Some new sets, Topologies and Decomposition of weaker forms of continuity via Fuzzy Ideals

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Abstract

The purpose of this paper is to introduce fuzzy L-perfect, fuzzy R*-perfect, fuzzy C*-perfect and fuzzy f_I -sets in fuzzy ideal topological spaces. Furthermore, we have introduced R* topology using fuzzy ideal topological spaces. The characterization for compatible ideals via fuzzy R*-perfect sets and a fuzzy topology via ideals which is finer than τ using fuzzy R* -perfect sets on a finite set is obtained. Moreover, the fuzzy weakly γ-I -open set is introduced. Some related concepts like fuzzy weakly γ-I-continuous functions, fuzzy weakly γ-I-open functions and fuzzy weakly γ-I-closed functions are also defined. The characterizations along with their properties concerning all these concepts are discussed. Finally, a new decomposition of weaker forms of fuzzy continuity is obtained using fuzzy weakly γ-I-open sets.*

Keywords: *Fuzzy L*-perfect set, Fuzzy R*-perfect set, Fuzzy C*-perfect set, Fuzzyf_I -set, Fuzzy weakly γI-open sets, Fuzzy weakly γ-I-continuous functions, Fuzzy weakly γ-I-open functions, Fuzzy weakly γ-Iclosed functions.*

1. Introduction and preliminaries

Topology is concerned with the analysis of space and continuity in Mathematics. Since it involve in the research of continuous deformations of a space, it is also called as rubber sheet geometry. In 19th century, Johann Benedict introduced the term topology. Fuzzy is one of the most significant and valuable ideas in the modern scientific world. In 1965, Zadeh[1] first contributed his works to introduce the fuzzy sets. In 1945, Vaidyanathaswamy [2] presented his idea to introduce ideal topological spaces. The concept of I-open set over local function in ideal topological space were defined by Jankovic and Hamlett[3] in 1990. Mahmoud [4] and Sarkar[5] autonomously introduced a portion of ideal concepts in fuzzy pattern and concentrated in numerous properties. The notion of decomposition in fuzzy topology is one of the interesting problems. It is also done through fuzzy ideal topological space to attain a new decomposition.

A membership function in fuzzy is defined as $A \le X$, where X be the non-empty fuzzy set and A be the fuzzy subset of X. I^X ,0 and 1 are the fundamental fuzzy sets which are known as the set of all fuzzy subsets of X, the empty set and the whole set. Chang [6] defined that the topology of fuzzy sets on I^X will be characterized by the subfamily τ of I^X. The fuzzy topological space is denoted by means of (X, τ). X_α represents a fuzzy point in X where x belongs to X ($x \in X$) and with the value of alpha as $\alpha(0 < \alpha \le 1)$. Then the fuzzy closure(Cl(A)), fuzzy interior(Int(A)) and fuzzy complement of A $(1 - A)$ are belongs to the properties of fuzzy subset A of a nonempty fuzzy set
X.A fuzzy ideal[5] I has to satisfy the given below two conditions. They are

1) $B \in I$ and $A \leq B$, then $A \in I$(heredity),

2) if $A \in I$ and $B \in I$ then $A \vee B \in I$ (finite additivity).
where I is the non-empty collection of fuzzy subsets of X.

The fuzzy topological space is denoted by means of the triple (X, τ, I) with a fuzzy ideal I and fuzzy topology τ. The fuzzy local function of $A \leq X$ for (X, τ, I) with respect to τ and I indicated as $A_*(\tau, I)$(briefly A_*)and is defined as $A_*(\tau, I)=\vee\{x \in X: A \wedge U \notin I \text{ for every } U \in \tau(x)\}$. The union of the fuzzy points x is known as A_* such that if $U \in \tau(x)$ and $E \in I$, then at least one $y \in X$ for which $U(y) + A(y) - 1 > E(y)$. $Cl_*(A) = A \vee A_*$ is called as Kuratowski closure operator of a fuzzy set in (X, τ, I). The collection $\tau_*(I)$ is an extension of fuzzy topological space (X, τ, I) than τ via fuzzy ideal which is constructed by the class $\beta = \{U - E: U \in \tau, E \in I\}$as a base[7]. This kind of topology of fuzzy sets is defined as generalization of the ordinary one. Let us we recall some definitions which are used in this sequel.

Lemma 1.1. [5] Let (X, τ, I) be fuzzy ideal topological space and A, B subsets of X.

The following properties hold:

(a) if $U \in \tau$, then $U \wedge A^* \leq (U \wedge A)^*$,

(b) if $U \in \tau$, then $U \wedge Cl_*(A) \leq Cl_*(U \wedge A)$,

(c) If $A \leq B$, then $A^* \leq B^*$, (d) $(A \vee B)^* = A^* \vee B^*$,

(e) $A^* = Cl(A^*) \leq Cl(A)$.

Let A be a subset of a fuzzy topological space (X, τ). The complement of a semi-open fuzzy set is said to be semi-closed[11] fuzzy set. The intersection of all semi-closed fuzzy sets containing A is called the semi-closure[14] of a fuzzy set A and denoted by $sCl(A)$. The semi-interior[14] of a fuzzy set A, denoted by $sInt(A)$, is defined by the union of all semi-open fuzzy sets contained in A.

Definition 1.1. A subset of A of a fuzzy ideal topological space (X, τ, I) is said to be fuzzy τ_*closed[11] if $A = Cl_*(A)$.

Definition 1.2. A fuzzy subset A of a fuzzy ideal topological space (X, τ, I) is said to be fuzzy regular-I–closed[7] if $A = (Int(A))_*$.

Definition 1.3. A fuzzy subset A of a fuzzy ideal topological space (X, τ, I) is said to be

(a) fuzzy$_*$-dense-in-itself[7] if $A \leq A_*$,

(b) fuzzy - I -open[8] if $A \leq Int(A_*)$,

(c) fuzzy almost- I–open[9] *if* $A \leq Cl(Int(A_*))$.

Lemma 1.2. [14] For a subset A of a fuzzy topological space (X, τ), the following property holds:

(a) sCl(A) = A ∨ Int(Cl(A)),

(b) sCl(A) = Int(Cl(A)) if A is fuzzy open.

Lemma 1.3. [13] Let (X, τ) be an ideal topological space with an arbitrary index Δ, I an ideal of subsets of X and ρ(X) the power set of X. If $\{A_\alpha: \alpha \in \Delta\} \leq \rho(X)$, then the following property holds:

$\vee_{\alpha\in\Delta}(A^*_{\alpha}) \leq (\vee_{\alpha\in\Delta}A_\alpha)^*$

Definition1.4.[10]A subset A of a fuzzy ideal topological space(X,τ,I)is said to be fuzzy strong t-I-set if $sCl(Int(Cl_*(A))) = Int(A)$.

Definition 1.5. [10] A subset A of a fuzzy ideal topological space (X, τ, I) is said to be fuzzy strong $B_I$set if A=U∧V, where U∈τ and V is a fuzzy strong -I-set. **Definition 1.6.** [10]

(a) A function f:(X,τ,I)→(Y,σ)is called weakly fuzzy α-I-continuous if inverse image of each fuzzy open set of Y is weakly fuzzy α - I -open in X.

(b) A function f:(X,τ,I)→(Y,σ) is called fuzzy strong B_I-continuous if inverse image of each fuzzy strong B_I-set in X.

Definition 1.7. A subset A of a space (X, τ, I) is said to be

(a) fuzzy α-I-open[15]if $A \leq Int(Cl_*(Int(A)))$,

(b) fuzzy γ-I-open[16]if $A \leq Cl^*(Int(A)) \vee Int(Cl_*(A))$.

Definition1.8.[12] A subset A of a space (X,τ,I) is said to be weakly fuzzy pre-I- open if $A \leq sCl(Int(Cl_*(A)))$, the complement of a weakly fuzzy pre-I-open set will be called weakly fuzzy pre-I-closed.

2.. Fuzzy L_*-perfect, Fuzzy R_*-perfect, Fuzzy C_*-perfect and Fuzzy f_I–sets

Definition 2.1. Let (X, τ, I) be a fuzzy ideal topological space. A subset A of X is said to be

(a) fuzzy L_*-perfect if $A - A_* \in I$,

(b) fuzzy R_*-perfect if $A_* - A \in I$,

(c) fuzzy C_*-perfect if *A is both fuzzy L_*-perfect and fuzzy R_*-perfect,*

(d) fuzzy f_I-set if $A \subseteq (int(A))_*$.

The collection of fuzzy L_* -perfect sets, fuzzy R_* -perfect sets, fuzzy C_* -perfect sets and fuzzy f_I-set in (X, τ, I) is denoted by L, R, C and f_I.

Proposition 2.1. If a subset A of an fuzzy ideal topological space (X, τ, I) is fuzzy C^*-perfect, then $A \Delta A^* \in I$.

Proof:

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Since A ∈I, $A_*= \varphi$. Then $A - A_* = A \in$ I and $A_* - A = \varphi \in$ I. Hence A is both a fuzzy L_*-perfect and fuzzy R_*-perfect set.

Proposition 2.2. In a fuzzy ideal topological space (X, τ, I), every fuzzy τ_*-closed set is fuzzy R_*perfect. **Proof:**

Let Abe a fuzzy τ^*-closed set. Therefore, $A^* \subseteq A$. Hence $A^* - A = \varphi$ =I. Therefore, A is a fuzzy R*-perfect set.

Corollary 2.1. In a fuzzy ideal topological space (X, τ, I),

(a) 0 and 1 are fuzzy R_*-perfect sets

(b) For any fuzzy subset of A of an fuzzy ideal topological space (X,τ,I),cl(A), A_*, $cl_*(A)$ are fuzzy R_*perfect sets

(c) Every fuzzy regular- I -closet is fuzzy R_* -perfect.

Proof:

The proof follows from proposition 2.2.

Proposition 2.3. If a fuzzy subset A of a fuzzy ideal topological space (X, τ, I) is such that A ∈I, then A is fuzzy C_*-perfect.

Proof:

Since A ∈I, $A_*= \varphi$. Then $A - A_* = A$ ∈I and $A_* - A = \varphi$ ∈I. Hence A is both a fuzzy L_* -perfect and fuzzy R_* -perfect set.

Corollary 2.2. Let A be a fuzzy subset of a fuzzy ideal topological space (X, τ, I).Consider the following.

(a) If A ∈I, then every fuzzy subset of A is a fuzzy C_*-perfect set.

(b) If A is fuzzy R_*-perfect, then $A_* - A$ is fuzzy C_*-perfect.

(c) If A is fuzzy L^*-perfect set, then $A - A^*$ is a fuzzy C^*-perfect set.

(d) If A is fuzzy C_*-perfect, then $A \Delta A_*$ is a fuzzy C_*-perfect set.

Proof :

The proof follows from proposition 2.3.

Proposition 2.4. In a fuzzy ideal topological space (X, τ, I), every fuzzy *-dense-in- itself is a fuzzy L_* -perfect set.

Proof :

Let A be a fuzzy *-dense-in-itself set of X. Then $A \subseteq A_*$. Therefore, $A - A_* = \varphi$ ∈I. Hence A is fuzzy L_* perfect set.

Corollary 2.3. In a fuzzy ideal topological space (X, τ, I),

(a) every fuzzy I -open set is fuzzy L_*-perfect set,

(b) every fuzzy almost I -open set is fuzzy L_*-perfect set,

(c) every fuzzy regular - I -closed set is fuzzy L_*-perfect,

(d) every fuzzy f_I-set is fuzzy L_*-perfect.

Proof:

Since all the above sets are fuzzy *-dense-in-itself, by proposition 2.4, these sets are fuzzy L_*-perfect.

Remark 2.1. The members of the fuzzy ideal of an fuzzy ideal space are fuzzy L_* perfect, but the non-empty members of the fuzzy ideal are not fuzzy *-dense-itself. Therefore, the converse of the above corollary and proposition 2.4 need not to be true.

Proposition 2.5. In a fuzzy ideal topological space (X, τ, I),

(a) empty set is an fuzzy L_*-perfect set,

(b) X is a fuzzy L_*-perfect set if the fuzzy ideal is codense.

Proof :

(a) Since $\varphi - \varphi_* = \varphi \in I$, the empty set is an L_* -perfect set.

(b) We know that $X = X_*$ if and only if the fuzzy ideal I is codense. Then $X - X_* = \varphi \in I$. Hence the result follows.

Proposition 2.6. Let (X, τ, I) be a fuzzy ideal topological space. Let A and B be two subsets of X such that $A \subseteq B$ and $A_* = B_*$, then

(a) B is fuzzy R_*-perfect if A is fuzzy R_*-perfect. (b) A is fuzzy L_*-perfect if B is fuzzy L_*-perfect.

Proof:

(a) Let A be a fuzzy R_* -perfect set. Then $A_* - A \in I$. Now, $B_* - B = A_* - B \subseteq A_* - A$. By heredity property of ideals, $B_* - B \in I$. Hence B is fuzzy R_*-perfect.

(b) Let B be a fuzzy L_* -perfect set. Then $B - B_* \in I$. Now, $A - A_* = A - B_* \subseteq B - B_*$. By heredity property of ideals, $A - A_* \in I$. Hence A is fuzzy L_*-perfect.

Corollary 2.4. Let (X, τ, I) be a fuzzy ideal topological space. Let A and B be two subsets of X such that $A \subseteq B \subseteq cl_*A$, then

(a) B is fuzzy R_*-perfect if A is fuzzy R_*-perfect.

(b) A is fuzzy L_*-perfect if B is fuzzy L_*-perfect. **Proof :**

Since $A \subseteq B \subseteq cl_*A$; $A_* \subseteq B_* \subseteq (cl_*A)* = A_*$. Hence $A_* = B_*$. Therefore, the result follows form proposition 2.6.

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Proposition 2.7. Let A be a subset of an fuzzy ideal topological space (X, τ, I) such that A is fuzzy L_*-perfect set and $A \wedge A_*$ is R_*-perfect; then both A and $A \wedge A_*$ are fuzzy C_*-perfect.

Proof:

Since A is fuzzy L_*-perfect, $A - A_* \in I$. By the condition for every $I \in I, (A \vee I)_* = A_* = (A - I)_*$. Therefore, $(A \vee (A - A^*))^* = A^* = (A - (A - A^*))^*$. This implies $A^* = (A \wedge A^*)^*$. Therefore, we have $A \wedge A_* \subseteq A$ with $(A \wedge A_*)_*$. By Proposition 2.6, A is fuzzy R_*-perfect if $A \wedge A_*$ is fuzzy R_*-perfect and $A \wedge A_*$ is fuzzy L_* -perfect if A is fuzzy L_* -perfect set. Hence A is fuzzy R_* -perfect and $A \wedge A_*$ is fuzzy L_* -perfect.

Proposition 2.8. If a subset A of a fuzzy ideal topological space (X, τ, I) is fuzzy R_* -perfect set and A_* is fuzzy L_* -perfect, then $A \wedge A_*$ is fuzzy L_* -perfect.

Proof:

Since A is fuzzy R_* -perfect, $A_* - A \in I$. By the condition for every $I \in I$, $(A \vee I)_* = A_* = (A - I)_*$. Therefore, $(A^* \vee (A^* - A))^* = A^{**} = (A^* - (A^* - A))^*$. This implies $A^{**} = (A \wedge A^*)^*$. Therefore, we have $A \wedge A^* \subseteq A^*$ with $(A \wedge A_*)_* = A_{**}$. By Proposition 2.6, $A \wedge A_*$ is fuzzy L_* -perfect if A_* is fuzzy L_* -perfect set. Hence $A \wedge A_*$ is fuzzy L_*-perfect.

Proposition 2.9. If A and B are fuzzy R_* -perfect sets, then $A \vee B$ is a fuzzy R_* -perfect set.

Proof:

Let A and B be fuzzy R_* -perfect sets. Then $A_* - A \in I$ and $B_* - B \in I$. By finite additive property of fuzzy ideals, $(A^* - A) \vee (B^* - B) \in I$. Since $(A^* \vee B^*) - (A \vee B) \subseteq (A^* - A) \vee (B^* - B)$, by heredity property $(A^* \vee B^*) - (A \vee B) \in I$. Hence $(A \vee B)^* - (A \vee B) \in I$. This proves the result.

Corollary 2.5. Finite union of fuzzy R_* -perfect sets is a fuzzy R_*-perfect set.

Proof: The proof follows from Proposition 2.9.

Proposition 2.10. If A and B are fuzzy L_* -perfect sets, then $A \vee B$ is a fuzzy L_* -perfect set.

Proof:

Since A and B be fuzzy L_* -perfect sets. Then $A - A_* \in I$ and $B - B_* \in I$. Hence by finite additive property of fuzzy ideals, $(A - A^*) \vee (B - B^*) \in I$. Since $(A \vee B) - (A \vee B)^* = (A \vee B) - (A^* \vee B^*) \subseteq (A - A^*) \vee (B - B_*)$, by heredity property $(A \vee B) - (A \vee B)_* \in I$. This proves that $A \vee B$ is a fuzzy L_* -perfect set. **Corollary 2.6.** Finite union of fuzzy L_* -perfect sets is a fuzzy L_* -perfect sets.

Proof:

The proof follows from Proposition 2.10.

Proposition 2.11. If A and B are fuzzy R_* -perfect sets, then $A \wedge B$ is a fuzzy R_* -perfect set.

Proof:

Suppose that A and B be fuzzy R_* -perfect sets. Then $A_* - A \in I$ and $B_* - B \in I$. By finite additive property of fuzzy ideals, $(A^* - A) \vee (B^* - B) \in I$. Since $(A^* \wedge B^*) - (A \wedge B) \subseteq (A^* - A) \vee (B^* - B)$, by heredity property $(A^* \wedge B^*) - (A \wedge B) \in I$. Also $(A \wedge B)^* - (A \wedge B) \subseteq (A^* \wedge B^*) - (A \wedge B) \in I$. This proves the result.

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Corollary 2.7. Finite intersection of fuzzy R_* -perfect sets is a fuzzy R_* -perfect set.

Proof:
The proof follows from Proposition 2.11.

Proposition 2.12. Finite union of fuzzy C_* -perfect sets is a fuzzy C_* -perfect set.

Proof:
The proof follows from Corollaries 2.6 and 2.7, finite union of fuzzy C_*-perfect sets is a fuzzy C_*perfect set.

Proposition 2.13. If (X, τ, I) is a fuzzy ideal topological space with X being finite, then the collection R is a fuzzy topology which is finer than the topology of fuzzy τ_*-closed sets.

Proof:
By Corollary 2.1, 0 and 1 are fuzzy R_* -perfect sets. By Corollary 2.5, finite union of fuzzy R_* perfect sets is a fuzzy R_* -perfect sets, and by Corollary 2.7, finite intersection of fuzzy R_* -perfect sets is fuzzy R_* -perfect. Hence the collection R is a fuzzy topology if X is finite. Also, by Proposition 2.2 every fuzzy τ^*-closed set is a fuzzy R^* -perfect set. Hence the fuzzy topology R is finer than the topology of fuzzy τ^*-closed sets if X is finite.

Proposition 2.14. In a fuzzy ideal topological space (X, τ, I), (fuzzy τ^* -closed sets) ∨ I ⊆R.

Proof:
The proof follows from Propositions 2.2 and 2.3. The following example shows that (fuzzy τ^*closed set) ∨I ⊆ R.

Example 2.1. Let (X, τ, I) be a fuzzy ideal topological space with X = (a, b, c) and A, B, and C be fuzzy subsets of X defined as follows:

A(a)=0.3, A(b)=0.5,A(c)=0.8

B(a)=0.2, B(b)=0.6,B(c)=0.9

C(a)=0.5, C(b)=0.7,C(c)=0.2

we have τ = {0, A, 1}. if we take I = ρ(x) ,then the collection of (fuzzy τ^*-closed sets) is A = cl^*A and R =0. Now (fuzzy τ^*-closed sets) ∨I = A f= R.

Proposition 2.15. Let (X, τ, I) be a fuzzy ideal topological space and A ⊆X. The set A is fuzzy R_*-perfect if and only if $F \subseteq A^* - A$ in X implies that F ∈I

Proof:
Assume that A is fuzzy R_* -perfect set. Then $A_* - A \in I$. By heredity property of fuzzy ideals, every set $F \subseteq A_* - A$ in X is also in I. Conversely assume that $F \subseteq A_* - A$ in X implies that F ∈I. Since $A^* - A$ is a subset of itself, by assumption $A^* - A \in I$. Hence A is fuzzy R^* -perfect.

Proposition 2.16. Let (X, τ, I) be a fuzzy ideal topological space and A ⊆X. The set A is fuzzy L_*-perfect if and only if $F \subseteq A - A_*$ in X implies that F ∈I.

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Proof:

Assume that A is fuzzy L_* -perfect set. Then $A - A_* \in I$. By heredity property of fuzzy ideals, every set F $\subseteq A - A_*$ in X is also in I. Conversely assume that $F \subseteq A - A_*$ in X implies that $F \in I$. Since $A - A^*$ is a subset of itself, by assumption $A - A^* \in I$. Hence A is fuzzy L^* -perfect.

Proposition 2.17. Let (X, τ) be a fuzzy topological space and $A \subseteq X$. Let I_1 and I_2 be two ideals on X with $I_1 \subseteq I_2$. Then A is fuzzy R_*-perfect with respect to I_2 if it is fuzzy R_*-perfect with respect to I_1.

Proof :

Since $I_1 \subseteq I_2$, $A_*(I_2) \subseteq A_*(I_1)$. Let A be fuzzy R_*-perfect with respect to I_1. Then $A_*(I_1) - A \in I_1$. Also, $A_*(I_2) - A \subseteq A_*(I_1) - A$. Hence by heredity property of fuzzy ideals, $A_*(I_2) - A \in I_1 \subseteq I_2$. Therefore A is fuzzy R_*-perfect with respect to I_2.

Theorem 2.1. Let (X, τ) be a fuzzy topological space with an fuzzy ideal I on X . Then the following are equivalent.

(a) $\tau \sim I$.

(b) If A has a cover of open sets each of whose intersection with A is I, then A is in I.

(c) If $A \subseteq X$, then $A \wedge A_* = \varphi \Longrightarrow A \in I$.

(d) If $A \subseteq X$, then $A - A^* \in I$.

(e) $A \subseteq X$ and A is fuzzy R_*-perfect set, then $A \Delta A_* \in I$.

(f) For every fuzzy τ^*-closed subset A, $A - A^* \in I$.

(g) For every $A \subseteq X$, if A contains no nonempty subset B with $B \subseteq B_*$, then

$A \in I$.

Proof:

To prove this theorem, it is enough to prove (d) ⇒ (e) ⇒(f). (d) ⇒(e) follows from Proposition 2.1. Suppose that $A \Delta A^* \in I$. Since $A - A^* \subseteq A \Delta A^*$, by heredity property $A - A^* \in I$. Hence (e) ⇒ (f).

3. Fuzzy weakly $\gamma - I$ –Open Sets

Definition3.1. AsubsetAof a fuzzy ideal topological space(X,τ,I)is said to be fuzzy weakly γ-Iopen if $A \leq Cl_*(Int(Cl(A))) \vee Cl(Int(Cl_*(A)))$, the complement of a fuzzy weakly γ-I-open set will be called fuzzy weakly γ-I-closed.

Theorem 3.1. In a fuzzy ideal topological space (X, τ, I), the following statements hold:

1) Every fuzzy α-I-open set is fuzzy weakly γ-I-open,

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2) Every fuzzy I open set is fuzzy weakly γ - I-open. **Proof :** It is obvious.

Remark 3.1. The converse of each part in the above theorem need not be true as the following examples show.

Example 3.1. Let X = {a, b, c} and A, B be fuzzy subsets of X defined as follows:

A(a) = 0.2, A(b) = 0.3, A(c) = 0.7, B(a) = 0.1,

B(b) = 0.2, B(c) = 0.2.

Let τ = {0, B, 1}. If we take I = {0}, then A is fuzzy weakly γ-I-open but A is not fuzzy α-Iopen and weakly fuzzy I open set.

Theorem 3.2. In a fuzzy ideal topological space (X, τ, I), the following statements hold:

1) Every weakly fuzzy α - I -open set is fuzzy weakly γ - I-open,

2) Every weakly fuzzy pre- I -open set is fuzzy weakly γ - I-open. **Proof:**

(1) Let A be a weakly fuzzy α - I -open set. Then, since sCl(A) ≤ Cl(A), we have A ≤ sCl(Int(Cl*(Int(A)))) ≤ Cl(Int(Cl*(A))) ≤ Cl(Int(Cl*(A))) ∨ Cl*(Int(Cl(A))). This shows that A is a fuzzy weakly γ- I -open set.

(2) Let A be a weakly fuzzy pre- I-open set. Then, since sCl(A) ≤ Cl(A), we have A ≤ sCl(Int(Cl*(A))) ≤ Cl(Int(Cl*(A))) ≤ Cl(Int(Cl*(A))) ∨Cl*(Int(Cl(A))). This shows that A is a fuzzy weakly γ- I -open set.

Remark 3.2. The converse of each part in the above theorem need not be true as the following examples show.

Example 3.2. Let X = {a, b, c} and A, B be fuzzy subsets of X defined as follows:

A(a) = 0.2, A(b) = 0.3, A(c) = 0.7, B(a) = 0.1,

B(b) = 0.2, B(c) = 0.2.

Let τ = {0, B, 1}. If we take I = {0}, then A is fuzzy weakly γ - I -open but A is not weakly fuzzy α - I-open set and A is not weakly fuzzy pre-I-open set.

Proposition 3.1. Let (X, τ, I) be a fuzzy ideal topological space. Let A,U and $A_\alpha(\alpha \in \Delta)$ be subsets of X. Then

(a) if A_α is fuzzy weakly γ - I -open for each α ∈Δ, then $\vee_{\alpha\in\Delta}A_\alpha$ is weakly fuzzy γ - I-open,

(b) if A is fuzzy weakly γ - I -open and U is fuzzy open i.e. U ∈τ, then (U ∧ A is fuzzy weakly γ- I-open

Lemma 3.1. [8] Let (X, τ, I) be a fuzzy ideal topological space and A, B subsets of X such that B ≤ A. Then $B^*(\tau_{|A}, I_{|A}) = B^*(\tau, I) \wedge A$.

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Proposition 3.2. Let (X, τ, I) be a fuzzy ideal topological space and A $\leq$ U $\in\tau$. If A is fuzzy weakly γ - I -open in (X, τ, I),then A is fuzzy weakly γ - I -open in $(U,\tau_{|U},I_{|U})$.

Proof : Let A be fuzzy weakly γ - I -open in (X, τ, I). Since U is fuzzy open, we have Int_U (A) = Int(A) for any subset A of U. By using this fact and lemmas 1.2,1.3 and 3.1, we have A $\leq$ $Cl^*(Int(Cl(A))) \vee Cl(Int(Cl^*(A)))$ and hence

$$A = A \wedge U \leq U \wedge [Cl^*(Int(Cl(A))) \vee Cl(Int(Cl^*(A)))]$$

$$\leq ClU^*[U\wedge Int(Cl(A))]\vee Cl_U[U\wedge Int(Cl_*(A))]$$

$$=ClU^*(Int_U(U\wedge Cl(A)))\vee Cl_U(Int_U(U\wedge Cl^*(A)))$$

$$=ClU^*(Int_U(Cl_U(A)))\vee Cl_U(Int_U(ClU^*(A)))$$

This shows that A is fuzzy weakly γ - I -open in $(U, \tau_{|U} , I_{|U})$.

Lemma3.2. Let (X,τ,I)be a fuzzy ideal topological space. If A is fuzzy weakly γ- I -open and U is fuzzy open i.e. U $\in\tau$, then (U ∧ A) is fuzzy weakly γ - I -open in$(U,\tau_{|U},I_{|U})$.

Proof :

Since U is fuzzy open i.e. U ∈ τ and A is fuzzy weakly γ - I -open, by proposition 3.2, A ∧ U is fuzzy weakly γ - I -open in (X, τ, I). Since U is fuzzy open

i.e. U ∈τ , by proposition 3.2, U ∧ A is fuzzy weakly γ - I -open in $(U, \tau_{|U} , I_{|U})$.

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4. Decomposition of Fuzzy Continuity

Proposition 4.1. For a subset A of a fuzzy ideal topological space (X, τ, I), the following conditions are equivalent:

(a) A is a fuzzy open set,
(b) A is fuzzy weakly γ-I-open set and fuzzy strong B_I-set.

Proof: The implications (a) ⇒ (b) is obvious.

(b) ⇒ (a). Let A be fuzzy weakly γ-I-open, we have $A \leq Cl^*(Int(Cl(A))) \vee Cl(Int(Cl^*(A)))$, also $A \leq Cl^*(Int(Cl(U \wedge V))) \vee Cl(Int(Cl^*(U \wedge V)))$, where $A = U \wedge V$, $U \in \tau$ and V is a fuzzy strong t -I-set. Hence $A=U \wedge A \leq U \wedge [Cl_*(Int(Cl(U\wedge V))) \vee Cl(Int(Cl_*(U\wedge V)))] \leq U\wedge Cl(Int(Cl_*(U\wedge V))) \leq U\wedge Cl(Int(Cl_*(U))) \wedge Cl(Int(Cl_*(V))) = U \wedge Cl(Int(Cl_*(V))) = Int(U\wedge Cl(Int(Cl_*(V)))) \leq Int(U) \wedge Int(Cl(Int(Cl_*(V)))) = Int(U) \wedge sCl(Int(Cl_*(V)))$ since V is fuzzy strong t-I-set, therefore $A = Int(U) \wedge Int(V) = Int(U \wedge V) = Int(A)$. This shows that A is fuzzy open.

Remark4.1. Fuzzy weakly γ-I-open and fuzzy strong B_I-set are independent to each other, as can be seen from the following examples.

Example 4.1. Let X = {a, b, c} and A, B be fuzzy subsets of X defined as follows:

A(a) = 0.2, A(b) = 0.3, A(c) = 0.2,

B(a) = 0.1, B(b) = 0.3, B(c) = 0.7,

C (a) = 0.1, C (b) = 0.3, C (c) = 0.2.

Let τ={0,B,1}.If we take I={0},then $C=A \wedge B$ is fuzzy strong B_I-set but C is not weakly fuzzy γ-Iopen set.

Example 4.2. Let X = {a, b, c} and A, B, C be fuzzy subsets of X defined as follows:

A(a) = 0.3, A(b) = 0.2, A(c) = 0.7,

B(a)=0.8,B(b)=0.8,B(c)=0.4,

C(a)=0.3,C(b)=0.2,C(c)=0.4.

Let τ = {0, A, B, A ∨B, C, 1}. If we take I = {0}, then C is weakly fuzzy γ- I-open set but $C=C \wedge B$ is not fuzzy strong B_I-set. Because $C \in \tau$ and B is not fuzzy strong t- I-set.

Definition4.1. A function f:(X,τ,I)→(Y,σ) is called fuzzy weakly γ-I-continuous if inverse image of each fuzzy open set of Y is fuzzy weakly γ - I -open in X.

Theorem4.1. If a function f: (X,τ,I)→(Y,σ)is weakly fuzzy α-I-continuous, then f is fuzzy weakly γ - I-continuous.

Proof: The proof is immediately follows from Theorem 3.1(a).

Proposition 4.2. For a function f: (X, τ, I) → (Y, σ), the following properties are equivalent.

(a) f is fuzzy weakly γ - I-Continuous.

(b) For each $x \in X$ and $V \in \sigma$ containing f (x), there exists fuzzy weakly γ - I -open set A containing x such that $f(A) \leq V$.

(c) The inverse image of each fuzzy closed set in (Y, σ) is fuzzy weakly γ - I - closed.

Proposition 4.3. If f: $(X, \tau, I) \rightarrow (Y, \sigma, J)$ is a fuzzy weakly γ - I -continuous function and g: $(Y, \sigma, J) \rightarrow (Z, \psi)$ is a fuzzy continuous function, then gof: $(X, \tau, I) \rightarrow (Z, \psi)$ is a fuzzy weakly γ - I continuous function.

Proof:

Let $V \in \psi$. Since g is fuzzy continuous, $g^{-1}(V) \in \sigma$. On the other hand, since f is fuzzy weakly γ-I-continuous, we have $f^{-1}(g^{-1}(V))$ is a fuzzy weakly γ- I -open set. Since $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$, we obtain that (gof) is fuzzy weakly γ-I-continuous.

Proposition 4.4. Let f: $(X, \tau, I) \rightarrow (Y, \sigma)$ is a fuzzy weakly γ - I –continuous and $U \in \tau$. Then the restriction $f_{|U}:(U,\tau_{|U},I_{|U})\rightarrow(Y,\sigma)$ is fuzzy weakly γ-I-continuous.

Proof:

Let V be any fuzzy open set of (Y, σ). Since f is fuzzy weakly γ - I - continuous, $f^{-1}(V)$ is a fuzzy weakly γ - I -open set. On the other hand, we have $(f_{|U})^{-1}(V) = f^{-1}(V) \wedge U$ is weakly b - I-open in $(U, \tau_{|U}, I_{|U})$. This shows that $f_{|U}: (U, \tau_{|U}, I_{|U}) \rightarrow (Y, \sigma)$ is fuzzy weakly γ - I-continuous.

Theorem 4.2. For a function f: $(X, \tau, I) \rightarrow (Y, \sigma)$ the following conditions are equivalent:

(a) f is fuzzy continuous,

(b) f is fuzzy weakly γ-I-continuous and fuzzy strong B_I-continuous.

Proof : This is an immediate consequence of Definition 1.4 and proposition 4.1.

5. Fuzzy R_*-Topology

By Corollary 2.1 and Proposition 2.11, we observe that the collection R satisfies the conditions of being a basis for some fuzzy topology and it will be called as $Rc^*(\tau,I)$. We define

$R_*(\tau,I) = \{A \subseteq X / X-A \in Rc^*(\tau,I)\}$ on an on-empty set X. Clearly, $R_*(\tau, I)$ is a fuzzy topology if the set X is finite. The members of the collection $R_*(\tau,I)$ will be called fuzzy R_*-open sets. If there is no confusion about the fuzzy topology τ and the ideal I, then we call $R^*(\tau, I)$ as R^* - fuzzy topology when X is finite.

Definition 5.1. A subset A of a fuzzy ideal topological space (X, τ, I) is said to be fuzzy R_* closed if it is a complement of a fuzzy R_*-open set.

Definition 5.2. Let A be a subset of a fuzzy ideal topological space (X, τ, I). One defines R_* - interior of the set A as the largest fuzzy R_* -open set contained in A. One will denote R^* -interior of a set A by $R^* - int(A)$.

Definition 5.3. Let A be a subset of a fuzzy ideal topological space (X, τ, I). A point $x \in A$ is said to be an R_* -interior point of the set A if there exists an fuzzy R_*-open set U of x such that $x \in U \subseteq A$.

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Definition5.4. Let (X,τ,I)be a fuzzy ideal topological space and x∈X. One defines R_*neighbourhood of x as a fuzzy R_*-open set containing x. One denotes the set of all R_*neighbourhoods of x by $R^* - N$ (x).

Proposition 5.1. In a fuzzy ideal topological space (X, τ, I), every fuzzy τ_*-open set is a fuzzy R_*open set.

Proof:
Let A be a fuzzy τ^*-open set. Therefore, X − A is a fuzzy τ^*-closed set. That implies that X − A is a fuzzy R_* -closed set. Hence A is a fuzzy R_* -open set.

Corollary 5.1. The fuzzy topology R_* (τ, I) on a finite set X is finer than the fuzzy topology τ_* (τ, I).

Proof :
The proof follows from Proposition 5.1.

Corollary 5.2. For any subset A of a fuzzy ideal topological space (X, τ, I), int(A) is an fuzzy R^* open set.

Proof :
The proof follows from Proposition 5.1.

Remark 5.1.

(a) Since every fuzzy open set is a fuzzy R_* -open set, every neighbourhood U of a point x ∈ X is an fuzzy R_* -neighbourhood of x .

(b) If x ∈ X is an interior point of a subset A of X , then x is an fuzzy R_* -interior point of A.

(c) From (b), we have $int(A) \subseteq int_*(A) \subseteq R_* - int(A)$, where $int_*(A)$ denotes interior of A with respect to the fuzzy topology τ^*.

Theorem 5.1. Let A and B be subsets of a fuzzy ideal topological space (X, τ, I) with X being finite. Then the following properties hold.

(a) $R^* - int(A) = \vee$ {U : U ⊆ A and U is an fuzzy R^*-open set}.

(b) $R^* - int(A)$ is the largest fuzzy R^*-open set of X contained in A.

(c) A is fuzzy $R^* -$ open if and only if $A = R^* - int(A)$.

(d) $R^* - int^*(R^* - int(A)) = R^* - int(A)$.

(e) If $A \subseteq B$, then $R^* - int(A) \subseteq R^* - int(B)$.

Proof :
The proof follows from Definitions 5.2,5.3,5.4.

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Definition5.5. Let A be subset of a fuzzy ideal topological space(X,τ,I).One defines fuzzy R_* closure of the set A as the smallest R_* -closed set containing A. One will denote fuzzy R_* -closure of a set A by $R^* - cl(A)$.

Remark 5.2. For any subset A of a fuzzy ideal topological space (X, τ, I), $R_* - cl(A) \subseteq cl_*(A) \subseteq cl(A)$.

Theorem 5.2. Let A and B be subsets of a fuzzy ideal topological space (X, τ, I)where X is finite. Then the following properties hold:

(a) $R^* - cl(A) = \wedge \{F : A \subseteq F$ and F is fuzzy R^* -closed set$\}$.

(b) A is fuzzy R_*-closed if and only if $A = R_* - cl(A)$.

(c) $R^* - cl(R^* - cl(A)) = R^* - cl(A)$

(d) If $A \subseteq B$, then $R^* - cl(A) \subseteq R^* - cl(B)$.

Proof :

The proof follows from Definition 5.5.

Theorem 5.3. Let A be subsets of a fuzzy ideal topological space (X, τ, I). Then the following properties hold:

(a) $R^* - int(X - A) = X - R^* - cl(A)$; (b)

$R^* - cl(X - A) = X - R^* - int(A)$. **Proof:**

The proof follows from Definitions 5.1,5.2,5.5.

References

1. L.A.Zadeh, Fuzzy sets, Information and control(Shenyang),8(1965),338-353.
2. R. Vaidyanathaswamy, The localization theory in set topology, Proceedings of the Indian National Science Academy, 20(1945),51-61.
3. D. Jankovic and T. R. Hamlett, New topologies from old via ideals, Amer. Math. Monthly, 97(1990), 295-310.
4. R. A. Mahmoud, Fuzzy ideals, fuzzy local function and fuzzy topology, J. Fuzzy Math. Los Angels, 5(1)(1997),117-123.
5. D. Sarkar, Fuzzy ideal theory, fuzzy local function and generated fuzzy topology, Fuzzy Sets and Systems, 5(1)(1997), 165-172.
6. C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl., 24(1968),182-190.

7. Fadhil ABBAS and Cemil YILDIZ, On Fuzzy Regular-I-Closed Sets, Fuzzy Semi-I-Regular Sets, Fuzzy AB_I-Sets and Decomposition of Fuzzy Regular-I-Continuity, Fuzzy A_I-Continuity, GU J Sci,24(4):731-738(2011).

8. A.A.NasefandR.A.Mahmoud, Some topological applications via fuzzyi deals, Chaos, Solitons and Fractrals, 13(2002), 825-831.

9. K. Malarvizhi, R. Srikirithiga and N. Arunselvam, Some new sets and a new decomposition of fuzzy continuity, fuzzy almost strong I -continuity via idealization, IJTSRD, (2018),2456-6470.

10. V. Chitra and K. Malarvizhi, Weakly fuzzy α−I -open sets a New Decomposition of Fuzzy Continuity via Ideals, IJMA, 7(1),2016,58-62.

11. Dr.V.Chitra, K.Malarvizhi and M.Narmadha Decomposition of weaker forms of continuity via fuzzy ideals, IJSRE, Vol. 4 Issue 08, (2016) ,5610-5617.

12. V. Chitra and M. Narmadha, Weakly fuzzy pre- I -open sets and a decomposition of fuzzy continuity, Annals of Fuzzy Mathematics and Informatics, Vol. 12, No. 6, (2016),825-834.

13. K.Kuratowski, Topology,Vol.1(transl.),Academicpress,NewYork,(1966).

14. T. H. Yalvac, Semi-interior and Semi-closure of a fuzzy set, J.Math. Anal. Appl., 132(1988), 356-364.

15. S.Yukesel, E. Gursel and A.Acikgoz, On fuzzy α-I-continuous and fuzzy α-I- open functions, Chaos, Solitons and Fractals, 41(2009),1691-1696.

16. M. K. Gupta and Rajneesh, Fuzzy γ-I-open sets and a New Decomposition of Fuzzy Semi-I-Continuity via Fuzzy Ideals, Int. Journal of Math. Analysis, Vol. 3, (2009), no. 28, 1349-1357.