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Numerical solution of Partial differential equations(PDE's) for nonlinear Local Fractional PDE's and Randomly generated grids

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Abstract:
In this paper, research work is extended numerical solution for Nonlinear local fractional partial differential equation over randomly generated grids and solution compared with other available methods. In this paper, results focussed on finite difference method over randomly generated grids, to check the practicability and feasibility, the new methods which are discussed in literature, apply and compare the results with this new algorithm to resolve the nonlinear local fractional equations (gas dynamics equation and Klein-Gordon equation), so we get the desired non-differentiable solutions. In this work, examples are solved and analyzed, we can expanded by applying randomly generated grids.

***Keywords**: Randomly generated grids, domain decomposition method, Sumudu transform method.*

1. Introduction

Numerical solution of partial differential equation (PDE) using randomly generated grids applied on fractional PDE's [1], Laplace Transform [2], Domain Decomposition method results compared with described methods as domain decomposition method the idea of randomly generated grids found best results than uniform meshes as well θ-Method [3] and the idea extendible standing local fractional (SDM) Sumudu decomposition methods [4].Author adopted numerical algorithms, which will developed, statistical test which were used to solve of non-linear PDE's [5]. Many scholars happening to improve the available methods, or discover new methods to solve them, or find approximate solutions for them. These efforts have affected this area in several methods, including the Adomain decomposition method and in the abbreviation (ADM), which is among the most famous methods developed recently, where it was developed by George Adomain [6,7]. Researcher used ADM method to solve Fractional PDE's [8-13], and ADM was solve another fractional equations which include, local fractional differential equations [14-16], local fractional partial differential equations [17-23], and local fractional integro-differential equations [24-26], or we find them benefit from the combined with some known transforms, such as: Laplace transform and Sumudu transform, in order to facilitate the solution of this type of equations, especially nonlinear ones. Among these works, we find local fractional laplace decomposition method [27] and local fractional SDM [22].The idea of this article is to work on the method proposed in [22] to solve linear local fractional partial differential equations in order to extend it to solve nonlinear partial differential equations within local fractional derivative. The value of the LFSDM is to enable us to combine two powerful methods to obtain exact solutions for nonlinear local fractional gas dynamics equation and nonlinear local fractional Klein-Gordon equation. Randomly generated grids presents a good results than existing methods, this method can be used to solve partial differential equation with the help of computational method, this method can be adopted for future engineering problem. Because this method is more efficient and applicable for better results. In this method we are only creating MATLAB coding for a given program, which creates various realizations for said problem and program. We can get the solution of a large number of meshes with in a seconds, which proves that idea will save the computational time, iterations and have good testing results which is applicable to use in a practical life

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problems. This idea can be extendible for more PDE's and dimensions like 3d TO n-dimensions. Here we have some methods solved PDE's.

2. Material and Method

In this Model we solve finite difference method (FDM) via Uniform meshes, random meshes also present the basic concepts on fractional local calculus, and in particular the local fractional derivative, local fractional integral and local fractional Sumudu transform.

2.1 Finite-difference methods.

1.1.1 Finite difference Method over Uniform meshes.

FDM consist of a discretemesh, $\delta\Delta := \{S_j\}$, and a grid function, $M\Delta := \{Mj\}$. The mesh$\delta\Delta$grid or mesh points graph$Sj \in \delta \subset R_y^d$and neighbors set, $S_{\boldsymbol{j_k}}, \boldsymbol{j_k} \in \aleph(j)$. The vectors $\{S_j - S_{j_k}\}_{\boldsymbol{j_k} \in \aleph(j)}$ form the stencil associated with S_j. Here, Δ abbreviates one or more discretization parameters of the underlying grid, $\delta\Delta$, which measure the clustering of these neighbors: the smaller Δ is, the closer $S_{\in \boldsymbol{j_k}}$ are to $S_{\boldsymbol{j}}$. Divided differences along appropriate discrete stencils are used to approximate the partial derivatives of the PDE. The resulting relations between the divided differences form a finite-difference scheme. Its solution, $\{Mj\}$,

$$D_{x^+}M_{jk} := \frac{M_{j+1,k}-M_{jk}}{\Delta x} \quad \text{Equation 1}$$

$$D_{y^-}M_{jk} := \frac{M_{jk}-M_{j,k-1}}{\Delta y} \quad \text{Equation 2}$$

$$D_{0x}M_{jk} := \frac{M_{jk}-M_{j,k-1}}{2\Delta x} \quad \text{Equation 3}$$

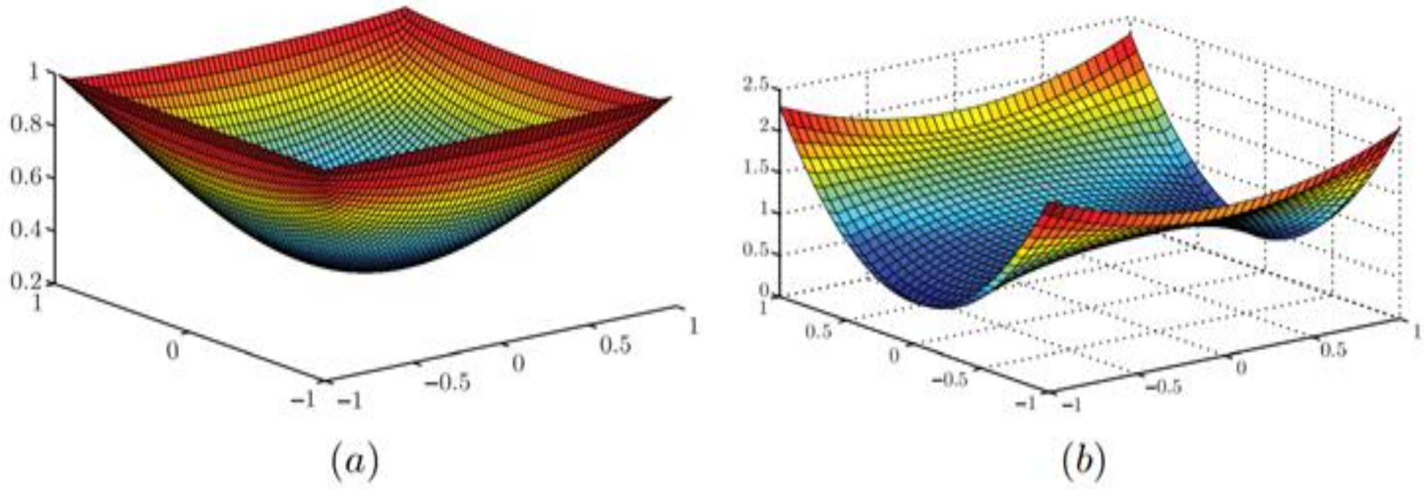
Where $D_{x^+}, D_{y^-}, D_{x^0}$are known as standard difference operators which were based on forward stencil, backword stencil and a centered stencils, there is a large principle of such difference operators to approximate first and higher derivatives.

We now ingredients into construction of finite difference approximation for Eikonal equation. $|\nabla M_{jk}| = h(x_j, y_k), (x_j, y_k) \in \Omega_\Delta$ Equation 4

where ∇M_{jk} stands for a approximate gradient,

Finite difference discretization of the denoising model reads

$$M_{jk} - \lambda\left[D_{x^-}\left(\frac{D_{x^+}M_{jk}}{\sqrt{\varepsilon^2 + |\nabla + M_{jk}|^2}}\right) + D_{y^-}\left(\frac{D_{y^+}M_{jk}}{\sqrt{\varepsilon^2 + |\nabla + M_{jk}|^2}}\right)\right] = (x_j, y_k), (x_j, y_k)\epsilon\Omega_\Delta \quad \text{Equation 5}$$



Figures 01 Figure 02

A finite-difference approximation based on a seventeen point stencil. (a) A finite-difference solution of the Monge–Ampere equation determined as $D^2M(y) = 1$ on Ω = the unit square, subject to the boundary condition, $m(y) = 1, y \in \partial\Omega$. (b) A finite-difference solution of Pucci problem, $2\lambda - (D^2m(y)) + \lambda + (D^2m(y)) = 0$, subject to the boundary condition, $m(y) = y^2{}_1 - y^2{}_2, y \in \partial\Omega$.

2.1.2 Finite difference method over Random meshes

Governing equation is chosen as 2D equation and applied on unit square with initial and boundary conditions as shown in the following figure.

$$\left.\begin{aligned}&\frac{\partial u}{dx} - \frac{\partial^2 u}{\partial y^2} = -f(y,x), -g \leq y \leq g, -g \leq x \leq g,\\ &\boldsymbol{g = 2, with\ u(x,y) = 0}\\ &u(0,y) = u(1,y) = c, u(x,0) = u(x,1) = d\end{aligned}\right\} \quad \textbf{Equation 6}$$

where u is dependent variable that depends on x and y coordinates, where f is a given forcing function, In this study, u represents any physical phenomenon say the distribution of steady temperature in metallic plate with constant applied temperature on left and right boundaries as $c = 0$and constant applied temperature on bottom and top boundaries as $u(x,0) = u^0(x)$.

$$\left.\begin{aligned}&\boldsymbol{\frac{u(j,i+1) - u(j,i)}{c} + \frac{u(j+1,i) - 2u(j,i) + u(j-1.i)}{d^2} = -f(j,i)}\\ &\boldsymbol{\forall i, \in \mathbb{N}, 1 \leq i \leq m, 1 \leq j \leq n.\ with\ u(i,j) = 0.}\\ &\boldsymbol{u(i,1) = c, u(i,n) = c, u(1,j) = d, u(m,j) = d}\end{aligned}\right\} \quad \textbf{Equation 7}$$

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Table 1 Data for Random mesh generation

Size	Standard deviation	Minimum cell size	Maximum cell size	Average cell size	Skewness	Correlation	Time	Iteration
10×10	0.81	-4.070	4.47	-0.04	0.23	0.48	0.01	105
20×20	0.6515553	-5.38	4.61	-0.002	-0.11	-0.5135	0.01	400
30×30	0.86	-6.25	5.91	0.002	-0.07	-0.1167	0.03	587
40×40	0.92	-10.12	7.20	-0.0003	0.05	-0.418	0.14	1238
50×50	0.73	-5.83	5.59	0.0000001	-0.04	-0.1125	0.21	1508

Numerical solution of random meshes is changed due to step size is not fixed as per uniform meshes step size as h and k are fix for every step but here in random mesh it varies so the Equation for random meshes is in three different approaches can be used to handle the unequal step size. Consider the shaded region in Figure 8 contains an interior node $u_{i,j}$ where solution is required to compute surrounded by different values of $h\,(h_1$ and $h_2)$ and $k\,(k_1 \text{and} k_2)$. h and k can be chosen as minimum, maximum or average of $(h_1$ and $h_2)$ and $(k_1 \text{and} k_2)$. respectively. It was roughly tested that among $h_{min}=$ min (h_1, h_2), $h_{(max)}=$ max(h_1, h_2) and $h_{avg}=$ mean (h_1, h_2) and $k_{(max)}=$ max(k_1, k_2) and $k_{avg}=$ mean (k_1, k_2) the $h_{max}$and k_{max} appear to be good selection for converged solution. Thus the Eq. 3 takes the following form and will be used for all the random meshes.

$$\boldsymbol{u^{p+1}(i,j) = \frac{k^2{}_{max}\left(h_{max} f(i,j) + u^p(i+1,j)\right) + h_{max}\left(u^p(i,j+1) - u^p(i,j-1)\right)}{k^2{}_{max} + 2h_{max}}} \quad \textbf{Equation 8}$$

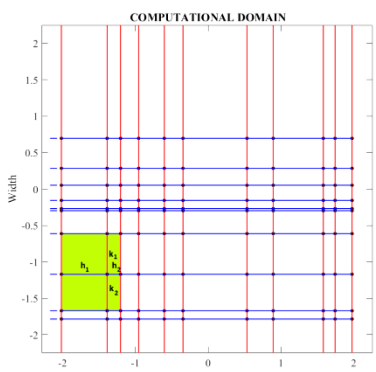

Figure 3 sample Random spacing

2.2 Generation of Data for domain decomposition method.

Initial stability data is used for the solution of domain decomposition method as a function of u. given in Figure. This is the steady state by using Galerkin procedure by using sub domain.

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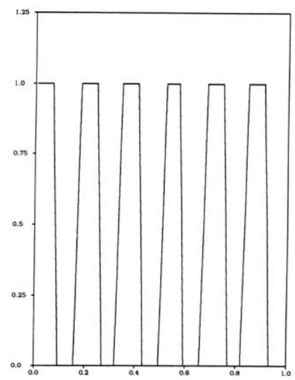

Figure 4 initial stability data

2.3.1 Local Fractional Derivative

Setting $l(q) \epsilon E_\sigma(\alpha, \beta)$, the local fractional derivative of $l(q)$ with order σ at $l = l_0$ is defined as [28,29]:

$$l^\sigma(q) = \frac{d^\sigma l}{dq^\sigma}\Big|_{l=l_0} = \lim_{l \to l_0} \frac{\Delta^\sigma(l(q) - l(q_0))}{l - l_0} \quad \text{Equation 9}$$

2.3.2 Local Fractional integral

$$l^\sigma(q) = \frac{1}{\Gamma(1+\sigma)} \int_\alpha^\beta l(\tau)(d\tau)^\sigma = \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta\tau \to 0} \sum_{j=0}^{N-1} l(\tau_j)(\Delta\tau_j), \quad \text{Equation 10}$$

where $\Delta\tau = \tau_{j+1} - \tau_j, \Delta\tau = max(\Delta\tau_0, \Delta\tau_1, \Delta\tau_2, \dots)$, and $[\tau_{j-1}, \tau_j], \tau_0 = \alpha, \tau_N = \beta$ is a partition interval $[\alpha, \beta]$.

2.4 Stability of domain decomposition solution

Domain decomposition methods results approximate v on 40 x 40, 80 x 80, and 160 x 160 meshes. In comparison of previous method, Domain decomposition error is smaller than other methods. Domain decomposition method is compared with Finite difference meshes to check the validity.

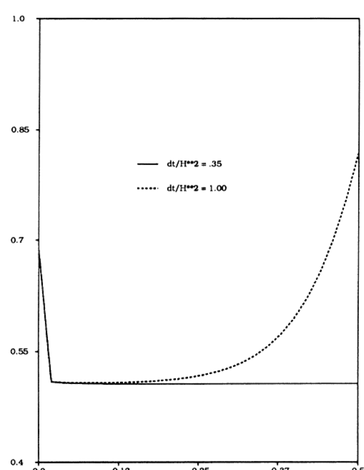

Figure 5 Stability of domain decomposition

3 Conclusion

Various methods were adopted, discussed and compared to solve partial differential equation. All methods are good. After comparison of theseabove methods as per results, the best method is to finite

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difference method using randomly generated grids than uniform meshes, domain decomposition method. As per randomly generated grids, it is observed that from different samples of random meshes about 1 to 5; random samples may provide faster convergence than uniform meshes and domain decompose method. However, at some places specific random samples have good results in terms of minimum iteration and minimum time with minimum error.

4 Future work

It is clearly available that randomly generated grids can be used for three dimensions and random meshes can be applied in domain decompose method. Randomly generated grids can be used in Method for solving Lane-Emden type differential equations by Coupling of wavelets and Laplace transform and other types of wavelets.

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