(uv)=1 \qquad \text{if } u^2-2uv+v^2=1$$
$$=0 \qquad \text{otherwise}$$

and $|e_f(0)-e_f(1)|\le 1$ where , $e_f(0)$ =number of edges with zero label and $e_f(1)$ =number of edges with one label.A graph which admits Difference perfect square cordial labeling is said to be Difference perfect square cordial graph. [5]

**Definition:** The triangular snake $T_n$ is obtained from the path $P_n$ by replacing each edge of the path by triangle $C_3$.[5]

**Definition:** A Quadrilateral snake $Q_n$ is obtained from a path $u_1,u_2,\ldots,u_n$ by joining $u_i$ and $u_{i+1}$ to a new vertex $v_i$ and $w_i$ respectively and then joining $v_i$ and $w_i$.That is every edge of a path is replaced by a cycle $C_4$.[5]

**Definition:** A double triangular snake $D(T_n)$ consist of two triangular snakes that have a common path.[5]

**Definition:** A double quadrilateral snake $D(Q_n)$ consist of two quadrilateral snakes that have a common path.[5]



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**Definition:** An alternate triangular snake $A(T_n)$ is obtained from the path $P_n$ by replacing every edge of path by a triangle $C_3$. That is it is obtained from a path $u_1, u_2, \dots u_n$ by joining $u_i u_{i+1}$(alternatively) to a new vertex $v_i$.[5]

**Definition:** An alternate quadrilateral snake $A(Q_n)$ is obtained from the path $P_n$= $u_1, u_2, \dots u_n$ by replacing every alternate edge of a path by cycle $C_4$ , in such a way that each pair of vertices $(u_i, u_{i+1})$ remains adjacent. That is it is obtained from a path by joining $u_i$ & $u_{i+1}$(alternatively) to a new vertex $v_i$ and $w_i$ respectively and then joining $v_i$ and $w_i$ by an edge.[5]

**Definition:** A double alternate triangular snake $DA(T_n)$ consists of two alternate triangular snakes that have a common path. [5]

**Definition:** A double alternate quadrilateral snake $DA(Q_n)$ consists of two alternate quadrilateral snakes that have a common path.[5]

**Definition:** The subdivision of a graph is the graph obtained by subdividing each edge of a graph $G$.

It is denoted by $S(G)$.

## II. MAIN RESULTS

**Theorem 2.1** $S(T_n)$ is a difference perfect square cordial graph.

**Proof:** Let $G = S(T_n)$

Let the edges $u_k u_{k+1}$ , $u_k v_k$ , $u_{k+1} v_k$ be subdivided by $r_k$ , $s_k$ , $t_k$ respectively.

$V(G) = \{u_k \ : 1 \le k \le n\} \cup \{v_k, r_k , s_k , t_k \ : 1 \le k \le n-1\}$

$E(G) = \{(u_k\, r_k): 1 \le k \le n-1\} \cup \{(r_k\, u_{k+1}): 1 \le k \le n-1\} \cup \{u_k s_k: 1 \le k \le n-1\}$
$\cup \{(s_k\, v_k): 1 \le k \le n-1\} \cup \{u_{k+1} t_k: 1 \le k \le n-1\} \cup \{(t_k\, v_k): 1 \le k \le n-1\}$

So, $|V(G)| = 5n - 4$ & $|E(G)| = 6n - 6$.

Define $f: V(G) \to \{1,2,3, \dots, 5n-4\}$ as follows.

**Case:1** $n = 2$

$f(u_1) = 1$

$f(u_2) = 6$

$f(r_1) = 5$

$f(s_1) = 2$

$f(v_1) = 3$

$f(t_1) = 4$

For this we define following edge function, $f^*: E(G) \to \{0,1\}$ by

$f^*(u_1 r_1) = 0$

$f^*(r_1 u_2) = 0$

$f^*(t_1 u_2) = 0$

$f^*(s_1 v_1) = 1$

$f^*(u_1 s_1) = 1$

$f^*(v_1 t_1) = 1$



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**Case 2:** $n > 2$

Define $f: V(G) \to \{1,2,3, \dots, 5n-4\}$ as follows.

$f(u_k) = 2k - 1 \quad 1 \le k \le n$

$f(r_k) = 2k \quad 1 \le k \le n-1$

$f(s_k) = 2n + 2k - 2 \quad 1 \le k \le n-1$

$f(t_k) = 4n - 3 + k \quad 1 \le k \le n-1$

$f(v_k) = 2n - 1 + 2k \quad 1 \le k \le n-1$

For this we define following edge function, $f^*: E(G) \to \{0,1\}$ by

$f^*(u_k r_k) = 1 \quad 1 \le k \le n-1$

$f^*(r_k u_{k+1}) = 1 \quad 1 \le k \le n-1$

$f^*(u_k s_k) = 0 \quad 1 \le k \le n-1$

$f^*(s_k v_k) = 1 \quad 1 \le k \le n-1$

$f^*(u_{k+1} t_k) = 0 \quad 1 \le k \le n-1$

$f^*(t_k v_k) = 0 \quad 1 \le k \le n-1$

| $\boldsymbol{n}$ | ***Edge conditions*** |
|---|---|
| $n \ge 2$ | $e_f(0) = 3n - 3 \quad , e_f(1) = 3n - 3$ |

We have $|e_f(0) - e_f(1)| \le 1$ , hence $S(T_n)$ is a difference perfect square cordial graph.

**Illustration:** A difference perfect square cordial labeling of $S(T_4)$ is shown in Figure-1.

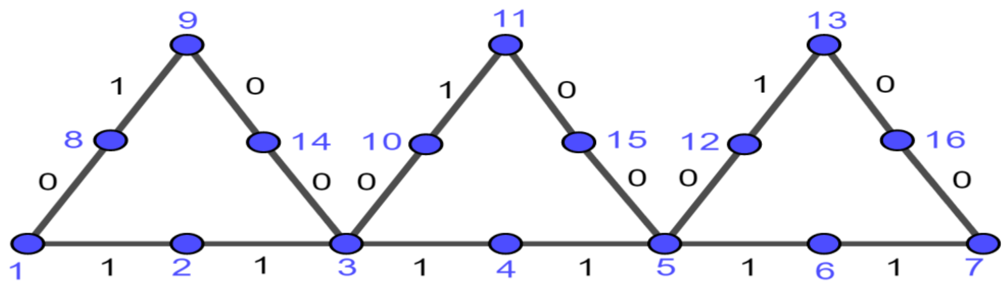


**Figure:1** $\boldsymbol{S(T_4)}$

**Theorem 2.2** The $S(Q_n)$ is a difference perfect square cordial graph.

**Proof:** Let $G = S(Q_n)$

Let the edges $u_k u_{k+1}, u_k v_k, v_k w_k, u_{k+1} w_k$ be subdivided by $r_k, s_k, x_k, t_k$ respectively.

$V(G) = \{u_k : 1 \le k \le n\} \cup \{v_k, r_k, s_k, t_k, w_k, x_k : 1 \le k \le n-1\}$

$E(G) = \{(u_k r_k): 1 \le k \le n-1\} \cup \{(r_k u_{k+1}): 1 \le k \le n-1\} \cup \{u_k s_k: 1 \le k \le n-1\} \cup \{(s_k v_k): 1 \le k \le n-1\} \cup \{u_{k+1} t_k: 1 \le k \le n-1\} \cup \{(v_k x_k): 1 \le k \le n-1\} \cup \{(x_k w_k): 1 \le k \le n-1\} \cup \{(t_k w_k): 1 \le k \le n-1\}$

So, $|V(G)| = 7n - 6$ & $|E(G)| = 8n - 8$..



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Define $f: V(G) \rightarrow \{1,2,3, \ldots, 7n-6\}$ as follows.

$f(u_k) = 2k - 1 \quad 1 \leq k \leq n$

$f(r_k) = 2k \quad 1 \leq k \leq n - 1$

$f(v_k) = 2n + 3k - 3 \quad 1 \leq k \leq n - 1$

$f(x_k) = 2n + 3k - 2 \quad 1 \leq k \leq n - 1$

$f(w_k) = 2n + 3k - 1 \quad 1 \leq k \leq n - 1$

$f(s_k) = 5n + k - 4 \quad 1 \leq k \leq n - 1$

$f(t_k) = 6n + k - 5 \quad 1 \leq k \leq n - 1$

For this we define following edge function, $f^*: E(G) \rightarrow \{0,1\}$ by

$f^*(u_k r_k) = 1 \quad 1 \leq k \leq n - 1$

$f^*(r_k u_{k+1}) = 1 \quad 1 \leq k \leq n - 1$

$f^*(v_k x_k) = 1 \quad 1 \leq k \leq n - 1$

$f^*(x_k w_k) = 1 \quad 1 \leq k \leq n - 1$

$f^*(u_k s_k) = 0 \quad 1 \leq k \leq n - 1$

$f^*(s_k v_k) = 0 \quad 1 \leq k \leq n - 1$

$f^*(u_{k+1} t_k) = 0 \quad 1 \leq k \leq n - 1$

$f^*(t_k w_k) = 0 \quad 1 \leq k \leq n - 1$

| $n$ | ***Edge condition*** |
|---|---|
| All $n \geq 2$ | $e_f(0) = 4n - 4$ , $e_f(1) = 4n - 4$ |

we have $|e_f(0) - e_f(1)| \leq 1$ , hence $S(Q_n)$ is a difference perfect square cordial graph.

**Illustration:** A difference perfect square cordial labeling of $S(Q_4)$ is shown in Figure-2.

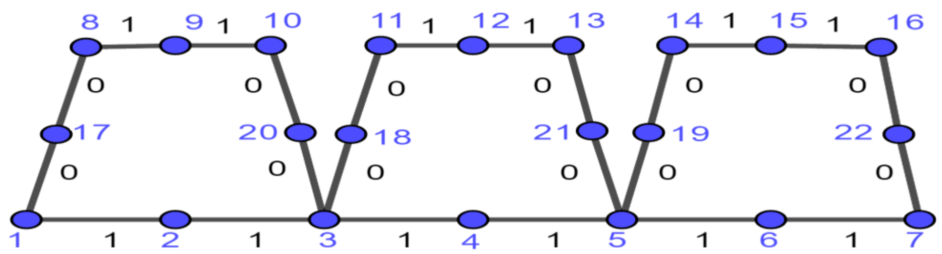


**Figure:2** $\boldsymbol{S(Q_4)}$

**Theorem 2.3** $S(DT_n)$ is a difference perfect square cordial graph.

**Proof:** Let $G = S(DT_n)$

Let the edges $u_k u_{k+1}, u_k v_k, u_{k+1} v_k, u_k w_k, u_{k+1} w_k$ be subdivided by $r_k, s_k, t_k, x_k, y_k$ respectively.

$V(G) = \{u_k : 1 \leq k \leq n\} \cup \{v_k, r_k, s_k, t_k, w_k, x_k, y_k : 1 \leq k \leq n - 1\}$



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$E(G) = \{(u_k\, r_k): 1 \leq k \leq n-1\} \cup \{(r_k\, u_{k+1}): 1 \leq k \leq n-1\} \cup \{u_k s_k: 1 \leq k \leq n-1\}$
$\cup\{(s_k\, v_k): 1 \leq k \leq n-1\} \cup \{u_{k+1} t_k: 1 \leq k \leq n-1\} \cup \{(u_k\, x_k): 1 \leq k \leq n-1\} \cup \{(x_k\, w_k): 1 \leq k \leq n-1\} \cup \{(t_k\, v_k): 1 \leq k \leq n-1\} \cup \{(u_{k+1}\, y_k): 1 \leq k \leq n-1\} \cup \{(y_k\, w_k): 1 \leq k \leq n-1\}$

So, $|V(G)| = 8n-7$ & $|E(G)| = 10n-10$.

Define $f: V(G) \to \{1,2,3, \dots, 8n-7\}$ as follows.

$f(u_k) = 4k-3 \quad 1 \leq k \leq n$

$f(r_k) = 7n-6+k \quad 1 \leq k \leq n-1$

$f(v_k) = 4k-1 \quad 1 \leq k \leq n-1$

$f(x_k) = 4n+2k-4 \quad 1 \leq k \leq n-1$

$f(w_k) = 4n+2k-3 \quad 1 \leq k \leq n-1$

$f(s_k) = 4k-2 \quad 1 \leq k \leq n-1$

$f(t_k) = 4k \quad 1 \leq k \leq n-1$

$f(y_k) = 6n-5+k \quad 1 \leq k \leq n-1$

For this we define following edge function, $f^*: E(G) \to \{0,1\}$ by

$f^*(u_k r_k) = 0 \quad 1 \leq k \leq n-1$

$f^*(r_k u_{k+1}) = 0 \quad 1 \leq k \leq n-1$

$f^*(u_k s_k) = 1 \quad 1 \leq k \leq n-1$

$f^*(u_{k+1} t_k) = 1 \quad 1 \leq k \leq n-1$

$f^*(s_k v_k) = 1 \quad 1 \leq k \leq n-1$

$f^*(t_k v_k) = 1 \quad 1 \leq k \leq n-1$

$f^*(u_k x_k) = 0 \quad 1 \leq k \leq n-1$

$f^*(x_k w_k) = 1 \quad 1 \leq k \leq n-1$

$f^*(u_{k+1} y_k) = 0 \quad 1 \leq k \leq n-1$

$f^*(y_k w_k) = 0 \quad 1 \leq k \leq n-1$

| $n$ | ***Edge condition*** |
|---|---|
| $n \geq 2$ | $e_f(0) = 5n-5$ , $e_f(1) = 5n-5$ |

We have $|e_f(0) - e_f(1)| \leq 1$ , hence $S(DT_n)$is a difference perfect square cordial graph.

**Illustration:** A difference perfect square cordial labeling of $S(DT_4)$ is shown in Figure-3.



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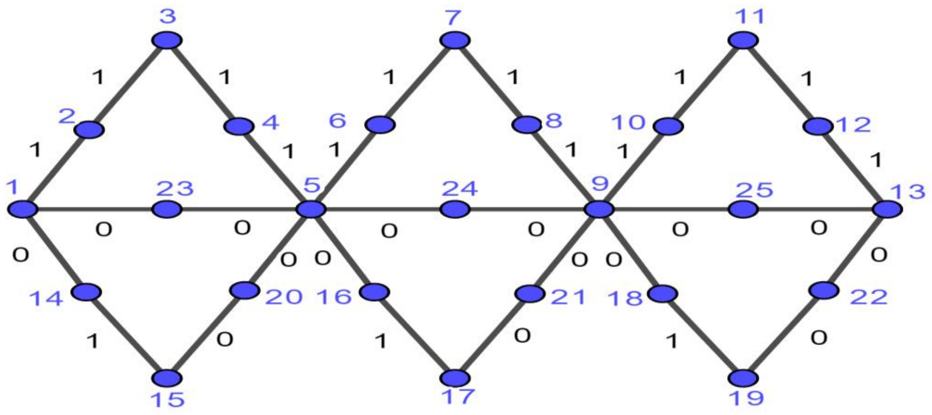


**Figure:3** $\boldsymbol{S(DT_4)}$

**Theorem 2.4** The $S(DQ_n)$ is a difference perfect square cordial graph.

**Proof:** Let $G = S(DQ_n)$

Let the edges $u_k u_{k+1}, u_k v_k, v_k w_k, u_{k+1} w_k,\ u_k x_k, x_k y_k, u_{k+1} y_k$ be subdivided by $r_k, s_k, n_k, t_k, z_k, m_k, j_k$ respectively.

$V(G) = \{u_k : 1 \le k \le n\} \cup \{v_k, r_k, s_k, t_k, w_k, x_k, y_k, n_k, z_k, m_k, j_k : 1 \le k \le n-1\}$

$E(G) = \{(u_k\, r_k) : 1 \le k \le n-1\} \cup \{(r_k\, u_{k+1}) : 1 \le k \le n-1\} \cup \{u_k s_k : 1 \le k \le n-1\} \cup \{(s_k\, v_k) : 1 \le k \le n-1\} \cup \{u_{k+1} t_k : 1 \le k \le n-1\} \cup \{(t_k\, w_k) : 1 \le k \le n-1\} \cup \{(v_k\, n_k) : 1 \le k \le n-1\} \cup \{(n_k\, w_k) : 1 \le k \le n-1\} \cup \{(u_k\, z_k) : 1 \le k \le n-1\} \cup \{(z_k\, x_k) : 1 \le k \le n-1\} \cup \{(x_k\, m_k) : 1 \le k \le n-1\} \cup \{(m_k\, y_k) : 1 \le k \le n-1\} \cup \{(u_{k+1}\, j_k) : 1 \le k \le n-1\} \cup \{(j_k\, y_k) : 1 \le k \le n-1\}$

So, $|V(G)| = 12n - 11$ & $|E(G)| = 14n - 14$.

Define $f : V(G) \to \{1,2,3,\dots,12n-11\}$ as follows.

$f(u_k) = 6k - 5 \qquad 1 \le k \le n$

$f(s_k) = 6k - 4 \qquad 1 \le k \le n-1$

$f(v_k) = 6k - 3 \qquad 1 \le k \le n-1$

$f(n_k) = 6k - 2 \qquad 1 \le k \le n-1$

$f(w_k) = 6k - 1 \qquad 1 \le k \le n-1$

$f(t_k) = 6k \qquad 1 \le k \le n-1$

$f(z_k) = 6n - 6 + 2k \qquad 1 \le k \le n-1$

$f(x_k) = 6n - 5 + 2k \qquad 1 \le k \le n-1$

$f(m_k) = 8n - 7 + k \qquad 1 \le k \le n-1$

$f(y_k) = 9n - 8 + k \qquad 1 \le k \le n-1$

$f(j_k) = 10n - 9 + k \qquad 1 \le k \le n-1$

$f(r_k) = 11n - 10 + k \qquad 1 \le k \le n-1$

For this we define following edge function, $f^* : E(G) \to \{0,1\}$ by

$f^*(u_k r_k) = 0 \qquad 1 \le k \le n-1$



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$f^*(r_k u_{k+1}) = 0$ $\quad 1 \le k \le n-1$

$f^*(u_k s_k) = 1$ $\quad 1 \le k \le n-1$

$f^*(s_k v_k) = 1$ $\quad 1 \le k \le n-1$

$f^*(v_k n_k) = 1$ $\quad 1 \le k \le n-1$

$f^*(n_k w_k) = 1$ $\quad 1 \le k \le n-1$

$f^*(u_{k+1} t_k) = 1$ $\quad 1 \le k \le n-1$

$f^*(t_k w_k) = 1$ $\quad 1 \le k \le n-1$

$f^*(u_k z_k) = 0$ $\quad 1 \le k \le n-1$

$f^*(z_k x_k) = 1$ $\quad 1 \le k \le n-1$

$f^*(x_k m_k) = 0$ $\quad 1 \le k \le n-1$

$f^*(m_k y_k) = 0$ $\quad 1 \le k \le n-1$

$f^*(y_k j_k) = 0$ $\quad 1 \le k \le n-1$

$f^*(u_{k+1} j_k) = 0$ $\quad 1 \le k \le n-1$

| $n$ | *Edge conditions* |
|---|---|
| $n \ge 2$ | $e_f(0) = 7n-7$ , $e_f(1) = 7n-7$ |

We have $|e_f(0) - e_f(1)| \le 1$ , hence $S(DQ_n)$ is a difference perfect square cordial graph.

**Illustration:** A difference perfect square cordial labeling of $S(DQ_4)$ is shown in Figure-4.

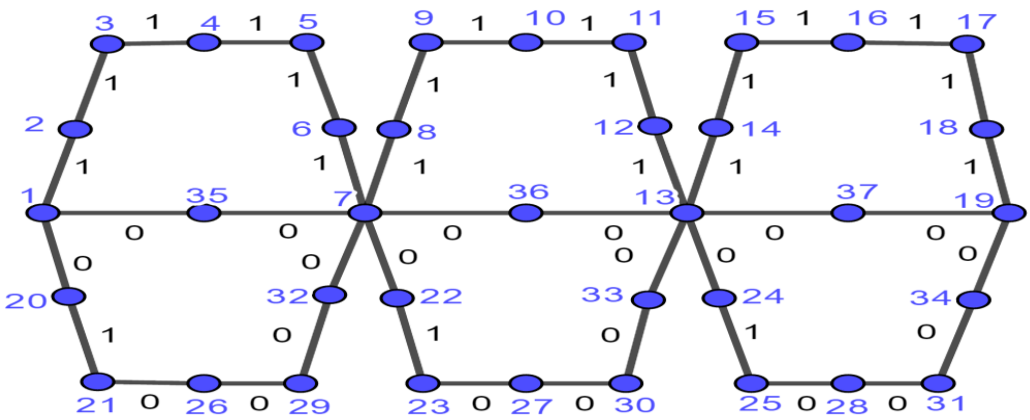

**Figure:4** $\boldsymbol{S(DQ_4)}$

**Theorem 2.5** $S(A(T_n))$ is a difference perfect square cordial graph.

**Proof:** Let $G = S(A(T_n))$

Let the edges $u_k u_{k+1}$ , $u_k v_k$ , $u_{k+1} v_k$ be subdivided by $r_k$ , $s_k$ , $t_k$ respectively.

**Case:1** Let the first triangle starts from $u_2$ and the last ends with $u_{n-1}$.

$V(G) = \{u_k : 1 \le k \le n\} \cup \{r_k : 1 \le k \le n-1\} \cup \{s_k, v_k, t_k : 1 \le k \le \frac{n-2}{2}\}$



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$E(G) = \{(u_1 r_1)\} \cup \{(u_{k+1}\, r_{k+1}): 1 \le k \le n-2\} \cup \{(r_k\, u_{k+1}): 1 \le k \le n-1\} \cup \{u_{2k}s_k: 1 \le k \le \frac{n-2}{2}\} \cup \{s_k v_k: 1 \le k \le \frac{n-2}{2}\} \cup \{u_{2k+1}t_k: 1 \le k \le \frac{n-2}{2}\} \cup \{t_k v_k: 1 \le k \le \frac{n-2}{2}\}$

So, $|V(G)| = \frac{7n-8}{2}$ & $|E(G)| = 4n-6$.

Define $f: V(G) \to \{1,2,3,\dots,\frac{7n-8}{2}\}$ as follows.

$f(u_1) = 3n-3$

$f(u_k) = 2k-2 \qquad 2 \le k \le n$

$f(r_k) = 2k-1 \qquad 1 \le k \le n-1$

$f(s_k) = 2n+k-2 \qquad 1 \le k \le \frac{n-2}{2}$

$f(t_k) = \frac{5n-6}{2} + k \qquad 1 \le k \le \frac{n-2}{2}$

$f(v_k) = 3n-3+k \qquad 1 \le k \le \frac{n-2}{2}$

For this we define following edge function, $f^*: E(G) \to \{0,1\}$ by

$f^*(u_1 r_1) = 0$

$f^*(u_{k+1} r_{k+1}) = 1 \qquad 1 \le k \le n-2$

$f^*(r_k u_{k+1}) = 1 \qquad 1 \le k \le n-1$

$f^*(u_{2k} s_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(s_k v_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(u_{2k+1} t_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(t_k v_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

| $n$ | *Edge conditions* |
|---|---|
| $n \ge 4$ | $e_f(0) = 2n-3$ , $e_f(1) = 2n-3$ |

**Case:2** Let the first triangle starts from $u_2$ and the last ends with $u_n$.

$V(G) = \{u_k : 1 \le k \le n\} \cup \{r_k : 1 \le k \le n-1\} \cup \{s_k, v_k, t_k : 1 \le k \le \frac{n-1}{2}\}$

$E(G) = \{(u_k\, r_k): 1 \le k \le n-1\} \cup \{(r_k\, u_{k+1}): 1 \le k \le n-1\} \cup \{u_{2k}s_k: 1 \le k \le \frac{n-1}{2}\} \cup \{s_k v_k: 1 \le k \le \frac{n-1}{2}\} \cup \{u_{2k+1}t_k: 1 \le k \le \frac{n-1}{2}\} \cup \{t_k v_k: 1 \le k \le \frac{n-1}{2}\}$

So, $|V(G)| = \frac{7n-5}{2}$ & $|E(G)| = 4n-4$.

**Sub Case :1** $n = 3$

Define $f: V(G) \to \{1,2,3,\dots,\frac{7n-5}{2}\}$ as follows.

$f(u_1) = 1$

$f(u_{k+1}) = 4k-2 \qquad 1 \le k \le n-1$

$f(r_k) = 4n-6+k \qquad 1 \le k \le n-1$



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$f(s_1) = 3$

$f(v_1) = 4$

$f(t_1) = 5$

For this we define following edge function, $f^*: E(G) \to \{0,1\}$ by

$f^*(u_k r_k) = 0 \qquad 1 \le k \le n-1$

$f^*(r_k u_{k+1}) = 0 \qquad 1 \le k \le n-1$

$f^*(u_2 s_1) = 1$

$f^*(s_1 v_1) = 1$

$f^*(v_1 t_1) = 1$

$f^*(t_1 u_3) = 1$

**Sub Case :2** $n > 3$

Define $f: V(G) \to \{1,2,3, \dots, \frac{7n-5}{2}\}$ as follows.

$f(u_k) = 2k - 1 \qquad 1 \le k \le n$

$f(r_k) = 2k \qquad 1 \le k \le n-1$

$f(s_k) = 2n + k - 1 \qquad 1 \le k \le \frac{n-1}{2}$

$f(t_k) = \frac{5n-3}{2} + k \qquad 1 \le k \le \frac{n-1}{2}$

$f(v_k) = 3n - 2 + k \qquad 1 \le k \le \frac{n-1}{2}$

For this we define following edge function, $f^*: E(G) \to \{0,1\}$ by

$f^*(u_k r_k) = 1 \qquad 1 \le k \le n-1$

$f^*(r_k u_{k+1}) = 1 \qquad 1 \le k \le n-1$

$f^*(u_{2k} s_k) = 0 \qquad 1 \le k \le \frac{n-1}{2}$

$f^*(s_k v_k) = 0 \qquad 1 \le k \le \frac{n-1}{2}$

$f^*(u_{2k+1} t_k) = 0 \qquad 1 \le k \le \frac{n-1}{2}$

$f^*(t_k v_k) = 0 \qquad 1 \le k \le \frac{n-1}{2}$

| $n$ | *Edge conditions* |
|---|---|
| $n \ge 3$ | $e_f(0) = 2n - 2$ , $e_f(1) = 2n - 2$ |

**Case:3** Let the first triangle starts from $u_1$ and the last ends with $u_n$.

$V(G) = \{u_k : 1 \le k \le n\} \cup \{r_k : 1 \le k \le n-1\} \cup \{s_k, v_k, t_k : 1 \le k \le \frac{n}{2}\}$

$E(G) = \{(u_k r_k): 1 \le k \le n-1\} \cup \{(r_k u_{k+1}): 1 \le k \le n-1\} \cup \{u_{2k-1} s_k: 1 \le k \le \frac{n}{2}\} \cup \{s_k v_k: 1 \le k \le \frac{n}{2}\} \cup \{u_{2k} t_k: 1 \le k \le \frac{n}{2}\} \cup \{t_k v_k: 1 \le k \le \frac{n}{2}\}$

So, $|V(G)| = \frac{7n-2}{2}$ & $|E(G)| = 4n - 2$.



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Define $f: V(G) \rightarrow \{1,2,3, \ldots, \frac{7n-2}{2}\}$ as follows.

$f(u_k) = 2k - 1 \qquad 1 \leq k \leq n$

$f(r_k) = 2k \qquad 1 \leq k \leq n - 1$

$f(s_k) = 2n + k - 1 \qquad 1 \leq k \leq \frac{n}{2}$

$f(t_k) = \frac{5n-2}{2} + k \qquad 1 \leq k \leq \frac{n}{2}$

$f\left(v_{\frac{n+2}{2}-k}\right) = 3n - 1 + k \qquad 1 \leq k \leq \frac{n}{2}$

For this we define following edge function, $f^*: E(G) \rightarrow \{0,1\}$ by

$f^*(u_k r_k) = 1 \qquad 1 \leq k \leq n - 1$

$f^*(r_k u_{k+1}) = 1 \qquad 1 \leq k \leq n - 1$

$f^*(u_{2k-1} s_k) = 0 \qquad 1 \leq k \leq \frac{n}{2}$

$f^*(s_k v_k) = 0 \qquad 1 \leq k \leq \frac{n}{2}$

$f^*(u_{2k} t_k) = 0 \qquad 1 \leq k \leq \frac{n}{2}$

$f^*(t_k v_k) = 0 \qquad 1 \leq k \leq \frac{n}{2} - 1$

$f^*\left(t_{\frac{n}{2}} v_{\frac{n}{2}}\right) = 1$

| $n$ | $Edge\ conditions$ |
|---|---|
| $n \geq 4$ | $e_f(0) = 2n - 1 \qquad , e_f(1) = 2n - 1$ |

In all cases we have $|e_f(0) - e_f(1)| \leq 1$ , hence $S(A(T_n))$ is a difference perfect square cordial graph.

**Illustration:** A difference perfect square cordial labeling of $S(A(T_5))$ is shown in Figure-5.

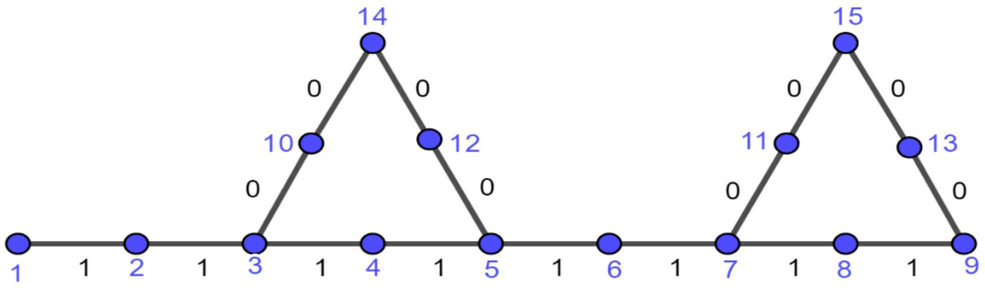


**Figure-5** $S(A(T_5))$

**Theorem 2.6** $SA(Q_n)$ is a difference perfect square cordial graph.

**Proof:** Let $G = SA(Q_n)$

Let the edges $u_k u_{k+1}, u_k v_k, v_k w_k, u_{k+1} w_k$ be subdivided by $r_k, s_k, x_k, t_k$ respectively.



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**Case:1** Let the first quadrilateral starts from $u_2$ and the last ends with $u_{n-1}$.

$V(G) = \{u_k \ : 1 \le k \le n\} \cup \{r_k \ : 1 \le k \le n-1\} \cup \{s_k, v_k, t_k, x_k, w_k \ : 1 \le k \le \frac{n-2}{2}\}$

$E(G) = \{(u_k r_k): 1 \le k \le n-1\} \cup \{(r_k u_{k+1}): 1 \le k \le n-1\} \cup \{u_{2k} s_k: 1 \le k \le \frac{n-2}{2}\}$
$\cup \{(s_k v_k): 1 \le k \le \frac{n-2}{2}\} \cup \{(u_{2k+1} t_k): 1 \le k \le \frac{n-2}{2}\} \cup \{(t_k w_k): 1 \le k \le \frac{n-2}{2}\} \cup \{v_k x_k: 1 \le k \le \frac{n-2}{2}\} \cup \{(x_k w_k): 1 \le k \le \frac{n-2}{2}\}$

So, $|V(G)| = \frac{9n-12}{2}$ & $|E(G)| = 5n-8$.

Define $f: V(G) \to \{1,2,3, \dots, \frac{9n-12}{2}\}$ as follows.

$f(u_1) = \frac{9n-12}{2}$

$f(u_k) = 2k-2 \qquad 2 \le k \le n$

$f(r_k) = 2k-1 \qquad 1 \le k \le n-1$

$f(s_k) = 2n+2k-3 \qquad 1 \le k \le \frac{n-2}{2}$

$f(x_k) = \frac{7n-10}{2} + k \qquad 1 \le k \le \frac{n-2}{2}$

$f(v_k) = 2n-2+2k \qquad 1 \le k \le \frac{n-2}{2}$

$f(t_k) = 3n-4+k \qquad 1 \le k \le \frac{n-2}{2}$

$f(w_k) = 4n-6+k \qquad 1 \le k \le \frac{n-2}{2}$

For this we define following edge function, $f^*: E(G) \to \{0,1\}$ by

$f^*(u_1 r_1) = 0$

$f^*(u_{k+1} r_{k+1}) = 1 \qquad 1 \le k \le n-2$

$f^*(r_k u_{k+1}) = 1 \qquad 1 \le k \le n-1$

$f^*(u_{2k} s_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(s_k v_k) = 1 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(u_{2k+1} t_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(t_k w_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(v_k x_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(x_k w_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

| $n$ | *Edge conditions* |
|---|---|
| $n \ge 4$ | $e_f(0) = \frac{5n-8}{2}$ , $e_f(1) = \frac{5n-8}{2}$ |

**Case:2** Let the first quadrilateral starts from $u_2$ and the last ends with $u_n$.

$V(G) = \{u_k \ : 1 \le k \le n\} \cup \{r_k \ : 1 \le k \le n-1\} \cup \{s_k, v_k, t_k, w_k, x_k : 1 \le k \le \frac{n-1}{2}\}$



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$E(G) = \{(u_k\, r_k): 1 \le k \le n-1\} \cup \{(r_k\, u_{k+1}): 1 \le k \le n-1\} \cup \{(u_{2k} s_k): 1 \le k \le \frac{n-1}{2}\}$ $\cup \{(s_k v_k): 1 \le k \le \frac{n-1}{2}\} \cup \{(u_{2k+1} t_k): 1 \le k \le \frac{n-1}{2}\} \cup \{(t_k w_k): 1 \le k \le \frac{n-1}{2}\} \cup \{(v_k x_k): 1 \le k \le \frac{n-1}{2}\} \cup \{(x_k w_k): 1 \le k \le \frac{n-1}{2}\}$

So, $|V(G)| = \frac{9n-7}{2}$ & $|E(G)| = 5n - 5$.

**Sub Case :1** $n = 3$

Define $f: V(G) \to \{1,2,3, \dots, \frac{9n-7}{2}\}$ as follows.

$f(u_1) = 1$

$f(u_{k+1}) = 3n + 1 - k \qquad 1 \le k \le n-1$

$f(r_k) = 3k + 4 \qquad 1 \le k \le n-1$

$f(s_1) = 2$

$f(v_1) = 3$

$f(x_1) = 4$

$f(w_1) = 5$

$f(t_1) = 6$

For this we define following edge function, $f^*: E(G) \to \{0,1\}$ by

$f^*(r_k u_{k+1}) = 0 \qquad 1 \le k \le n-1$

$f^*(u_1 r_1) = 0$

$f^*(u_2 r_2) = 1$

$f^*(u_2 s_1) = 0$

$f^*(s_1 v_1) = 1$

$f^*(v_1 x_1) = 1$

$f^*(x_1 w_1) = 1$

$f^*(w_1 t_1) = 1$

$f^*(t_1 u_3) = 0$

**Sub Case :1** $n > 3$

Define $f: V(G) \to \{1,2,3, \dots, \frac{9n-7}{2}\}$ as follows.

$f(u_k) = 2k - 1 \qquad 1 \le k \le n$

$f(r_k) = 2k \qquad 1 \le k \le n-1$

$f(s_k) = 2n + 2k - 2 \qquad 1 \le k \le \frac{n-1}{2}$

$f(w_k) = \frac{7n-5}{2} + k \qquad 1 \le k \le \frac{n-1}{2}$

$f(x_k) = 3n - 2 + k \qquad 1 \le k \le \frac{n-1}{2}$



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$f(v_k) = 2n - 1 + 2k \qquad 1 \le k \le \frac{n-1}{2}$

$f(t_k) = 4n - 3 + k \qquad 1 \le k \le \frac{n-1}{2}$

For this we define following edge function, $f^*: E(G) \to \{0,1\}$ by

$f^*(u_k r_k) = 1 \qquad 1 \le k \le n-1$

$f^*(r_k u_{k+1}) = 1 \qquad 1 \le k \le n-1$

$f^*(u_{2k} s_k) = 0 \qquad 1 \le k \le \frac{n-1}{2}$

$f^*(s_k v_k) = 1 \qquad 1 \le k \le \frac{n-1}{2}$

$f^*(u_{2k+1} t_k) = 0 \qquad 1 \le k \le \frac{n-1}{2}$

$f^*(t_k w_k) = 0 \qquad 1 \le k \le \frac{n-1}{2}$

$f^*(v_k x_k) = 0 \qquad 1 \le k \le \frac{n-1}{2}$

$f^*(x_k w_k) = 0 \qquad 1 \le k \le \frac{n-1}{2}$

| $\boldsymbol{n}$ | ***Edge conditions*** |
|---|---|
| $n \ge 3$ | $e_f(0) = \frac{5n-5}{2}$ , $e_f(1) = \frac{5n-5}{2}$ |

**Case:3** Let the first quadrilateral starts from $u_1$ and the last ends with $u_n$.

$V(G) = \{u_k : 1 \le k \le n\} \cup \{r_k : 1 \le k \le n-1\} \cup \{s_k, v_k, t_k, x_k, w_k : 1 \le k \le \frac{n}{2}\}$

$E(G) = \{(u_k r_k): 1 \le k \le n-1\} \cup \{(r_k u_{k+1}): 1 \le k \le n-1\} \cup \{u_{2k-1} s_k: 1 \le k \le \frac{n}{2}\}$ $\cup \{s_k v_k: 1 \le k \le \frac{n}{2}\} \cup \{u_{2k} t_k: 1 \le k \le \frac{n}{2}\} \cup \{t_k w_k: 1 \le k \le \frac{n}{2}\} \cup \{v_k x_k: 1 \le k \le \frac{n}{2}\} \cup \{x_k w_k: 1 \le k \le \frac{n}{2}\}$

So, $|V(G)| = \frac{9n-2}{2}$ & $|E(G)| = 5n-2$.

Define $f: V(G) \to \{1,2,3, \dots, \frac{9n-2}{2}\}$ as follows.

$f(u_k) = 2k - 1 \qquad 1 \le k \le n$

$f(r_k) = 2k \qquad 1 \le k \le n-1$

$f(s_k) = 2n + 2k - 2 \qquad 1 \le k \le \frac{n}{2}$

$f(x_k) = \frac{7n-2}{2} + k \qquad 1 \le k \le \frac{n}{2}$

$f(t_k) = 3n + k - 1 \qquad 1 \le k \le \frac{n}{2}$

$f(v_k) = 2n + 2k - 1 \qquad 1 \le k \le \frac{n}{2}$

$f\left(w_{\frac{n+2}{2}-k}\right) = 4n - 1 + k \qquad 1 \le k \le \frac{n}{2}$

For this we define following edge function, $f^*: E(G) \to \{0,1\}$ by

$f^*(u_k r_k) = 1 \qquad 1 \le k \le n-1$



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$f^*(r_k u_{k+1}) = 1 \qquad 1 \le k \le n-1$

$f^*(u_{2k-1} s_k) = 0 \qquad 1 \le k \le \frac{n}{2}$

$f^*(s_k v_k) = 1 \qquad 1 \le k \le \frac{n}{2}$

$f^*(u_{2k} t_k) = 0 \qquad 1 \le k \le \frac{n}{2}$

$f^*(t_k w_k) = 0 \qquad 1 \le k \le \frac{n}{2}$

$f^*(v_k x_k) = 0 \qquad 1 \le k \le \frac{n}{2}$

$f^*(x_k w_k) = 0 \qquad 1 \le k \le \frac{n}{2} - 1$

$f^*\left(x_{\frac{n}{2}} w_{\frac{n}{2}}\right) = 1$

| $n$ | *Edge conditions* |
|---|---|
| $n \ge 4$ | $e_f(0) = \dfrac{5n-2}{2}$ , $e_f(1) = \dfrac{5n-2}{2}$ |

In all cases we have $|e_f(0) - e_f(1)| \le 1$ , hence $S(AQ_n)$ is a difference perfect square cordial graph.

**Illustration:** A difference perfect square cordial labeling of $S(AQ_4)$ is shown in Figure-6.

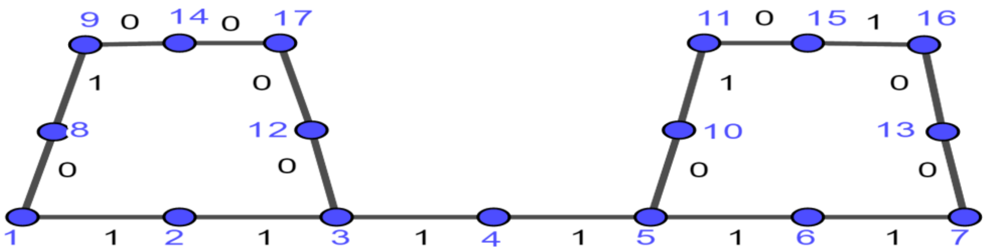


**Figure:6** $\boldsymbol{S(AQ_4)}$

**Theorem 2.7** The $SDA(T_n)$ is a difference perfect square cordial graph.

**Proof:** Let $G = SDA(T_n)$

Let the edges $u_k u_{k+1}, u_k v_k, u_{k+1} v_k, u_k w_k, u_{k+1} w_k$ be subdivided by $r_k, s_k, t_k, x_k, y_k$ respectively.

**Case:1** Let the first triangle starts from $u_2$ and the last ends with $u_{n-1}$.

$V(G) = \{u_k : 1 \le k \le n\} \cup \{r_k : 1 \le k \le n-1\} \cup \{s_k, v_k, t_k, x_k, y_k, w_k : 1 \le k \le \frac{n-2}{2}\}$

$E(G) = \{(u_k r_k) : 1 \le k \le n-1\} \cup \{(r_k u_{k+1}) : 1 \le k \le n-1\} \cup \{u_{2k} s_k : 1 \le k \le \frac{n-2}{2}\} \cup \{s_k v_k : 1 \le k \le \frac{n-2}{2}\} \cup \{u_{2k+1} t_k : 1 \le k \le \frac{n-2}{2}\} \cup \{t_k v_k : 1 \le k \le \frac{n-2}{2}\} \cup \{u_{2k} x_k : 1 \le k \le \frac{n-2}{2}\} \cup \{x_k w_k : 1 \le k \le \frac{n-2}{2}\} \cup \{u_{2k+1} y_k : 1 \le k \le \frac{n-2}{2}\} \cup \{w_k y_k : 1 \le k \le \frac{n-2}{2}\}$

So, $|V(G)| = 5n - 7$ & $|E(G)| = 6n - 10$.

Define $f: V(G) \to \{1,2,3,\dots,5n-7\}$ as follows.

$f(u_1) = 2n - 1$



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$f(u_k) = 2k - 2 \qquad 2 \le k \le n$

$f(r_k) = 2k - 1 \qquad 1 \le k \le n - 1$

$f(s_k) = 2n + 2k - 2 \qquad 1 \le k \le \frac{n-2}{2}$

$f(y_k) = \frac{9n-12}{2} + k \qquad 1 \le k \le \frac{n-2}{2}$

$f(v_k) = 2n - 1 + 2k \qquad 1 \le k \le \frac{n-2}{2}$

$f(t_k) = 4n - 5 + k \qquad 1 \le k \le \frac{n-2}{2}$

$f(x_k) = 3n - 4 + 2k \qquad 1 \le k \le \frac{n-2}{2}$

$f(w_k) = 3n - 3 + 2k \qquad 1 \le k \le \frac{n-2}{2}$

For this we define following edge function, $f^*: E(G) \to \{0,1\}$ by

$f^*(u_1 r_1) = 0$

$f^*(u_{k+1} r_{k+1}) = 1 \qquad 1 \le k \le n - 2$

$f^*(r_k u_{k+1}) = 1 \qquad 1 \le k \le n - 1$

$f^*(u_{2k} s_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(s_k v_k) = 1 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(u_{2k+1} t_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(t_k v_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(u_{2k} x_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(x_k w_k) = 1 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(u_{2k+1} y_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(y_k w_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

| $\boldsymbol{n}$ | ***Edge conditions*** |
|---|---|
| $n \ge 4$ | $e_f(0) = 3n - 5 \qquad , e_f(1) = 3n - 5$ |

**Case:2** Let the first triangle starts from $u_2$ and the last ends with $u_n$.

$V(G) = \{u_k : 1 \le k \le n\} \cup \{r_k : 1 \le k \le n - 1\} \cup \{s_k, v_k, t_k, x_k, y_k, w_k : 1 \le k \le \frac{n-1}{2}\}$

$E(G) = \{(u_k\, r_k): 1 \le k \le n - 1\} \cup \{(r_k\, u_{k+1}): 1 \le k \le n - 1\} \cup \{u_{2k} s_k: 1 \le k \le \frac{n-1}{2}\} \cup \{s_k v_k: 1 \le k \le \frac{n-1}{2}\} \cup \{u_{2k+1} t_k: 1 \le k \le \frac{n-1}{2}\} \cup \{t_k v_k: 1 \le k \le \frac{n-1}{2}\} \cup \{u_{2k} x_k: 1 \le k \le \frac{n-1}{2}\} \cup \{x_k w_k: 1 \le k \le \frac{n-1}{2}\} \cup \{u_{2k+1} y_k: 1 \le k \le \frac{n-1}{2}\} \cup \{w_k y_k: 1 \le k \le \frac{n-1}{2}\}$

So, $|V(G)| = 5n - 4$ & $|E(G)| = 6n - 6$.

Define $f: V(G) \to \{1,2,3, \dots, 5n - 4\}$ as follows.



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$f(u_k) = 2k - 1 \qquad 1 \le k \le n$

$f(r_k) = 2k \qquad 1 \le k \le n - 1$

$f(s_k) = 2n + 2k - 2 \qquad 1 \le k \le \frac{n-1}{2}$

$f(y_k) = \frac{9n-7}{2} + k \qquad 1 \le k \le \frac{n-1}{2}$

$f(v_k) = 2n - 1 + 2k \qquad 1 \le k \le \frac{n-1}{2}$

$f(t_k) = 4n - 3 + k \qquad 1 \le k \le \frac{n-1}{2}$

$f(x_k) = 3n - 3 + 2k \qquad 1 \le k \le \frac{n-1}{2}$

$f(w_k) = 3n - 2 + 2k \qquad 1 \le k \le \frac{n-1}{2}$

For this we define following edge function, $f^*: E(G) \to \{0,1\}$ by

$f^*(u_k r_k) = 1 \qquad 1 \le k \le n - 1$

$f^*(r_k u_{k+1}) = 1 \qquad 1 \le k \le n - 1$

$f^*(u_{2k} s_k) = 0 \qquad 1 \le k \le \frac{n-1}{2}$

$f^*(s_k v_k) = 1 \qquad 1 \le k \le \frac{n-1}{2}$

$f^*(u_{2k+1} t_k) = 0 \qquad 1 \le k \le \frac{n-1}{2}$

$f^*(t_k v_k) = 0 \qquad 1 \le k \le \frac{n-1}{2}$

$f^*(u_{2k} x_k) = 0 \qquad 1 \le k \le \frac{n-1}{2}$

$f^*(x_k w_k) = 1 \qquad 1 \le k \le \frac{n-1}{2}$

$f^*(u_{2k+1} y_k) = 0 \qquad 1 \le k \le \frac{n-1}{2}$

$f^*(y_k w_k) = 0 \qquad 1 \le k \le \frac{n-1}{2}$

| $n$ | *Edge conditions* |
|---|---|
| $n \ge 3$ | $e_f(0) = 3n - 3 \quad , e_f(1) = 3n - 3$ |

**Case:3** Let the first triangle starts from $u_1$ and the last ends with $u_n$.

$V(G) = \{u_k : 1 \le k \le n\} \cup \{r_k : 1 \le k \le n - 1\} \cup \{s_k, v_k, t_k, x_k, y_k, w_k : 1 \le k \le \frac{n}{2}\}$

$E(G) = \{(u_k r_k): 1 \le k \le n - 1\} \cup \{(r_k u_{k+1}): 1 \le k \le n - 1\} \cup \{u_{2k-1} s_k: 1 \le k \le \frac{n}{2}\} \cup \{s_k v_k: 1 \le k \le \frac{n}{2}\} \cup \{u_{2k} t_k: 1 \le k \le \frac{n}{2}\} \cup \{t_k v_k: 1 \le k \le \frac{n}{2}\} \cup \{u_{2k-1} x_k: 1 \le k \le \frac{n}{2}\} \cup \{x_k w_k: 1 \le k \le \frac{n}{2}\} \cup \{u_{2k} y_k: 1 \le k \le \frac{n}{2}\} \cup \{w_k y_k: 1 \le k \le \frac{n}{2}\}$

So, $|V(G)| = 5n - 1$ & $|E(G)| = 6n - 2$.

Define $f: V(G) \to \{1,2,3, \dots, 5n - 1\}$ as follows.

$f(u_k) = 2k - 1 \qquad 1 \le k \le n$

$f(r_k) = 2k \qquad 1 \le k \le n - 1$



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$f(s_k) = 2n + 2k - 2 \qquad 1 \le k \le \frac{n}{2}$

$f(x_k) = \frac{7n-4}{2} + 2k \qquad 1 \le k \le \frac{n}{2}$

$f(v_k) = 2n - 1 + 2k \qquad 1 \le k \le \frac{n}{2}$

$f\left(t_{\frac{n+2}{2}-k}\right) = 3n - 1 + k \qquad 1 \le k \le \frac{n}{2}$

$f(w_k) = \frac{7n-2}{2} + 2k \qquad 1 \le k \le \frac{n}{2}$

$f(y_k) = \frac{9n-2}{2} + k \qquad 1 \le k \le \frac{n}{2}$

For this we define following edge function, $f^*: E(G) \to \{0,1\}$ by

$f^*(u_k r_k) = 1 \qquad 1 \le k \le n-1$

$f^*(r_k u_{k+1}) = 1 \qquad 1 \le k \le n-1$

$f^*(u_{2k-1} s_k) = 0 \qquad 1 \le k \le \frac{n}{2}$

$f^*(s_k v_k) = 1 \qquad 1 \le k \le \frac{n}{2}$

$f^*(u_{2k} t_k) = 0 \qquad 1 \le k \le \frac{n}{2}$

$f^*(t_k v_k) = 0 \qquad 1 \le k \le \frac{n}{2} - 1$

$f^*(u_{2k-1} x_k) = 0 \qquad 1 \le k \le \frac{n}{2}$

$f^*(x_k w_k) = 1 \qquad 1 \le k \le \frac{n}{2}$

$f^*(u_{2k} y_k) = 0 \qquad 1 \le k \le \frac{n}{2}$

$f^*(y_k w_k) = 0 \qquad 1 \le k \le \frac{n}{2}$

$f^*\left(t_{\frac{n}{2}} v_{\frac{n}{2}}\right) = 1$

| $\boldsymbol{n}$ | $\boldsymbol{Edge\ conditions}$ |
|---|---|
| $n \ge 4$ | $e_f(0) = 3n - 1 \qquad , e_f(1) = 3n - 1$ |

In all cases we have $\left|e_f(0) - e_f(1)\right| \le 1$ , hence $S(D(AT_n))$ is a difference perfect square cordial graph.

**Illustration:** A difference perfect square cordial labeling of $SD(AT_4)$ is shown in Figure-7.



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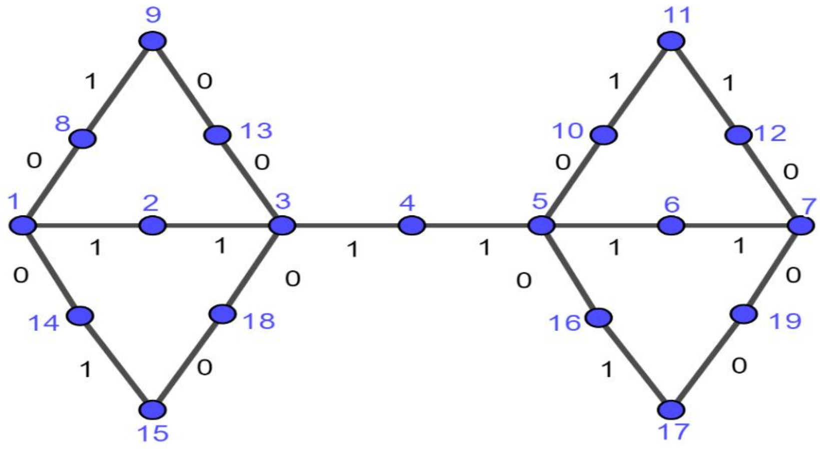


**Figure:7** $\boldsymbol{SD(AT_4)}$

**Theorem 2.8** $SDA(Q_n)$ is a difference perfect square cordial graph.

**Proof:** Let $G = SDA(Q_n)$

Let the edges $u_k u_{k+1}, u_k v_k, v_k w_k, u_{k+1} w_k, u_k j_k, j_k m_k, u_{k+1} m_k$ be subdivided by $r_k, s_k, z_k, t_k, x_k, l_k, y_k$ respectively.

**Case:1** Let the first quadrilateral starts from $u_2$ and the last ends with $u_{n-1}$.

$V(G) = \{u_k : 1 \le k \le n\} \cup \{r_k : 1 \le k \le n-1\} \cup \{s_k, v_k, t_k, z_k, w_k, x_k, j_k, l_k, m_k, y_k : 1 \le k \le \frac{n-2}{2}\}$

$E(G) = \{(u_k r_k): 1 \le k \le n-1\} \cup \{(r_k u_{k+1}): 1 \le k \le n-1\} \cup \{u_{2k} s_k: 1 \le k \le \frac{n-2}{2}\} \cup \{(s_k v_k): 1 \le k \le \frac{n-2}{2}\} \cup \{(u_{2k+1} t_k): 1 \le k \le \frac{n-2}{2}\} \cup \{(t_k w_k): 1 \le k \le \frac{n-2}{2}\} \cup \{v_k z_k: 1 \le k \le \frac{n-2}{2}\} \cup \{(z_k w_k): 1 \le k \le \frac{n-2}{2}\} \cup \{u_{2k} x_k: 1 \le k \le \frac{n-2}{2}\} \cup \{(x_k j_k): 1 \le k \le \frac{n-2}{2}\} \cup \{(u_{2k+1} y_k): 1 \le k \le \frac{n-2}{2}\} \cup \{(y_k m_k): 1 \le k \le \frac{n-2}{2}\} \cup \{j_k l_k: 1 \le k \le \frac{n-2}{2}\} \cup \{(l_k m_k): 1 \le k \le \frac{n-2}{2}\}$

So, $|V(G)| = 7n - 11$ & $|E(G)| = 8n - 14$.

Define $f: V(G) \to \{1,2,3,\dots,7n-11\}$ as follows.

$f(u_1) = 7n - 11$

$f(u_k) = 2k - 2 \qquad 2 \le k \le n$

$f(r_k) = 2k - 1 \qquad 1 \le k \le n-1$

$f(s_k) = 5n + k - 8 \qquad 1 \le k \le \frac{n-2}{2}$

$f(t_k) = \frac{11n-18}{2} + k \qquad 1 \le k \le \frac{n-2}{2}$

$f(v_k) = 2n - 4 + 3k \qquad 1 \le k \le \frac{n-2}{2}$

$f(z_k) = 2n - 3 + 3k \qquad 1 \le k \le \frac{n-2}{2}$

$f(w_k) = 2n - 2 + 3k \qquad 1 \le k \le \frac{n-2}{2}$

$f(j_k) = \frac{7n-14}{2} + 3k \qquad 1 \le k \le \frac{n-2}{2}$



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$f(l_k) = \frac{7n-12}{2} + 3k \qquad 1 \le k \le \frac{n-2}{2}$

$f(m_k) = \frac{7n-10}{2} + 3k \qquad 1 \le k \le \frac{n-2}{2}$

$f(y_k) = \frac{13n-22}{2} + k \qquad 1 \le k \le \frac{n-2}{2}$

$f(x_k) = 6n - 10 + k \qquad 1 \le k \le \frac{n-2}{2}$

For this we define following edge function, $f^*: E(G) \to \{0,1\}$ by

$f^*(u_1 r_1) = 0$

$f^*(u_{k+1} r_{k+1}) = 1 \qquad 1 \le k \le n-2$

$f^*(r_k u_{k+1}) = 1 \qquad 1 \le k \le n-1$

$f^*(u_{2k} s_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(s_k v_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(u_{2k+1} t_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(t_k w_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(v_k z_k) = 1 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(z_k w_k) = 1 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(u_{2k} x_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(x_k j_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(u_{2k+1} y_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(y_k m_k) = 0 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(j_k l_k) = 1 \qquad 1 \le k \le \frac{n-2}{2}$

$f^*(l_k m_k) = 1 \qquad 1 \le k \le \frac{n-2}{2}$

| $n$ | ***Edge conditions*** |
|---|---|
| $n \ge 4$ | $e_f(0) = 4n-7$ , $e_f(1) = 4n-7$ |

**Case:2** Let the first quadrilateral starts from $u_2$ and the last ends with $u_n$.

$V(G) = \{u_k : 1 \le k \le n\} \cup \{r_k : 1 \le k \le n-1\} \cup \{s_k, v_k, t_k, z_k, w_k, x_k, j_k, l_k, m_k, y_k : 1 \le k \le \frac{n-1}{2}\}$

$E(G) = \{(u_k r_k): 1 \le k \le n-1\} \cup \{(r_k u_{k+1}): 1 \le k \le n-1\} \cup \{u_{2k} s_k: 1 \le k \le \frac{n-1}{2}\} \cup \{(s_k v_k): 1 \le k \le \frac{n-1}{2}\} \cup \{(u_{2k+1} t_k): 1 \le k \le \frac{n-1}{2}\} \cup \{(t_k w_k): 1 \le k \le \frac{n-1}{2}\} \cup \{v_k z_k: 1 \le k \le \frac{n-1}{2}\} \cup \{(z_k w_k): 1 \le k \le \frac{n-1}{2}\} \cup \{u_{2k} x_k: 1 \le k \le \frac{n-1}{2}\} \cup \{(x_k j_k): 1 \le k \le \frac{n-1}{2}\} \cup \{(u_{2k+1} y_k): 1 \le k \le \frac{n-1}{2}\} \cup \{(y_k m_k): 1 \le k \le \frac{n-1}{2}\} \cup \{j_k l_k: 1 \le k \le \frac{n-1}{2}\} \cup \{(l_k m_k): 1 \le k \le \frac{n-1}{2}\}$



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So, $|V(G)| = 7n - 6$ & $|E(G)| = 8n - 8$.

Define $f: V(G) \rightarrow \{1,2,3, \dots, 7n - 6\}$ as follows.

$f(u_k) = 2k - 1 \qquad 1 \leq k \leq n$

$f(r_k) = 2k \qquad 1 \leq k \leq n - 1$

$f(s_k) = 5n + k - 4 \qquad 1 \leq k \leq \frac{n-1}{2}$

$f(t_k) = \frac{11n-9}{2} + k \qquad 1 \leq k \leq \frac{n-1}{2}$

$f(v_k) = 2n - 3 + 3k \qquad 1 \leq k \leq \frac{n-1}{2}$

$f(z_k) = 2n - 2 + 3k \qquad 1 \leq k \leq \frac{n-1}{2}$

$f(w_k) = 2n - 1 + 3k \qquad 1 \leq k \leq \frac{n-1}{2}$

$f(j_k) = \frac{7n-9}{2} + 3k \qquad 1 \leq k \leq \frac{n-1}{2}$

$f(l_k) = \frac{7n-7}{2} + 3k \qquad 1 \leq k \leq \frac{n-1}{2}$

$f(m_k) = \frac{7n-5}{2} + 3k \qquad 1 \leq k \leq \frac{n-1}{2}$

$f(y_k) = \frac{13n-11}{2} + k \qquad 1 \leq k \leq \frac{n-1}{2}$

$f(x_k) = 6n - 5 + k \qquad 1 \leq k \leq \frac{n-1}{2}$

For this we define following edge function, $f^*: E(G) \rightarrow \{0,1\}$ by

$f^*(u_k r_k) = 1 \qquad 1 \leq k \leq n - 1$

$f^*(r_k u_{k+1}) = 1 \qquad 1 \leq k \leq n - 1$

$f^*(u_{2k} s_k) = 0 \qquad 1 \leq k \leq \frac{n-1}{2}$

$f^*(s_k v_k) = 0 \qquad 1 \leq k \leq \frac{n-1}{2}$

$f^*(u_{2k+1} t_k) = 0 \qquad 1 \leq k \leq \frac{n-1}{2}$

$f^*(t_k w_k) = 0 \qquad 1 \leq k \leq \frac{n-1}{2}$

$f^*(v_k z_k) = 1 \qquad 1 \leq k \leq \frac{n-1}{2}$

$f^*(z_k w_k) = 1 \qquad 1 \leq k \leq \frac{n-1}{2}$

$f^*(u_{2k} x_k) = 0 \qquad 1 \leq k \leq \frac{n-1}{2}$

$f^*(x_k j_k) = 0 \qquad 1 \leq k \leq \frac{n-1}{2}$

$f^*(u_{2k+1} y_k) = 0 \qquad 1 \leq k \leq \frac{n-1}{2}$

$f^*(y_k m_k) = 0 \qquad 1 \leq k \leq \frac{n-1}{2}$



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$f^*(j_k l_k) = 1$ $\quad 1 \le k \le \frac{n-1}{2}$

$f^*(l_k m_k) = 1$ $\quad 1 \le k \le \frac{n-1}{2}$

| $n$ | ***Edge conditions*** |
|---|---|
| $n \ge 3$ | $e_f(0) = 4n - 4 \quad , e_f(1) = 4n - 4$ |

**Case:3** Let the first quadrilateral starts from $u_1$ and the last ends with $u_n$.

$V(G) = \{u_k \ : 1 \le k \le n\} \cup \{r_k \ : 1 \le k \le n-1\} \cup \{s_k, v_k, t_k, z_k, w_k, x_k, j_k, l_k, m_k, y_k \ : 1 \le k \le \frac{n}{2}\}$

$E(G) = \{(u_k\, r_k): 1 \le k \le n-1\} \cup \{(r_k\, u_{k+1}): 1 \le k \le n-1\} \cup \{u_{2k-1} s_k: 1 \le k \le \frac{n}{2}\} \cup \{(s_k v_k): 1 \le k \le \frac{n}{2}\} \cup \{(u_{2k} t_k): 1 \le k \le \frac{n}{2}\} \cup \{(t_k w_k): 1 \le k \le \frac{n}{2}\} \cup \{v_k z_k: 1 \le k \le \frac{n}{2}\} \cup \{(z_k w_k): 1 \le k \le \frac{n}{2}\} \cup \{u_{2k-1} x_k: 1 \le k \le \frac{n}{2}\} \cup \{(x_k j_k): 1 \le k \le \frac{n}{2}\} \cup \{(u_{2k} y_k): 1 \le k \le \frac{n}{2}\} \cup \{(y_k m_k): 1 \le k \le \frac{n}{2}\} \cup \{j_k l_k: 1 \le k \le \frac{n}{2}\} \cup \{(l_k m_k): 1 \le k \le \frac{n}{2}\}$

So, $|V(G)| = 7n - 1$ & $|E(G)| = 8n - 2$.

Define $f: V(G) \to \{1,2,3,\ldots,7n-1\}$ as follows.

$f(u_k) = 2k - 1$ $\quad 1 \le k \le n$

$f(r_k) = 2k$ $\quad 1 \le k \le n-1$

$f(s_k) = 6n + k - 1$ $\quad 1 \le k \le \frac{n}{2}$

$f(t_k) = \frac{13n-2}{2} + k$ $\quad 1 \le k \le \frac{n}{2}$

$f(v_k) = 2n - 3 + 3k$ $\quad 1 \le k \le \frac{n}{2}$

$f(z_k) = 2n - 2 + 3k$ $\quad 1 \le k \le \frac{n}{2}$

$f(w_k) = 2n - 1 + 3k$ $\quad 1 \le k \le \frac{n}{2}$

$f(j_k) = \frac{7n-6}{2} + 3k$ $\quad 1 \le k \le \frac{n}{2}$

$f(l_k) = \frac{7n-4}{2} + 3k$ $\quad 1 \le k \le \frac{n}{2}$

$f(m_k) = \frac{7n-2}{2} + 3k$ $\quad 1 \le k \le \frac{n}{2}$

$f(x_k) = \frac{11n-2}{2} + k$ $\quad 1 \le k \le \frac{n}{2}$

$f\left(y_{\frac{n+2}{2}-k}\right) = 5n - 1 + k$ $\quad 1 \le k \le \frac{n}{2}$

For this we define following edge function, $f^*: E(G) \to \{0,1\}$ by

$f^*(u_k r_k) = 1$ $\quad 1 \le k \le n-1$

$f^*(r_k u_{k+1}) = 1$ $\quad 1 \le k \le n-1$

$f^*(u_{2k-1} s_k) = 0$ $\quad 1 \le k \le \frac{n}{2}$

$f^*(s_k v_k) = 0$ $\quad 1 \le k \le \frac{n}{2}$



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$f^*(u_{2k}t_k) = 0$ $\quad 1 \le k \le \frac{n}{2}$

$f^*(t_k w_k) = 0$ $\quad 1 \le k \le \frac{n}{2}$

$f^*(v_k z_k) = 1$ $\quad 1 \le k \le \frac{n}{2}$

$f^*(z_k w_k) = 1$ $\quad 1 \le k \le \frac{n}{2}$

$f^*(u_{2k-1}x_k) = 0$ $\quad 1 \le k \le \frac{n}{2}$

$f^*(x_k j_k) = 0$ $\quad 1 \le k \le \frac{n}{2}$

$f^*(u_{2k}y_k) = 0$ $\quad 1 \le k \le \frac{n}{2}$

$f^*(y_k m_k) = 0$ $\quad 1 \le k \le \frac{n}{2} - 1$

$f^*(j_k l_k) = 1$ $\quad 1 \le k \le \frac{n}{2}$

$f^*(l_k m_k) = 1$ $\quad 1 \le k \le \frac{n}{2}$

$f^*\left(y_{\frac{n}{2}} m_{\frac{n}{2}}\right) = 1$

| $n$ | *Edge conditions* |
|---|---|
| $n \ge 4$ | $e_f(0) = 4n - 1$ , $e_f(1) = 4n - 1$ |

In all cases we have $|e_f(0) - e_f(1)| \le 1$ , hence $SD(AQ_n)$ is a difference perfect square cordial graph.

**Illustration:** A difference perfect square cordial labeling of $SD(AQ_4)$ is shown in Figure-8.

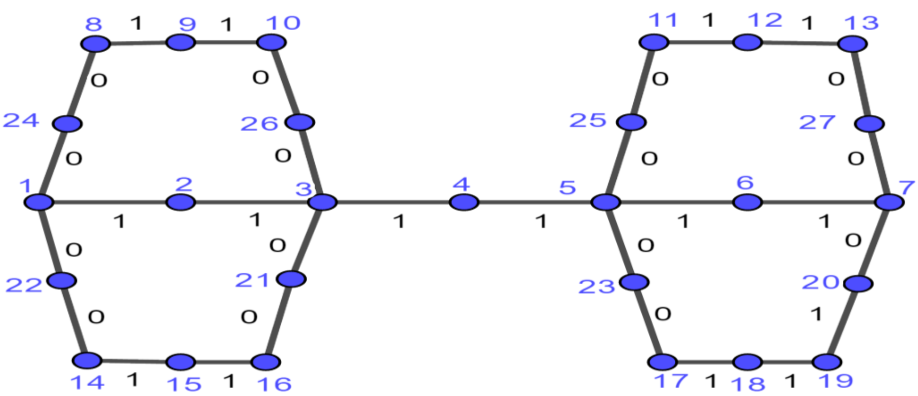

**Figure:8 $SD(AQ_4)$**

## III. CONCLUSION

In this paper we have proved that $S(T_n)$, $S(Q_n)$, $S(DT_n)$, $S(DQ_n)$, $S(AT_n)$, $S(AQ_n)$, $S(DAT_n)$, $S(DAQ_n)$ graphs are difference perfect square cordial graphs. We can discuss more similar results for various graphs.

## REFERENCES



ISSN: 2233-7857 IJFGCN

[1] J A Gallian, A dynamic survey of graph labeling, The electronic Journal of Combinatories , #DS6, 2018.

[2] J. Gross and J. Yellen, Graph theory and its application, CRC Press, 1999.

[3] U. V. Vaghela, D.B. Parmar, Difference perfect square cordial labeling, Journal of The Gujarat Research Society, 21(5), 2019.

[4] U. V. Vaghela, D.B. Parmar, Difference perfect square cordial labeling of some graphs, Journal of Xidian University, 14(2), 2020.

[5] U. V. Vaghela, D.B. Parmar, Difference perfect square cordial labeling of snake graphs, Zeichen Journal, 6(3), 2020.



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