

Intuitionistic Fuzzy Contra Weakly Generalized Closed Mappings

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Abstract.

The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy contra weakly generalized closed mappings, intuitionistic fuzzy almost contra weakly generalized closed mappings and intuitionistic fuzzy contra weakly generalized * closed mappings in intuitionistic fuzzy topological space. Some of their properties are explored.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy contra weakly generalized closed mappings, intuitionistic fuzzy contra weakly generalized * closed mappings and intuitionistic fuzzy almost contra weakly generalized closed mappings.

1. Introduction

Fuzzy set (FS) as proposed by Zadeh [14] in 1965, is a framework to encounter uncertainty, vagueness and partial truth and it represents a degree of membership for each member of the universe of discourse to a subset of it. After the introduction of fuzzy topology by Chang [2] in 1968, there have been several generalizations of notions of fuzzy sets and fuzzy topology. By adding the degree of non-membership to FS, Atanassov [1] proposed intuitionistic fuzzy set (IFS) in 1986 which looks more accurate to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. In 1997, Coker [3] introduced the concept of intuitionistic fuzzy topological space.

In this paper we introduce intuitionistic fuzzy contra weakly generalized closed mappings, intuitionistic fuzzy contra weakly generalized * closed mappings and intuitionistic fuzzy almost contra weakly generalized closed mappings in intuitionistic fuzzy topological space. We provide some characterization of intuitionistic fuzzy contra weakly generalized closed mappings and establish the relationships with other classes of early defined forms of intuitionistic mappings.

2. Preliminaries

Definition 2.1: [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ where the functions $\mu_A(x) : X \rightarrow [0, 1]$ and $\nu_A(x) : X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2: [1] Let A and B be IFSs of the forms $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$.

Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,

(c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$,

(d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$,

(e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$.

For the sake of simplicity, the notation $A = \langle x, \mu_A, \nu_A \rangle$ shall be used instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0_{\sim} = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_{\sim} = \{ \langle x, 1, 0 \rangle / x \in X \}$ are the *empty set* and the *whole set* of X , respectively.

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on a non empty set X is a family τ of IFSs in X satisfying the following axioms:

- (a) $0_{\sim}, 1_{\sim} \in \tau$,
- (b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
- (c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i \in J\} \subseteq \tau$.

In this case, the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an *intuitionistic fuzzy open set* (IFOS in short) in X .

The complement A^c of an IFOS A in an IFTS (X, τ) is called an *intuitionistic fuzzy closed set* (IFCS in short) in X .

Definition 2.4: [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X . Then the *intuitionistic fuzzy interior* and an *intuitionistic fuzzy closure* are defined by

$$\text{int}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \},$$

$$\text{cl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}.$$

Note that for any IFS A in (X, τ) , we have $\text{cl}(A^c) = (\text{int}(A))^c$ and $\text{int}(A^c) = (\text{cl}(A))^c$.

Definition 2.5: An IFS $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ in an IFTS (X, τ) is said to be

- (a) *intuitionistic fuzzy semi closed set* [4] (IFSCS in short) if $\text{int}(\text{cl}(A)) \subseteq A$,
- (b) *intuitionistic fuzzy α -closed set* [4] (IF α CS in short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$,
- (c) *intuitionistic fuzzy pre-closed set* [4] (IFPCS in short) if $\text{cl}(\text{int}(A)) \subseteq A$,
- (d) *intuitionistic fuzzy regular closed set* [4] (IFRCS in short) if $\text{cl}(\text{int}(A)) = A$,
- (e) *intuitionistic fuzzy generalized closed set* [13] (IFGCS in short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,
- (f) *intuitionistic fuzzy generalized semi closed set* [11] (IFGSCS in short) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS,
- (g) *intuitionistic fuzzy α generalized closed set* [9] (IF α GCS in short) if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS.

An IFS A is called *intuitionistic fuzzy semi open set, intuitionistic fuzzy α -open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set and intuitionistic fuzzy α generalized open set* (IFSOS, IF α OS, IFPOS, IFROS, IFGOS, IFGSOS and IF α GOS) if the complement A^c is an IFSCS, IF α CS, IFPCS, IFRCS, IFGCS, IFGSCS and IF α GCS respectively.

Definition 2.6: [5] An IFS $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ in an IFTS (X, τ) is said to be an *intuitionistic fuzzy weakly generalized closed set* (IFWGCS in short) if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X .

The family of all IFWGCSs of an IFTS (X, τ) is denoted by IFWGC(X).

Definition 2.7: [5] An IFS $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ in an IFTS (X, τ) is said to be an *intuitionistic fuzzy weakly generalized open set* (IFWGOS in short) if the complement A^c is an IFWGCS in X .

The family of all IFWGOSs of an IFTS (X, τ) is denoted by IFWGO(X).

Result 2.8: [5] Every IFCS, IF α CS, IFGCS, IFRCS, IFPCS, IF α GCS is an IFWGCS but the converses need not be true in general.

Definition 2.9: [6] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X . Then the *intuitionistic fuzzy weakly generalized interior* and an *intuitionistic fuzzy weakly generalized closure* are defined by

$$\text{wgint}(A) = \cup \{ G / G \text{ is an IFWGOS in } X \text{ and } G \subseteq A \},$$

$$\text{wgcl}(A) = \cap \{ K / K \text{ is an IFWGCS in } X \text{ and } A \subseteq K \}.$$

Definition 2.10: [3] Let f be a mapping from an IFTS X into an IFTS Y . If $B = \{ \langle y, \mu_B(y), \nu_B(y) \rangle / y \in Y \}$ is an IFS in Y , then the *pre-image* of B under f denoted by $f^{-1}(B)$, is the IFS in X defined by $f^{-1}(B) = \{ \langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x)) \rangle / x \in X \}$, where $f^{-1}(\mu_B(x)) = \mu_B(f(x))$.

If $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ is an IFS in X , then the *image* of A under f denoted by $f(A)$ is the IFS in Y defined by $f(A) = \{ \langle y, f(\mu_A(y)), f_-(\nu_A(y)) \rangle / y \in Y \}$ where $f_-(\nu_A) = 1 - f(1 - \nu_A)$.

Definition 2.11: Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be

- (a) *intuitionistic fuzzy closed mapping* [12] (IFCM for short) if $f(A)$ is an IFCS in Y for every IFCS A in X ,
- (b) *intuitionistic fuzzy weakly generalized closed mapping* [7] (IFWGCM in short) if $f(A)$ is an IFWGCS in (Y, σ) for every IFCS A of (X, τ) ,
- (c) *intuitionistic fuzzy weakly generalized * closed mapping* [7] (IFWG*CM in short) if $f(A)$ is an IFWGCS in (Y, σ) for every IFWGCS A of (X, τ) ,
- (d) *intuitionistic fuzzy almost weakly generalized closed mapping* [8] (IFAWGCM in short) if $f(A)$ is an IFWGCS in (Y, σ) for every IFRCs A of (X, τ) ,
- (e) *intuitionistic fuzzy contra α generalized closed mapping* [10] (IFC α GCM in short) if $f(A)$ is an IF α GOS in (Y, σ) for every IFCS A of (X, τ) .

Definition 2.12: [5] An IFTS (X, τ) is said to be an *intuitionistic fuzzy $wT_{1/2}$* (IF $wT_{1/2}$ in short) *space* if every IFWGCS in X is an IFCS in X .

Definition 2.13: [5] An IFTS (X, τ) is said to be an *intuitionistic fuzzy wgT_q* (IF wgT_q in short) *space* if every IFWGCS in X is an IFPCS in X .

3. Intuitionistic Fuzzy Contra Weakly Generalized Closed Mappings

In this section, we introduce intuitionistic fuzzy contra weakly generalized closed mappings and study some of their properties.

Definition 3.1: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be an *intuitionistic fuzzy contra weakly generalized closed mapping* (IFCWGCM in short) if $f(A)$ is an IFWGOS in (Y, σ) for every IFCS A of (X, τ) .

Theorem 3.2: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a surjective mapping from an IFTS X into an IFTS Y . Then the following statements are equivalent.

- (a) f is an intuitionistic fuzzy contra weakly generalized closed mapping,
- (b) $f(A)$ is an IFWGCS in Y for every IFOS A in X .

Proof:

(a) \Rightarrow (b): Let A be an IFOS in X . Then A^c is an IFCS in X . By hypothesis, $f(A^c) = (f(A))^c$ is an IFWGOS in Y . Hence $f(A)$ is an IFWGCS in Y .

(b) \Rightarrow (a): Let A be an IFCS in X . Then A^c is an IFOS in X . By hypothesis, $f(A^c) = (f(A))^c$ is an IFWGOS in Y . Hence $f(A)$ is an IFWGOS in Y . Thus f is an intuitionistic fuzzy contra weakly generalized closed mapping.

Example 3.3: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $T_1 = \langle x, (0.8, 0.7), (0.2, 0.2) \rangle$, $T_2 = \langle y, (0.2, 0.3), (0.8, 0.7) \rangle$. Then $\tau = \{0_-, T_1, 1_-\}$ and $\sigma = \{0_-, T_2, 1_-\}$ are IFTs on X and Y respectively. Consider a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ defined as $f(a) = u$ and $f(b) = v$. This mapping f is an intuitionistic fuzzy contra weakly generalized closed mapping.

Theorem 3.4: Every intuitionistic fuzzy contra α generalized closed mapping is an intuitionistic fuzzy contra weakly generalized closed mapping but not conversely.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy contra α generalized closed mapping. Let A be an IFCS in X . By hypothesis, $f(A)$ is an IF α GOS in Y . Since every IF α GOS is an IFWGOS, $f(A)$ is an IFWGOS in Y . Hence f is an intuitionistic fuzzy contra weakly generalized closed mapping.

Example 3.5: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $T_1 = \langle x, (0.3, 0.5), (0.6, 0.5) \rangle$, $T_2 = \langle y, (0.5, 0.6), (0.5, 0.4) \rangle$. Then $\tau = \{0_-, T_1, 1_-\}$ and $\sigma = \{0_-, T_2, 1_-\}$ are IFTs on X and Y respectively. Consider a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ defined as $f(a) = u$ and $f(b) = v$. This mapping f is an intuitionistic fuzzy contra weakly generalized closed mapping but not an intuitionistic fuzzy contra α generalized closed mapping, since the IFS $T_1 = \langle x, (0.3, 0.5), (0.6, 0.5) \rangle$ is an IFOS in X but $f(T_1) = \langle y, (0.3, 0.5), (0.6, 0.5) \rangle$ is not an IF α GCS in Y , since $\alpha cl(f(T_1^c)) = 1_- \not\subseteq T_2$.

Theorem 3.6: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a bijective mapping from an IFTS X into an IFTS Y . Suppose that one of the following properties hold.

- (a) $f^{-1}(wgcl(A)) \subseteq int(f^{-1}(A))$ for each IFS A in Y ,
- (b) $wgcl(f(B)) \subseteq f(int(B))$ for each IFS B in X ,
- (c) $f(cl(B)) \subseteq wgint(f(B))$ for each IFS B in X .

Then f is an intuitionistic fuzzy contra weakly generalized closed mapping.

Proof:

(a) \Rightarrow (b): Let B be an IFS in X . Then $f(B)$ is an IFS in Y . By hypothesis, $f^{-1}(wgcl(f(B))) \subseteq int(f^{-1}(wgcl(f(B)))) = int(B)$. This implies $wgcl(f(B)) = f(f^{-1}(wgcl(f(B)))) \subseteq f(int(B))$. Hence $wgcl(f(B)) \subseteq f(int(B))$ for each IFS B in X .

(b) \Rightarrow (c): It can be proved by using the complement.

Suppose (c) holds: Let A be an IFCS in X . By our assumption, we have $f(A) = f(cl(A)) \subseteq wgint(f(A)) \subseteq f(A)$. Hence $wgint(f(A)) = f(A)$. This implies $f(A)$ is an IFWGOS in Y . Hence $f(A)$ is an intuitionistic fuzzy contra weakly generalized closed mapping.

Theorem 3.7: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a bijective mapping from an IFTS X into an IFTS Y . Then f is an intuitionistic fuzzy contra weakly generalized closed mapping if $cl(f^{-1}(A)) \subseteq f^{-1}(wgint(A))$ for every IFS A in Y .

Proof: Let A be an IFCS in X . Then $cl(A) = A$ and $f(A)$ is an IFCS in Y . By hypothesis, $cl(f^{-1}(f(A))) \subseteq f^{-1}(wgint(f(A)))$. Since f is one to one, $f^{-1}(f(A)) = A$. Therefore $A = cl(A) = cl(f^{-1}(f(A))) \subseteq f^{-1}(wgint(f(A)))$. This implies $f(A) \subseteq f(f^{-1}(wgint(f(A)))) = wgint(f(A)) \subseteq f(A)$. Hence $f(A)$ is an IFWGOS in Y . Thus f is an intuitionistic fuzzy contra weakly generalized closed mapping.

Theorem 3.8: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y . Then f is an intuitionistic fuzzy contra weakly generalized closed mapping if $f(wgcl(B)) \subseteq int(f(B))$ for every IFS B in X .

Proof: Let B be an IFCS in X . Then $cl(B) = B$. Since every IFCS is an IFWGCS, we have $wgcl(B) = B$. By hypothesis, $f(B) = f(wgcl(B)) \subseteq int(f(B)) \subseteq f(B)$. This implies $f(B)$ is an IFOS in Y . Therefore $f(B)$ is an IFWGOS in Y . Hence f is an intuitionistic fuzzy contra weakly generalized closed mapping.

Theorem 3.9: An intuitionistic fuzzy closed mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be an intuitionistic fuzzy contra weakly generalized closed mapping if $IFWGCS(Y) = IFWGO(Y)$.

Proof: Let $A \subseteq X$ be an IFCS in X . By hypothesis, $f(A)$ is an IFCS in Y . Since every IFCS is an IFWGCS, $f(A)$ is an IFWGCS in Y . Thus $f(A)$ is an IFWGOS in Y . Hence f is an intuitionistic fuzzy contra weakly generalized closed mapping.

Theorem 3.10: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy contra weakly generalized closed mapping and Y an $IF_w T_{1/2}$ space. Then $f(A)$ is an IFOS in Y for every IFCS A in X .

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping and let A be an IFCS in X . By hypothesis, $f(A)$ is an IFWGOS in Y . Since Y is an $IF_wT_{1/2}$ space, $f(A)$ is an IFOS in Y . Hence $f(A)$ is an IFOS in Y for every IFCS A in X .

Theorem 3.11: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a surjective intuitionistic fuzzy contra weakly generalized closed mapping. Then the following conditions hold.

- (a) $wgcl(f(B)) \subseteq f(int(wgcl(B)))$ for every IFOS B in X ,
- (b) $f(cl(wgint(B))) \subseteq wgint(f(B))$ for every IFCS B in X .

Proof:

(a) Let B be an IFOS in X . By hypothesis, $f(B)$ is an IFWGCS in Y . This implies $wgcl(f(B)) = f(B) = f(int(B)) \subseteq f(int(wgcl(B)))$. Hence $wgcl(f(B)) \subseteq f(int(wgcl(B)))$ for every IFOS B in X .

(b) It can be proved by taking the complement.

Theorem 3.12: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \delta)$ be any two mappings. Then the following statements hold.

- (a) Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy closed mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ an intuitionistic fuzzy contra weakly generalized closed mapping. Then their composition $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is an intuitionistic fuzzy contra weakly generalized closed mapping.
- (b) Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy closed mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ an intuitionistic fuzzy contra α generalized closed mapping. Then their composition $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is an intuitionistic fuzzy contra weakly generalized closed mapping.

Proof:

(a) Let A be an IFCS in X . According to the hypothesis, $f(A)$ is an IFCS in Y . Since g is an intuitionistic fuzzy contra weakly generalized closed mapping, $g(f(A)) = g \circ f(A)$ is an IFWGOS in Z . Hence $g \circ f: (Y, \sigma) \rightarrow (Z, \delta)$ is an intuitionistic fuzzy contra weakly generalized closed mapping.

(b) Let A be an IFCS in X . Hypothetically stating, $f(A)$ is an IFCS in Y . Since g is an intuitionistic fuzzy contra α generalized closed mapping, $g(f(A)) = g \circ f(A)$ is an $IF\alpha$ GOS in Z . Thus $g \circ f(A)$ is an IFWGOS in Z . Hence $g \circ f: (Y, \sigma) \rightarrow (Z, \delta)$ is an intuitionistic fuzzy contra weakly generalized closed mapping.

4. Intuitionistic Fuzzy Contra Weakly Generalized * Closed Mappings and Intuitionistic Fuzzy Almost Contra Weakly Generalized Closed Mappings

In this section, we introduce intuitionistic fuzzy contra weakly generalized*closed mappings and intuitionistic fuzzy almost contra weakly generalized closed mappings. We investigate some of their properties.

Definition 4.1: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be an *intuitionistic fuzzy contra weakly generalized*closed mapping* (IFCWG*CM in short) if $f(A)$ is an IFWGOS in (Y, σ) for every IFWGCS A of (X, τ) .

Theorem 4.2: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a surjective mapping from an IFTS X into an IFTS Y . Then the following statements are equivalent.

- (a) f is an intuitionistic fuzzy contra weakly generalized*closed mapping,
- (b) $f(A)$ is an IFWGCS in Y for every IFWGOS A in X .

Proof:

(a) \Rightarrow (b): Let A be an IFWGOS in X . Then A^c is an IFWGCS in X . By hypothesis, $f(A^c) = (f(A))^c$, is an IFWGOS in Y . Hence $f(A)$ is an IFWGCS in Y .

(b) \Rightarrow (a): Let A be an IFWGCS in X . Then A^c is an IFWGOS in X . By hypothesis, $f(A^c) = (f(A))^c$ is an IFWGOS in Y . This implies $f(A)$ is an IFWGOS in Y . Hence f is an intuitionistic fuzzy contra weakly generalized*closed mapping.

Theorem 4.3: Every intuitionistic fuzzy contra weakly generalized*closed mapping is an intuitionistic fuzzy contra weakly generalized closed mapping but not conversely.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy contra weakly generalized*closed mapping. Let A be an IFCS in X . Since every IFCS is an IFWGCS, A is an IFWGCS in X . By hypothesis, $f(A)$ is an IFWGS in Y . Hence f is an intuitionistic fuzzy contra weakly generalized closed mapping.

Example 4.4: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $T_1 = \langle x, (0.6, 0.7), (0.4, 0.3) \rangle$, $T_2 = \langle y, (0.5, 0.5), (0.2, 0.1) \rangle$, $T_3 = \langle y, (0.4, 0.1), (0.6, 0.2) \rangle$. Then $\tau = \{0_-, T_1, T_2, 1_-\}$ and $\sigma = \{0_-, T_3, 1_-\}$ are IFTs on X and Y respectively. Consider a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ defined as $f(a) = u$ and $f(b) = v$. This f is an intuitionistic fuzzy contra weakly generalized closed mapping but not an intuitionistic fuzzy contra weakly generalized*closed mapping since the IFS $A = \langle x, (0.5, 0.2), (0.4, 0.2) \rangle$ is an IFWGS in X but $f(A) = \langle y, (0.5, 0.2), (0.4, 0.2) \rangle$ is not an IFWGCS in Y , since $\text{cl}(\text{int}(f(A))) = T_3^c \not\subseteq T_2$.

Definition 4.5: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be an *intuitionistic fuzzy almost contra weakly generalized closed mapping* (IFACWGCM in short) if $f(A)$ is an IFWGS in (Y, σ) for every IFRCs A of (X, τ) .

Theorem 4.6: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y . Then f is said to be an intuitionistic fuzzy almost contra weakly generalized closed mapping if the image of each IFROS in X is an IFWGCS in Y .

Proof: Let A be an IFROS in X . This implies A^c is an IFRCs in X . Since f is an intuitionistic fuzzy almost contra weakly generalized closed mapping, $f(A^c)$ is an IFWGS in Y . since $f(A^c) = (f(A))^c$, $f(A)$ is an IFWGCS in Y .

Example 4.7: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $T_1 = \langle x, (0.5, 0.6), (0.4, 0.3) \rangle$, $T_2 = \langle y, (0.3, 0.3), (0.6, 0.7) \rangle$. Then $\tau = \{0_-, T_1, 1_-\}$ and $\sigma = \{0_-, T_2, 1_-\}$ are IFTs on X and Y respectively. Consider a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ defined as $f(a) = u$ and $f(b) = v$. This mapping f is an intuitionistic fuzzy almost contra weakly generalized closed mapping.

Theorem 4.8: Every intuitionistic fuzzy contra weakly generalized * closed mapping is an intuitionistic fuzzy almost contra weakly generalized closed mapping but not conversely.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy contra weakly generalized * closed mapping. Let A be an IFRCs in X . Since every IFRCs is an IFWGCS, A is an IFWGCS in X . By hypothesis, $f(A)$ is an IFWGS in Y . Hence f is an intuitionistic fuzzy almost contra weakly generalized closed mapping.

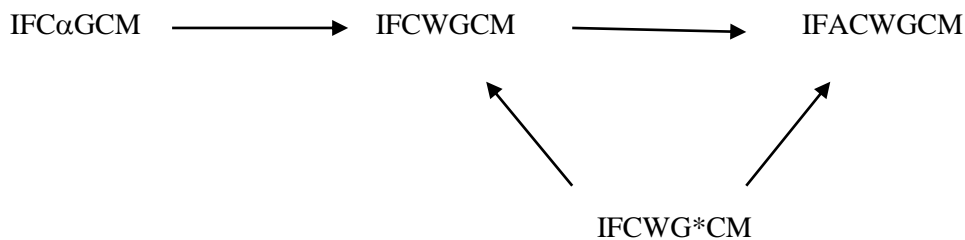
Example 4.9: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $T_1 = \langle x, (0.5, 0.4), (0.5, 0.6) \rangle$, $T_2 = \langle y, (0.6, 0.7), (0.4, 0.3) \rangle$, $T_3 = \langle y, (0.4, 0.7), (0.6, 0.3) \rangle$. Then $\tau = \{0_-, T_1, T_2, 1_-\}$ and $\sigma = \{0_-, T_3, 1_-\}$ are IFTs on X and Y respectively. Consider a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ defined as $f(a) = u$ and $f(b) = v$. This f is an intuitionistic fuzzy almost contra weakly generalized closed mapping but not an intuitionistic fuzzy contra weakly generalized * closed mapping since the IFS $A = \langle x, (0.5, 0.7), (0.5, 0.3) \rangle$ is an IFWGS in X but $f(A) = \langle y, (0.5, 0.7), (0.5, 0.3) \rangle$ is not an IFWGCS in Y , since $\text{cl}(\text{int}(f(A))) = 1_-\not\subseteq T_2$.

Theorem 4.10: Every intuitionistic fuzzy contra weakly generalized closed mapping is an intuitionistic fuzzy almost contra weakly generalized closed mapping but not conversely.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy contra weakly generalized closed mapping. Let A be an IFRCs in X . Since every IFRCs is an IFCS, A is an IFCS in X . By hypothesis, $f(A)$ is an IFWGS in Y . Hence f is an intuitionistic fuzzy almost contra weakly generalized closed mapping.

Example 4.11: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $T_1 = \langle x, (0.2, 0.2), (0.4, 0.4) \rangle$, $T_2 = \langle x, (0.5, 0.4), (0.2, 0) \rangle$, $T_3 = \langle y, (0.5, 0.6), (0.2, 0) \rangle$, $T_4 = \langle y, (0.5, 0.4), (0.5, 0.1) \rangle$. Then $\tau = \{0_-, T_1, T_2, 1_-\}$ and $\sigma = \{0_-, T_3, T_4, 1_-\}$ are IFTs on X and Y respectively. Consider a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ defined as $f(a) = u$ and $f(b) = v$. This f is an intuitionistic fuzzy almost contra weakly generalized closed mapping but not an intuitionistic fuzzy contra weakly generalized closed mapping, since the IFS $T_2 = \langle x, (0.5, 0.4), (0.2, 0) \rangle$ is an IFOS in Y but $f(T_2) = \langle y, (0.5, 0.4), (0.2, 0) \rangle$ is not an IFWGCS in Y , since $cl(int(f(T_2))) = 1_- \not\subseteq T_3$.

The relations among various types of intuitionistic fuzzy contra closedness are given in the following diagram.



The reverse implications are not true in general in the above diagram. In this diagram “ $A \rightarrow B$ ” we mean A implies B but not conversely.

Theorem 4.12: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \delta)$ be any two mappings. Then the following statements hold.

- Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy contra weakly generalized closed mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ an intuitionistic fuzzy weakly generalized * closed mapping. Then their composition $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is an intuitionistic fuzzy contra weakly generalized closed mapping.
- Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy weakly generalized closed mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ an intuitionistic fuzzy contra weakly generalized * closed mapping. Then their composition $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is an intuitionistic fuzzy contra weakly generalized closed mapping.
- Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy contra weakly generalized closed mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ an intuitionistic fuzzy contra weakly generalized * closed mapping. Then their composition $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is intuitionistic fuzzy weakly generalized closed mapping.
- Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy almost contra weakly generalized closed mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ an intuitionistic fuzzy contra weakly generalized * closed mapping. Then their composition $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is intuitionistic fuzzy almost weakly generalized closed mapping.

Proof:

- Let A be an IFCS in X . Since f is an intuitionistic fuzzy contra weakly generalized closed mapping, $f(A)$ is an IFWGOS in Y . By hypothesis, $g(f(A)) = g \circ f(A)$ is an IFWGOS in Z . Hence $g \circ f: (Y, \sigma) \rightarrow (Z, \delta)$ is an intuitionistic fuzzy contra weakly generalized closed mapping.
- Let A be an IFCS in X . Since f is an intuitionistic fuzzy weakly generalized closed mapping, $f(A)$ is an IFWGCS in Y . By hypothesis, $g(f(A)) = g \circ f(A)$ is an IFWGOS in Z . Hence $g \circ f: (Y, \sigma) \rightarrow (Z, \delta)$ is an intuitionistic fuzzy contra weakly generalized closed mapping.
- Let A be an IFCS in X . Since f is an intuitionistic fuzzy contra weakly generalized closed mapping, $f(A)$ is an IFWGOS in Y . By hypothesis, $g(f(A)) = g \circ f(A)$ is an IFWGCS in Z . Hence $g \circ f: (Y, \sigma) \rightarrow (Z, \delta)$ is an intuitionistic fuzzy weakly generalized closed mapping.
- Let A be an IFRC in X . Since f is an intuitionistic fuzzy almost contra weakly generalized closed mapping, $f(A)$ is an IFWGOS in Y . By hypothesis, $g(f(A)) = g \circ f(A)$ is an IFWGCS in Z . Hence $g \circ f: (Y, \sigma) \rightarrow (Z, \delta)$ is an intuitionistic fuzzy almost weakly generalized closed mapping.

5. References

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