

## Openness of some subfunctors of functor probability measures in categories $Comp$ and topological properties subspaces of space of probability measures.

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### Abstract.

Are shown, that subfunctors  $P_f$  and  $P_{f,n}$  of the functor  $P$  of probability measures are open subfunctors and topological properties of everywhere density subspaces of the convex space of probability measures  $P(X)$  defined in the infinite compact  $X$  of type of homotopy density of homotopy negligible, boundary sets and homomorphity pairs. We also investigate in which cases the homeomorphism of the spaces  $F(X)$  and  $F(Y)$  follow the homeomorphisms of the spaces  $X$  and  $Y$ , where  $F$  is a subfunctor of the functor  $P$  of the probability measures.

**Keywords:** Probability measures, homotopy dense and homotopy negligible sets,  $C$ -embedding.

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### 1. Introduction

It is known, that functor  $P$  probability measures is opened functor operating in category  $Comp$  compacts and  $c$  subfunctors continuous displays in itself [1]. In this note some are shown subfunctors functor  $P$  probability measures too are opened functor. It means, that these functor translate open displays between compacts in open displays. On the other hand it is known, that for any infinite компакта  $X$  space  $P(X)$  homeomorphisms hybert cube  $Q$ . Naturally there is a question: in what cases from homomorphity spaces  $F(X)$  and  $F(Y)$  follows homomorphity compacts  $X$  and  $Y$ , for normal функторов  $F: Comp \rightarrow Comp$ . And also in this note it is shown, that for функтора  $P_f: Comp \rightarrow Comp$  from homeomorphisms  $P_f(X) \setminus \delta(X)$  and  $P_f(Y) \setminus \delta(Y)$  follows homomorphity compacts  $X$  and  $Y$ . Further it is shown, that for compact  $X$  hereditary normal

spaces  $P_f(X)$  it is equivalent merization  $X$ . It is investigated everywhere dense subspaces spaces  $P(X)$  of probability measures being boundary, homotopy dense and homotopy neglects by sets.

## 2. Preliminary part

Let's result definition and some properties of normality covariation of functor  $F: Comp \rightarrow Comp$ , operating in a category compacts and continuous displays in themselves. It is said that functor  $F$ :

1. Keeps empty set and a point, if  $F(\emptyset) = \emptyset$  and  $F(\{1\}) = \{1\}$  where through  $\{k\}$ ,  $k \geq 0$  we designate set of non-negative whole numbers- $\{0, 1, \dots, k-1\}$ , smaller  $k$ . In this terminology  $\{0\} = \emptyset$ ;

2. Monomorphen, if for any (topological) investment.  $f: A \rightarrow X$  display  $F(f): F(A) \rightarrow F(X)$  is an investment;

3. Epimorphem if for any display  $f: X \rightarrow Y$  on  $Y$  display  $F(f): F(X) \rightarrow F(Y)$  is also display "on";

4. Keeps crossings if for any family  $\{A_\alpha: \alpha \in A\}$  of the closed subsets bicompsacts  $X$  and identical investments  $i_\alpha: A_\alpha \rightarrow X$ , display  $F(i): \bigcap \{F(A_\alpha): \alpha \in A\} \rightarrow X$  defined by equality  $F(i)(\alpha) = F(i_\alpha)(\alpha)$ , is an investment for everyone  $\alpha \in A$ ;

5. Keeps prototypes if for any display  $f: X \rightarrow Y$  and any closed set  $A \subset Y$  display  $F(f|_{f^{-1}(A)})(f^{-1}(A)) \rightarrow F(A)$  is homomorphity;

6. Keeps weight, if  $\omega(F(X)) = \omega(X)$  for infinite bicompsacts  $X$ ;

7. It is continuous, if for any return spectrum  $S = \{X_\alpha; \pi_\beta^\alpha: \alpha \in A\}$  from bicompsacts, homomorphity display  $f: F(\lim S) \rightarrow \lim F(S)$  the limit of displays  $F(\pi_\alpha)$  if  $\pi_\alpha: \lim S \rightarrow X_\alpha$  - through projections of spectrum  $S$  is which is.

Further we assume, that all considered functors of homomorphity and keep crossings. We assume also, that all functors are kept by nonempty spaces. This restriction is insignificant, as it we exclude from consideration only empty functors, i.e. functors  $F$  which translates any space in empty set.

Really, let  $F(X) = \emptyset$  for some nonempty bicompsacts  $X$ .

Then  $F(X) = F(1) = \emptyset$  in force homomorphity  $F$ . Let now  $Y$  - random nonempty of bicompsacts. We will consider constant display  $f: Y \rightarrow 1$ . Then  $F(f)(F(Y)) \subset F(1) = \emptyset$ . Hence, space  $F(Y)$  is empty, as it is displayed in empty set. So, we have proved, that exists unique monomorphes functor, keeping nonempty sets.

Let  $F : Comp \rightarrow Comp$  – функтор. Through  $C(X, Y)$  the space of continuous displays from  $X$  and  $Y$  in the bicomact-opened topology is designated. In particular,  $C(\{k\}, Y)$  it is natural homeomorphe  $k$ -ой degree  $Y^k$  of space  $Y$ .

To display  $\xi : \{k\} \rightarrow Y$  point  $(\xi(0), \dots, \xi(k-1)) \in Y^k$  is put in conformity.

For functor  $F$ , of bicomact  $X$  natural number  $k$ , we will define display  $\pi_{F, X, k} : C(\{k\}, X) \times F(\{k\}) \rightarrow F(X)$  by equality  $\pi_{F, X, k}(\xi, \alpha) = F(\xi)(\alpha)$ , where  $\xi \in C(\{k\}, X)$ ,  $\alpha \in F(\{k\})$ .

When it is clear, about what functor and about what is bicomact  $Y$  there is a speech, we will designate display  $\pi_{F, X, k}$  through  $\pi_{F, k}$  or  $\pi_k$ .

Under E.V.Shepina's [1] theorem, display  $F : C(Z, Y) \rightarrow F(F(Z), F(Y))$  is continuous for everyone continuous functor  $F$  and bicomacts  $Z$  and  $Y$ .

Therefore it takes place.

**The offer 1 [2].** For continuous functor  $F$ , bicomact  $X$  and natural number  $k$  display  $F_{\pi_{F, X, k}}$  is continuous.

Let's define subfunctor  $F_k$  functor  $F$  as follows: for bicomact  $X$  space  $F_k(X)$  is an image of display  $\pi_{F, X, k}$ , and for display  $f : X \rightarrow Y$  display  $F_k(f)$  is narrowing of display  $F(f)$  on  $F_k(X)$ . From easily checked communicativeness of diagrammes

$$\begin{array}{ccc} C(\{k\}) \times F(\{k\}) & \xrightarrow{\bar{f} \times id} & C(\{k\}, Y) \times F(\{k\}) \\ \pi_{X, k} \downarrow & & \downarrow \pi_{X, k} \\ F(X) & \xrightarrow{F(f)} & F(Y) \end{array}$$

Where  $\bar{f}(\xi) = f \circ \xi$ , investment  $F(f)(F_k(X)) \subset F_k(Y)$  and, hence, functorial of designs  $F_k$  follows.

Functor  $F$  is called as functor of degrees  $n$ , if  $F_n(X) = F(X)$  for everyone bicomact  $X$ , but  $F_{n-1}(X) \neq F(X)$  for some  $X$ .

Let  $X$  – is some bicomact;  $F$  – is some normal functor and  $x \in F(X)$ . As degree of point  $x$  is called such least natural number  $n$ , that  $x$  belonging to an image  $F(f)$  for some display  $f : K \rightarrow X$   $n$ - dotted of space  $K$ . If such final number  $n$  does not exist that degree of point  $x$  it is considered infinite. As steppe functor  $F$  is called the maximum of every possible points  $x \in F(X)$  for every possible bicomacts [1]

### 3. About functor of $P$ probability measures in category $Comp$ - of compacts and their continuous displays in itself.

For compacts  $X$  through  $P(X)$  spaces of probability measures are designated. It is known, that for infinite compact  $X$ , this space  $P(X)$  is homeomorphous gibert to cube  $\mathcal{Q}$  [3]. For natural number  $n \in \mathbb{N}$  through  $P_n(X)$  the set of all probability measures no more than with  $n$  carriers i.e.  $P_n(X) = \{\mu \in P(X) : |\text{supp}\mu| \leq n\}$  is designated. Compact  $P_n(X)$  is a convex linear combination of measures of Dirak of a kind:

$$\mu = m_1\delta_{x_1} + m_2\delta_{x_2} + \dots + m_n\delta_{x_n}, \sum_{i=1}^n m_i = 1, m_i \geq 0, x_i \in X,$$

$\delta_{x_i}$  – is a measure of Dirak in point  $x_i$ . Through  $\delta(X)$  the set of all measures of Diract compact  $X$  is designated.

Let's remind, that space  $P_f(X) \subset P(X)$ , consists of all probability measures  $\mu$  of a kind:

$$\mu = m_1\delta_{x_1} + m_2\delta_{x_2} + \dots + m_k\delta_{x_k} \text{ with final carriers, for each of which } m_i \geq \frac{k}{k+1} \text{ at some } i \text{ [3,4].}$$

For natural number  $n$  we will put  $P_{f,n}(X) = \{\mu \in P_f(X) : |\text{supp}\mu| \leq n\}$ . Obviously, that for any

$$P_f(X) = \bigcup_{n=1}^{\infty} P_{f,n}(X)$$

compact  $X$  takes place:

For compact  $X$  through  $P^C(X)$  the set of all measures  $\mu \in P(X)$  is designated, the carrier of each of which lays in one of connectivity components of compact  $X$  [4].

**Definition [5].** Seminormal functor  $F : Comp \rightarrow Comp$  is called retract  $\eta_F$  as steady if for any compact  $X \in Comp$  subspace  $\eta_F(X)$  is retract for compact  $F(X)$ . I.e. exists continuous retract  $r_{\eta_F}^X : F(X) \rightarrow \eta_F(X)$ .

On the other hand, investment  $f : X \rightarrow Y$  is called coretract if exists retraction  $r : Y \rightarrow X$  [3].

**Offer 3 [2].** Display  $f : X \rightarrow Y$  is coretraction, in only case when, when there is a multiplicate operator of continuation for  $f$ .

It is obvious, that the following takes place

**Offer 4.** Seminormal functor  $F : Comp \rightarrow Comp$  is retracted  $\eta_F$  it is steady, in only case when, when investment  $\eta_F : X \rightarrow F(X)$  is coretraction for any  $X \in Comp$ .

It is obvious, that for convex compact functor  $P$  probability measures is retracted steady [4]. Hence,  $AR$  – compacts are retracted steady for any seminormalfunctor  $F$ . In works [3,5] it is shown, that subfunctors  $P_f$  and  $P_{f,n}$  functor  $P$  probability measures retracted are steady. From definition retracted steady functors follows, that retraction  $r_{\eta_F}^X : F(X) \rightarrow \eta_F(X)$  it is closed and perfect.

**Offer 5 [2].** If compact  $X$  contains in  $Y$  banahov's space  $C(X)$  is supposed by the linear and multiplicate operator of continuation in  $C(Y)$  in that and only in that case when  $X$  retract spaces  $Y$ . In this case we receive

**Consequence 1.** For any retract  $\eta_F$  steady functor  $F : Comp \rightarrow Comp$  subspace  $\eta_F(X)$  spaces  $F(X) \subset C$  – include in  $F(X)$ .

If  $X$  merization compact then  $X^n \times F(n)$  too merization compact, and display  $\pi_{F,X,n} : X^n \times F(n) \rightarrow F(X)$  is perfect. From here  $F(X)$  merization, where  $F$  retract  $\eta_F$  steady functor degrees  $\leq n$ . Using resulted properties retracted  $\eta_F$  steady functor final degree  $\leq n$  and properties of perfect displays [2]. We can confirm.

**The theorem 1.** For compact  $X$  and retracted  $\eta_F$  steady functor  $F$  degrees  $\leq n$  are equivalent following conditions:

- 1)  $X$  merizationed;
- 2)  $F(X)$  merizationed

**Consequence 2.** For functor  $F = P_f$  and  $P_{f,n}$  following conditions are equivalent:

- 1)  $X$  merizationed;
- 2)  $F(X)$  merizationed

#### 4. About topology on subspaces of probability measures.

Let  $\mathcal{F}$  - subfunctor of functor  $P$ , having final carriers. Then base of vicinities of measure  $\mu_0 = m_1^0 \delta(x_1) + \dots + m_s^0 \delta(x_s) \in \overline{f(X)}$  forms sets of a kind

$$O < \mu_0, U_1, \dots, U_s, \varepsilon > = \{ \mu \in \mathcal{F}(X) : \mu = \sum_{i=1}^{s+1} \mu_i \}$$

Where  $\mu_i \in M^+(X)$  - set of all non-negative functionals and  $\| \mu_{i+1} \| < \varepsilon$ ,  $\text{supp } \mu_i \subset U_i$ ,  $\| \mu_i \| - m_i^0 < \varepsilon$  for  $i = 1, \dots, s$  where  $U_1, \dots, U_s$  - vicinities of points  $x_1, x_2, \dots, x_s$  with disjunctive short circuits.

Really, at first we will show, that set  $O < \mu_0, U_1, \dots, U_s, \varepsilon >$  contains a vicinity of measure  $\mu_0$  in weak topology. For everyone  $i = 1, \dots, s$  for example let's take function  $\varphi_i : X \rightarrow I$ , satisfying to

conditions:  $\varphi_i([U_i]) = 1$ ,  $\varphi_i(\bigcup_{j \neq i} [U_j]) = 0$ . Besides, we take function  $\varphi_{s+1} : X \rightarrow I$  so that  $\varphi_{s+1}(X \setminus U_1 \cup \dots \cup U_s) = 1$  and  $\varphi_{s+1}(\{x_1, x_2, \dots, x_s\}) = 0$ . Now we will check up inclusion

$$O(\mu_1, \varphi_1, \dots, \varphi_{s+1}, \frac{\varepsilon}{2}) \subset O < \mu_0, U_1 \cup \dots \cup U_s, \varepsilon > \quad (1)$$

Measure  $\mu \in O(\mu_1, \varphi_1, \dots, \varphi_{s+1}, \frac{\varepsilon}{2})$  it is represented in the form of  $\mu = \mu_1 + \dots + \mu_s + \mu_{s+1}$ , where  $\text{supp } \mu_i \subset U_i$  for  $i = 1, \dots, s$  and  $\text{supp } \mu_{s+1} \subset X \setminus U_1 \cup \dots \cup U_s$ . Then  $\mu_{s+1} \leq \mu$  whence

$\mu_{s+1}(\varphi_{s+1}) < \frac{\varepsilon}{2}$ . At the same time by definition of function  $\varphi_{s+1}$  it is had

$\mu_{s+1}(\varphi_{s+1}) = \mu_{s+1}(1_x) = \|\mu_{s+1}\|$ . So,  $\|\mu_{s+1}\| < \frac{\varepsilon}{2} < \varepsilon$  for check (1) it is necessary to show, that

$$\|\mu_i\| - m_i^0 < \varepsilon \quad \text{We have} \quad \frac{\varepsilon}{2} > |\mu_0(\varphi_i) - \mu(\varphi_i)| \geq |\mu_0(\varphi_i)| - |\mu(\varphi_i)| =$$

$m_i^0 - |(\mu_1 + \dots + \mu_s + \mu_{s+1})(\varphi_i)| =$  / identified functions

$$\varphi_i / = m_i^0 - (\mu_i + \mu_{s+1})(\varphi_i) = m_i^0 - \mu_i(\varphi_i) - \mu_{s+1}(\varphi_i) = m_i^0 - \|\mu_i\| - \mu_{s+1}(\varphi_i)$$

Hence,

$$m_i^0 - \|\mu_i\| < \frac{\varepsilon}{2} + \mu_{s+1}(\varphi_i) \leq \frac{\varepsilon}{2} + \mu_{s+1}(1_x) = \frac{\varepsilon}{2} + \|\mu_{s+1}\| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$$

On the other hand  $\frac{\varepsilon}{2} > \mu_i(\varphi_i) + \mu_{s+1}(\varphi_i) - m_i^0 < \frac{\varepsilon}{2}$ . Inequality  $\|\mu_i\| - m_i^0 < \varepsilon$ , and together with it

and inclusion (1) are proved. Now we will show, that in any basic vicinity  $O(\mu_0, \varphi_1, \dots, \varphi_k, \varepsilon)$  the

vicinity of kind  $O < \mu_0, U_1, \dots, U_s, \delta >$  contains. For this purpose it is enough to consider a vicinity of

kind  $O(\mu_0, \varphi, \varepsilon)$  as family of vicinities of measure  $\mu_0$  of kind  $O < \mu_0, U_1, \dots, U_s, \delta >$  directed

downwards on inclusion/crossing of final number of vicinities of such kind contains a vicinity of such kind/. It follows from justice of inclusion

$$O < \mu_0, U_1^1 \cap U_1^2, \dots, U_s^1 \cap U_s^2, \frac{1}{2} \min\{\delta_1, \delta_2\} > \subset \\ \subset O < U_0, U_1^1, \dots, U_s^1, \delta_1 > \cap O < \mu_0, U_1^2, \dots, U_s^2, \delta > \quad (2)$$

The basic which part of check consists in the following:

$$\mu(U_i^j) = \mu(U_i^1 \cap U_i^2) + \mu(U_i^j \setminus U_i^1 \cap U_i^2) \leq \mu(U_i^1 \cap U_i^2) + \mu(X \setminus \bigcup_{e=1}^s (U_e^1 \cap U_e^2)) < \\ < \mu(U_i^1 \cap U_i^2) + \frac{1}{2} \min\{\delta_1, \delta_2\} \leq \mu(U_i^1 \cap U_i^2) + \frac{1}{2} \delta_j$$

On it for measure  $\mu$  belonging to the left part of proved inclusion (2), we have

$$\mu_0(U_i^j) - \mu(U_i^j) \leq \mu_0(U_i^j) - \mu(U_i^1 \cap U_i^2) = m_i^0 - \mu(U_i^1 \cap U_i^2) \leq \frac{1}{2} \min\{\delta_1, \delta_2\} < \delta_j$$

And on the other hand

$$\mu(U_i^j) - \mu_0(U_i^j) < \mu(U_i^j \cap U_i^j) + \frac{1}{2} \delta_j - m_i^0 < \frac{1}{2} \min\{\delta_1, \delta_2\} + \frac{1}{2} \delta_j \leq \delta_j$$

It is necessary  $O(\mu_0, \varphi, \varepsilon)$  to find a vicinity of kind  $O < \mu_0, U_1, \dots, U_s, \delta >$  in vicinity . As

$O(\mu_0, \alpha\varphi, \alpha\varepsilon) = O(\mu_0, \varphi, \varepsilon)$  for  $\alpha > 0$ , it is possible to consider, that  $\|\varphi\| \leq 1$ . Besides, it is

possible to consider, that  $\varphi \geq 0$ . For  $\delta > 0$  we take not crossed vicinities  $U_i$  of points  $x_i$  so that

fluctuations of function  $\varphi$  on  $U_i$  was less  $\delta$ . Then

$$|\mu_0(\varphi) - \mu(\varphi)| \leq \left| m_1^0 \varphi(x_1) - \int_{u_1} \varphi d\mu \right| + \dots + \left| m_s^0 \varphi(x_s) - \int_{u_s} \varphi d\mu \right| + \left| \int_{X \setminus U_1 \cup \dots \cup U_s} \varphi d\mu \right|$$

Further

$$\begin{aligned} & \left| m_i^0 \varphi(x_i) - \int_{u_i} \varphi d\mu \right| = \left| m_i^0 \varphi(x_s) - \int_{u_i} \varphi(x_i) d\mu + \int_{u_i} \varphi(x_i) d\mu - \int_{u_i} \varphi d\mu \right| \leq \\ & \leq \left| m_i^0 \varphi(x_i) - \int_{u_i} \varphi(x_i) d\mu \right| + \left| \int_{u_i} [\varphi(x_i) - \varphi] d\mu \right| \leq \varphi(x_i) \cdot |m_i^0 - \|\mu_i\|| + \left| \int_{u_i} [\varphi(x_i) - \varphi] d\mu \right| \leq \\ & \leq \varphi(x_i) \delta + \delta \|\mu_i\| \leq 2 \cdot \delta \end{aligned}$$

$$\delta < \frac{\varepsilon}{(2s+1)}$$

On it for inclusion is carried out.

$$O < \mu_0, U_1, \dots, U_s, \delta > \subset O(\mu_0, \varphi, \varepsilon)$$

Let  $Q^-$  is some topological property. We will say, that space  $X$  possesses property  $Q$  out of set  $A$ , in space  $X$  if space  $X \setminus A$ , possesses property  $Q$  where  $A \subset X, A \neq \emptyset$ . It is known, that normality out of diagonal  $\square$  compact  $X$ , satisfies to the first axiom of count ability [5].

**The theorem 2.** For compact  $X$  and  $Y$  spaces  $P_f(X)$  and  $P_f(Y)$  homeomorphous accordingly out of sets  $\delta(X)$  and  $\delta(Y)$  in only case when, when  $\delta(X)$  and  $\delta(Y)$  homeomorphous.

**The proof.** Let  $X$  and  $Y$  such compacts, that  $P_f(X) \setminus \delta(X)$  homeomorphous  $P_f(Y) \setminus \delta(Y)$ .  $P_f(X) \setminus \delta(X) \cong P_f(Y) \setminus \delta(Y)$ . Through,  $h$  we will designate this

homeomorphous  $h: P_f(X) \setminus \delta(X) \rightarrow P_f(Y) \setminus \delta(Y)$ . Now we will establish

homeomorphous  $h': \delta(X) \rightarrow \delta(Y)$ . It is known, that for any  $\delta_x \in \delta(X)$  prototype  $(r_f^X)^{-1}(\delta_x)$

contains point  $\delta_x$ . Homeomorphous  $h$  displays set  $(r_f^X)^{-1}(\delta_x) \setminus \delta_x$  on some set

$B_y \subset P_f(Y) \setminus \delta(Y)$ . This set  $B_y$  coincides with set  $(r_f^Y)^{-1}(\delta_y) \setminus \delta_y$  for some  $\delta_y \in \delta(Y)$ . .t.e. Exists

$y \in Y$  such, that  $B_y = (r_f^Y)^{-1}(\delta_y) \setminus \delta_y$ . As a result point  $\delta_x \in \delta(X)$  we put in accordance to point

$\delta_y \in \delta(Y)$ . .t.e.  $h'(\delta_x) = \delta_y$ . Owing to a continuity of displays  $r_f^X, r_f^Y$  and a continuity

homeomorphous  $h$  the continuity of display  $h': \delta(X) \rightarrow \delta(Y)$  is easily checked. The Converse is obvious. The theorem 2 is proved.

Similarly as the theorem 2, is proved following for функтора  $F = P_{f,n}$ .

**The theorem 3.** For compacts  $X$  and  $Y$  spaces  $F(X)$  and homeomorphous  $F(Y)$  accordingly out of sets  $\eta_F(X)$  and  $\eta_F(Y)$  in only case when, when  $\eta_F(X)$  and  $\eta_F(Y)$  homeomorphous.

Let  $\tau$  – unlimited cardinal number. Directed set  $A$  is called  $\tau$  – full if any chain of its elements containing no more  $\tau$  – parts, has in  $A$  an exact top side. The continuous spectrum set over  $\tau$  – full by directed set, is called  $\tau$  – полным.

The return spectrum is called  $\tau$  – specters if it  $\tau$  – full and weight of all spaces entering into it do not surpass  $\tau$ .

For  $\mathcal{X}_0$  a  $\sigma$ -spectrum and  $\mathcal{X}_0$ -completeness is called sigma spectrum and sigma completeness. Compacts, homeomorphous are to limiting space of some sigma-spectrum with open projections, is called opened originated.

In work [4] it is resulted the following.

**The theorem [4].** Let  $X$  and  $Y$  openly generated compacts without points counted character, and  $h: P_n(X) \rightarrow P_n(Y)$  homeomorphous. Then  $h(P_k(X)) = P_k(Y)$  for any natural  $k < n$ , and particulars,  $X$  homeomorphous  $Y$ .

From the theorem 2 follows, following a consequence 3 which is generalization of the theorem [4].

**Consequence 3.** Let  $X$  and  $Y$  infinite compacts, and  $h: P_{f,n}(X) \rightarrow P_{f,n}(Y)$  homeomorphous. Then  $h(P_{f,k}(X)) = P_{f,k}(Y)$  for any natural  $k < n$ , in particular,  $X$  homeomorphous  $Y$ .

Let's remind, that  $Y \subset X$  is  $C$ -included in  $X$  if any continuous material function defined on  $Y$ , proceeds before continuous function on  $X$ [2].

**The theorem 4.** Let  $F: Comp \rightarrow Comp$  normal functor translating  $AR(\mathcal{M})$  spaces in  $AR(\mathcal{M})$  space. Then  $\eta_F C$ -вложено in  $F(X)$  for any  $X \in Comp$ .

**The proof.** Let  $X \in Comp$ , owing to a continuity functor  $F: Comp \rightarrow Comp$ . Compact  $\eta_F(X)$  it is enclosed in  $F(X)$ . We will consider continuous function  $f: X \rightarrow R$  - the valid straight line. Display  $F(f): F(X) \rightarrow F(R)$  тоже is continuous. As  $F$  keeps  $AR(\mathcal{M})$  spaces, on it exists retraction  $r_F^R: F(R) \rightarrow R$ . Required continuous continuations are compositions of display  $F(f)$  and retraction  $r_F^R$ . I.e.  $\bar{f} = r_F^R \circ F(f): F(X) \rightarrow R$ . The theorem 4 is proved.

### 5. About topological property homothopic dense subsets of space of probability measures.

Let's remind, that topological space  $Y$  is called as absolute/district/retracts in a class  $K$ /registers  $Y \in A(N)R(K)$  if  $Y \in K$  and for everyone homeomorphous  $h$ , displaying  $Y$  on closed subset  $h(Y)$  of space  $X$  from a class  $To$ , set  $h(y)$  is retract/district/spaces  $X$ .

From Keller-Keli results [6] follows, that space  $P(X)$  of probability measures on infinite compact

$X$  homeomorphous gibert to brick  $Q'$ ; where  $Q' = \prod_{n=1}^{\infty} [0, \frac{1}{2^n}]$ .  
 $Q$ -variety name separable metric space, locally homeomorphous gibert to cube  $Q$ , where  $Q = \prod_{i=1}^{\infty} [-1, 1]_i$  cube  $W_i^{\pm} = \{(g_j) \in Q \mid g_i = \pm 1\}$   $i$ -ая a side gibert cube  $Q$ , gibert  
 $BdQ = \bigcup_{i=1}^{\infty} W_i^{\pm}$  - is called pseudo-border of cube  $Q$ , and  $S = Q \setminus BdQ$  - pseudointrali cube  $Q$  [5].

In the theory unlimited measures varieties the important role is played by three objects: of giberts cube  $Q$ , separable gibert space  $\ell_2$  and  $\sum^-$  linear cover of standard brick  $Q'$  in gibert space  $\ell_2$ . Under the theorem of Anderson-Kadetsa  $\ell_2$  homeomorphous  $S$ . From results Bessagi-Pelchinskiy follows, that  $\sum$  homeomorphous  $\text{rint}Q$  [6]. Here through  $\text{rint}Q$  set  $\{x = (x_n) \in Q \mid |x_n| < t < 1$  for all  $n \in N\}$  is designated. Further,  $\text{rint}Q \approx BdQ$  [6], means  $BdQ \approx \sum$ . Through  $\ell_2^f$  pointed linear subspace of gibert spaces  $\ell_2$ , consisting of all points, only final which number of co-ordinates is distinct from zero.

**Definition [7].** The closed set *And* spaces  $X$  is called  $Z$  as -set in  $X$  if identical display  $id_X$  of space  $X$  can be as much as close approximated displays  $f: X \rightarrow X \setminus A$ . Counted association  $Z$  of -sets in  $X$  is called  $\sigma-Z$ -plurality  $X$ .

Following on [4].  $\sigma-Z$ -plurality,  $B$  giberts cube  $Q$  name boundary set in  $Q$  it/is designated through  $B(Q)/$  if  $Q \setminus B \approx \ell_2$ . Более in the general image, boundary set in  $Q$ -variety name  $\sigma-Z$ -plurality, addition to which is  $\ell_2^-$  varieties.

From resulted above follows, that pseudo-border  $BdQ$ . gibert cube  $Q$  is boundary set for gibert cube  $Q$ .

Let  $X$  topological space. Set  $A \subset X$  is called homeomorphous as dense in  $X$  if exists homotopes  $h(x, t): X \times [0, 1] \rightarrow X$  such, that  $h(x, 0) = id_X$  and  $h(X \times [0, 1]) \subset A$ .

$A$  Set  $A \subset X$  homotopes indignity in  $X$  if  $X \setminus A$  homotopes it is dense in  $X$ .

Investment  $e: Y \rightarrow X$  homotopes dense (accordingly, homotopes indignity), if  $e(Y)$  homotopes dense set (according homotopes indignity in  $X$ ).

**The theorem 5.** For infinite of compact  $X$  and any  $n \in N$  space  $P(X) \setminus P_n(X)$  homotopes is dense  $P(X)$ .

The proof. As we have noticed, that for infinite компакта  $X$  space  $P(X)$  homeomorphous gibert to cube  $Q$ . With another of the parties, for  $n \in N$  subspace  $P_n(X)$  is  $Z$ -plurality in  $P(X)$  [8].

Required homotopes  $h(\mu, t): P(X) \times [0, 1] \rightarrow P(X)$  строим the formula believing

$$h(\mu, t) = (1-t)\mu + \mu_0 t$$

Where,  $\mu_0 \in P(X) \setminus P_n(X)$  any fixed measure. For definiteness for  $\mu_0$  it is possible to take any measure concentrated in  $(n+1)$  points. I.e.  $\mu_0 = m_1 \delta_{x_1} + m_2 \delta_{x_2} + \dots + m_n \delta_{x_n} + m_{n+1} \delta_{x_{n+1}}$  where  $m_i > 0$

and  $\sum_{i=1}^{n+1} m_i = 1$ .

If  $t = 0$ ,  $m_0 h(\mu, 0) = (1-0)\mu + 0 \cdot \mu_0 = \mu \in P(X)$  i.e.  $h(\mu, 0) = id_{P(X)}$ .

If  $t > 0$  then  $h(\mu, t) = (1-t)\mu + t \cdot \mu_0 \in P(X) \setminus P_n(X)$  as  $\text{supp } h(\mu, t)$  consists more than  $(n+1)$  points. I.e.  $h(\mu, t) \notin P_n(X)$ , if  $t > 0$ . It means, that  $h(\mu_0, (0, 1]) \in P(X) \setminus P_n(X)$ . The theorem it is proved.

From this theorem 5 and from definition homotopes indignity in subsystem

**Consequence 3.** For any  $n \in N$  and infinite компакта  $X$  spaces  $P_n(X)$  homotopes indignity in  $P(X)$ .

By definition of space  $P_{f,n}(X)$  for any compact  $X$  space  $P_{f,n}(X)$  is subsystem compact  $P_n(X)$ .

From here, subspace  $P_{f,n}(X)$  is  $Z$  – plurality in  $P(X)$  as  $P_n(X)$  is  $Z$  – huge in  $P(X)$  [8].

In this case from the theorem 5, in particular, follows

**Consequence 4.** For any infinite compact  $X$  and for any  $n \in N$  subsystem  $P(X) \setminus P_{f,n}(X)$  homotopes is dense in  $P(X)$ .

In communication, with a consequence 3 and by definition homotopes indignity we have

**Consequence 5.** For any infinite compact  $X$  and for any  $n \in N$  space  $P_{f,n}(X)$  homotopes indignity in  $P(X)$ .

It is known, that for any compact  $X$  subspace  $P_\omega(X)$  spaces  $P(X)$  everywhere is dense in  $P(X)$ .

For infinite compact  $X$  space  $P_\omega(X)$  is convex, locally convex and  $\sigma$  – compact. It means, that space  $P_\omega(X)$  is  $AR$  space. On the other hand, for any infinite compact  $X$  subspace  $P_\omega(X)$  is  $\sigma$  –  $Z$  – multitude in  $P(X)$ .

**The theorem 6.** For any infinite compact  $X$  and for any  $n \in N$  subspace  $P_\omega(X) \setminus P_n(X)$  homotoped is dense in  $P_\omega(X)$ .

**The proof.** Let  $X$  infinite compact and  $n \in N$ . Required to homotope  $h(\mu, t) : P_\omega(X) \times [0, 1] \rightarrow P_\omega(X)$  we build believing

$$h(\mu, t) = (1-t)\mu + t \cdot \mu_0.$$

Where  $\mu_0$  any measure from set  $P_{n+2}(X) \setminus P_{n+1}(X)$  i.e.  $\mu_0 \in P_{n+2}(X) \setminus P_{n+1}(X)$ .

$$\mu_0 = m_1 \delta_{x_1} + m_2 \delta_{x_2} + \dots + m_{n+2} \delta_{x_{n+2}}, \quad \sum_{i=1}^{n+2} m_i = 1 \quad \text{and} \quad m_i > 0.$$

If  $t = 0$  then  $h(\mu, 0) = (1-0)\mu + 0 \cdot \mu_0 = \mu$  Means,  $h(\mu, 0) = id_{P_\omega(X)}$ .

If  $t > 0$ , then  $h(\mu, t) = (1-t)\mu + t \cdot \mu_0 \in P_n(X)$  as  $|\text{supp } h(\mu, t)| \geq n+1$ .  $t \in (0, 1]$  Measure  $h(\mu, t)$

means for any belongs to subspace  $P_\omega(X) \setminus P_n(X)$ . As were required to prove.

From the theorem 6 follows

From definition of space  $P_{f,n}(X)$  is a subset of space  $P_\omega(X)$  and in connection with the theorem 6, takes place

**Consequence 6.** For any infinite compact  $X$  and for any  $n \in N$  space  $P_\omega(X) \setminus P_{f,n}(X)$  homotoped is dense in  $P_\omega(X)$

**Consequence 7.** For any  $n \in N$  and any infinite compact  $X$  subspace  $P_n(X)$  homotoped indignity in  $P_\omega(X)$ .

From a consequence 6, in particular, follows

**Consequence 8.** For any  $n \in N$  and any infinite compact  $X$  subspace  $P_{f,n}(X)$  homotoped indignity in  $P_\omega(X)$ .

In work [10] there is a following

**The offer 2 [10].** We will admit  $X$  is *ANR* space  $Y \subset X$  homotoped densely in  $X$ . Then  $Y$  too *ANR* space.

From this offer 2 [10] and theorems 2 follows.

**Consequence 9.** For any infinite compact  $X$  and any  $n \in N$  subspace  $P_\omega(X) \setminus P_n(X)$  and  $P_\omega(X) \setminus P_{f,n}(X)$  is *ANR* space.

Hence, owing to camber of these subspaces they are pulled together *AR* spaces. I.e. for any  $n \in N$  and any infinite compact  $X$  spaces  $P(X) \setminus P_{f,n}(X)$  and  $P_\omega(X) \setminus P_{f,n}(X)$  is *AR* spaces. On the other hand takes place  $P_\omega(X) \setminus P_n(X) \subseteq P(X) \setminus P_n(X)$ .

**The theorem 7.** For any infinite compact  $X$  and for any  $n \in N$  subspace  $P_n(X)$  is  $Z$ -multitude in  $P_\omega(X)$ .

**The proof.** Let  $X$  infinite compact and  $n \in N$ . For any  $\varepsilon > 0$  required to display  $f_\varepsilon : P_\omega(X) \rightarrow P_\omega(X) \setminus P_n(X)$  we build believing

$$f_\varepsilon(\mu) = (1 - \varepsilon)\mu + \varepsilon\mu_0, \quad \mu_0 \in P_\omega(X) \setminus P_n(X)$$

Where  $\mu_0 = m_0\delta_{x_0} + m_1\delta_{x_1} + m_2\delta_{x_2} + \dots + m_{n+2}\delta_{x_{n+2}}$ ,  $\text{supp } \mu_0 = \{x_0, x_1, \dots, x_{n+2}\}$ ,  $\sum_{i=1}^{\infty} m_i = 1, m_i \geq 0$ ,

$|f_\varepsilon(\mu) - \mu| = |(1 - \varepsilon)\mu + \varepsilon\mu_0 - \mu| = |\mu - \varepsilon\mu + \varepsilon\mu_0 - \mu| = |\mu_0 - \mu| \cdot \varepsilon$ . It is obvious, that takes place  $f_\varepsilon(P_\omega(X)) \cap P_n(X) = \emptyset$  and  $(f_\varepsilon, id_{P_\omega(X)}) < U$  for everyone  $U \in \text{cov}(P_\omega(X))$ .

Let's admit  $U$  some family of subsets of space  $X$ . We say, that two displays  $f, g : Y \rightarrow X$   $U$ -sullenly (we write  $(f, g) < U$ ) if for everyone  $y \in Y$  takes place  $f(y) \neq g(y)$  and  $U \in U$ ,  $f(y), g(y) \in U$ . Through  $\text{cov}(X)$  the family of all opened covered spaces  $X$  is designated.

The following equivalent definitions  $Z$ -multitude of space  $X$  is sometimes used. Set  $A$  space  $X$  is called (strong)  $Z$ -множеством in  $X$  if  $A$  closed and for each covering  $U \in \text{cov}(X)$  there is display  $f : X \rightarrow Y$  the such. That  $(f, id_X) < U$  and  $f(X) \cap A = \emptyset$  (accordingly,  $C\ell_X f(A) \cap A = \emptyset$ ). Means,  $P_n(X)$  is  $Z$ -multitude in  $P_\omega(X)$ . The theorem it is proved.

**Consequence 10.** For any infinite compact  $X$  and for any  $n \in N$  subspace  $P_{f,n}(X)$  is  $Z$ -multitude in  $P_\omega(X)$ .

In work [10] the following has

**The offer 1.4.2 [10].** Let  $A$  topologically full subset *ANR* of space  $X$  which is  $A = \bigcup_{n=1}^{\infty} A_n$  and  $A_n$  is  $Z$ -multitude in  $X$ . Then  $A$  is  $Z$ -multitude in  $X$ .

**Consequence 11.** For any infinite compact  $X$  and for any  $n \in N$  subspace  $P_f(X)$  is  $Z$ -multitude in  $P_\omega(X)$ .

In communication, with a consequence 11 and under above resulted offer 2 [10] is has **Consequence 12.** For any infinite compact  $X$  and for any  $n \in N$  subspace  $P_\omega(X) \setminus P_f(X)$  is  $ANR$ -space.

From this the offer 1.4.2. [10] and theorems 3 we will receive

**The theorem 4.** For any infinite compact  $X$  space  $P_\omega(X)$  possesses compact  $Z$ -mean. I.e. any compact subset space  $P_\omega(X)$  is  $Z$ -multitude.

**Definition [10].** The topological space is called  $\text{ko} Z_\sigma^-$ -subspace if  $X$  contains homotoped dense topologically full subset  $G \subset X$ .

For any closed subset  $A$  infinite compact  $X$  distinct from  $X$  space  $S_p(A)$  is  $\text{ko} Z_\sigma^-$ -subspace and any open subset  $S_p(A)$  too is  $\text{ko} Z_\sigma^-$ -space. And also is space Bera.

Closed set  $A$  of space  $X$  is called  $Z_\infty^-$ -as set if for everyone  $U \in \text{cov}(X)$  and for each display  $f: I^n \rightarrow X$  ended measured cube  $I^n$  on  $X$ , for display  $\bar{f}: I^n \rightarrow X$  takes place  $(\bar{f}, f) \prec U$  and  $\bar{f}(I^n) \cap A = \emptyset$ .

In work [10] there is a following

**The theorem 1.4.4 [10].** Let  $X$  is  $ANR$  space. For each closed subset  $A \subset X$  following conditions equivalented:

And is  $Z$ -multitude in  $X$ ;

And is  $Z_\infty^-$ -multitude in  $X$ ;

And homotoped indignity in  $X$ .

From this theorem 1.4.4 and the proved theorem 3 we have

**The theorem 5.** For any infinite compact  $X$  and any  $n \in N$  subset  $P_n(X)$  is  $Z_\infty^-$ -set in  $P_\omega(X)$ .

**Definition [10].** Steams  $(M, X)$  and  $(M', X')$  is called homeomorphous if exists such homeomorphism  $f: M \rightarrow M'$  such, that  $h(X) = X'$ .

Let  $A$  counted everywhere a dense subset infinite compact  $X$  distinct from  $X$ . As already we

noticed, that equality takes place: 
$$P(A) = P\left(\bigcup_{i=1}^{\infty} A_i\right) = \bigcup_{i=1}^{\infty} P(A_i)$$
 where  $A_k = \{a_0, a_1, a_3, \dots, a_k\}$ ,  $A = \{a_0, a_1, \dots, a_n, \dots\}$ ,  $A \subset X$  and  $\bar{A} = X, A \neq X$ .

Подпространство  $P(A_k)$  homeomorphous  $k$ -measured to simplex  $\sigma^k$ -вершины which lay in point  $\{a_1, a_2, \dots, a_k\}$  i.e.  $\sigma^k = T(a_0, a_1, \dots, a_k)$ . Means, subspace  $\bigcup_{i=1}^{\infty} P(A_i)$  is counted association

ended measured simplexes, on the other hand  $\bigcup_{i=1}^{\infty} P(A_i)$  is convened and everywhere is dense in  $P(X) \sqcup Q$ . I.e.  $P(A)$  unlimited measured  $\sigma$ -fd-compact convex, everywhere dense subspace of gibert cube  $P(X) \cong Q$ .

In this case, in work [9] there is a following:

**The theorem [9].** If  $C$  unlimited measured  $\sigma - fd -$  compact convex subset of full metric linear space  $E$  then  $C$  possesses compact by  $Z -$  mean. I.e. any compact subset with is  $Z -$  multitude. Means has (owing to the theorem [9]) a place the following

**The theorem 6.** For any counted everywhere dense subset  $A$  infinite compact  $X$  is distinct from  $X$ , space  $P(A)$  possesses compact by  $Z -$  mean.

From this theorem 6, in particular, follows

**Consequence 13.** For any counted everywhere dense subset  $A$  infinite compact  $X$  distinct from  $X$ , subspace  $P(A)$  is  $\sigma - Z -$  set in  $P(X)$ .

In this case, owing to infinity compact  $X$  space  $P(X)$  homeomorphous  $Q$  and  $P(X)$  is  $AR$  space.

In this case, owing to the theorem 2 [9]. The following Takes place

**The theorem 8.** For any counted everywhere dense subset  $A$  infinite compact  $X$  distinct from  $X$ , pair  $(P(X), P(A))$  homeomorphous to steam  $(Q, Q^f)$ .

Where  $Q^f = \{(x_i) \in Q : x_i = 0 \text{ for all final } i\}$  and  $Q^f$  homeomorphous  $\ell_2^f = \{(x_i) \in \ell_2 : x_i = 0 \text{ for all final } i\}$ .

In work [9] there is a following:

**Consequence 4 [9].** Let  $W$  unlimited measured  $\sigma -$  compact convex subset of full linear space. If  $W$  not consisted of gibert cube  $Q$  and  $\overline{W} \in AR(M)$ . Then  $\overline{W} \setminus W \sqcup \ell_2$ .

On the other hand, for any count everywhere dense subset  $A$  infinite compact  $X$  distinct from  $X$  space  $P(X) \setminus P(A)$  homeomorphous  $\ell_2$ . Owing to this consequence 4 [9] it turns out.

If  $A$  everywhere dense counted a subset infinite compact  $X$  distinct from  $X$  then space  $P(A)$ . To equally space  $P_\omega(A)$  i.e.  $P(A) \sqcup P_\omega(A)$ . From here,  $\overline{P_\omega(A)} = P(X)$  and therefore truly following

**The theorem 9.** For any counted everywhere dense subset  $A$  infinite компакта  $X$  pair  $(P(X), P(A))$  homeomorphous to steam  $(Q, \ell_2^f)$ . Hence,  $P(X) \setminus P_\omega(A)$  homeomorphous  $\ell_2$ .

Let's remind some concepts from the theory shapes which to us are necessary further. Let  $A$  and  $B$  - компакты, laying in gibert cube  $Q$ . Shaped as display compact  $A$  in compact  $B$  is called such sequence of displays  $f_n : Q \rightarrow Q$ , that for any vicinity  $V$  compact  $B$  will be vicinity  $U$  compact  $A$  and natural  $N_0$ , such, that at  $n \in N_0$ ,  $f_n(U) \subset V$  and  $f_n|_U \sqcup f_{n+1}|_U (n \in V)$  /i.e. displays  $f_n|_U$  and  $f_{n+1}|_U$  homeomorphous as displays in space  $V$ /. It shaped display we will designate through:  $f = \{f_n, A, B\} : A \rightarrow B$ . Two, shaped displays  $f = \{f_n, A, B\}$  and  $g = \{g_n, A, B\}$  compact  $A$  in compact  $B$  are called homeomorphous  $(f \sqcup g)$  if for any vicinity  $V$  compact  $B$  will be vicinity  $U$  compact  $A$  and natural number  $N_0$ , such, that at  $n \geq N_0$ ,  $f_n|_U \sqcup g_n|_U$  /on  $V$ /. Shaped display  $id_A = \{id_A, A, A\} : A \rightarrow A$  is called as identical, if  $f$  and  $g$  two shaped displays, laying in gibert

cube  $Q$  compacts as their composition is called display  $gf = \{g_n, f_n, A, C\} : A \rightarrow C$ , where  $f = \{f_n, A, B\}$ ,  $g = \{g_n : B, C\} : B \rightarrow C$ .

Shaped display  $f : A \rightarrow B$  is called shaped as equivalence if there is such display  $g : B \rightarrow A$ , that  $fg \sqsubseteq id_B$  and  $gf \sqsubseteq id_A$ . If there is a display satisfying to this condition of compact  $A$  on compact  $B$  we say, that compacts  $A$  and  $B$  have identical shapes and we write  $Sh(A) = Sh(B)$ .

In work [7] there is a following

**The theorem 2 [7].** If compacts  $X$  and  $Y$  is  $Z$ -multitude gibert cube  $Q$  them shapes coincide ( $ShX = ShY$ ) in only case when homeomorphous in addition  $(Q \setminus X \sqsubseteq Q \setminus Y)$ .

**The theorem 10.** For any infinite compacts  $X$  and  $Y$  and for any  $n \in N$  spaces  $P(X) \setminus P_n(X)$  homeomorphous in only case when  $ShP_n(X) = ShP_n(Y)$ .

**The proof.** Let  $X$  and  $Y$  infinite compacts, then under Keller-Keli  $P(X)$  theorem and  $P(Y)$  homeomorphous of gibert cube  $Q$ . Under the theorem 2 [3] subspace  $P_n(X)$  and  $P_n(Y)$  are  $Z$ -multitude in  $P(X)$  and  $P(Y)$  accordingly.

If  $ShP_n(X) \sqsubseteq ShP_n(Y)$ , under the above-named theorem of 2 [7] spaces  $P(X) \setminus P_n(X)$  and  $P(Y) \setminus P_n(Y)$  homeomorphous. If  $P(X) \setminus P_n(X) \sqsubseteq P(Y) \setminus P_n(Y)$  on the same theorems 2 [7] shapes  $P_n(X)$  and  $P_n(Y)$  coincide. I.e.  $ShP_n(X) = ShP_n(Y)$  As these subspace are  $Z$ -multitude in spaces  $P(X)$  and  $P(Y)$  accordingly. The theorem 10 is proved.

For any compacts space  $P_{f,n}(X)$  is subspaces  $P_n(X)$ . On the other hand, if  $P_n(X)$  is  $Z$ -multitude in  $P(X)$   $P_{f,n}(X)$  too will be  $Z$ -multitude in  $P(X)$ .

**Consequence 14.** For any infinite compacts  $X, Y$  and for any  $n \in N$  it has:  $ShP_{f,n}(X) = ShP_{f,n}(Y)$  in only case when,  $P(X) \setminus P_{f,n}(X)$  and  $P(Y) \setminus P_{f,n}(Y)$  homeomorphous.

On the other hand for any compact  $X$  subspace  $P_f(X)$  is counted association  $Z$ -multitude and  $P_f(X)$  too compact. From here, compact  $P_f(X)$  too is  $Z$ -multitude in  $P_f(X)$ .

In this case, under the theorem 10 it is had

**Consequence 15.** For any infinite compacts  $X, Y$  shaped spaces  $P_f(X)$  and  $P_f(Y)$  coincide in only case when, when spaces  $P(X) \setminus P_f(X)$  and  $P(Y) \setminus P_f(Y)$  homeomorphous.

From definition  $Y$ -multitude and proved theorems concerning  $Y$ -multitude we can confirm.

**The theorem 11.** For any infinite compact  $X$  we take place the following:

For any  $n \in N$  spaces  $P(X) \setminus P_n(X)$ ,  $P(X) \setminus P_{f,n}(X)$  and  $P(X) \setminus P_f(X)$  are  $Q$ -of multitudes;

For any everywhere dense counted subsets  $A$  distinct from  $X$  spaces  $P_\omega(A)$  and  $P(A)$  are  $\ell_2^f$  varieties;

For ended measured compact  $X$  space  $P_\omega(X)$  is  $\ell_2^f$  variety;

If  $P_\omega(X)$  contains gibert cube  $Q$   $P_\omega(X)$  is  $\Sigma$ -multitude;

Space  $P(X) \setminus P_\omega(X)$  is  $\ell_2$ -variety;

For any everywhere dense open subset  $A$  distinct from  $X$  subspace:

- 1)  $P(A)$  is  $\ell_2$ -variety if  $P(A)$  does not contain gibert cube  $Q$ ;
- 2)  $P(A)$  is  $\sum$ -variety if  $P(A)$  contains gibert cube  $Q$ ;

In particular takes place

**Consequence 16.**  $P(Q) \setminus P_n(Q)$  is  $Q^-$ -multitude, for any  $n \in N$ ;

$P_\omega(Q)$  is  $\sum^-$ -multitude;

$P(Q) \setminus P_\omega(Q)$  is  $\ell_2$ -variety;

**The theorem 12.** For any infinite compact  $X$  and any closed subset  $A$  distinct from  $X$  space  $S_P(A)$  homeomorphous  $\ell_2$ .

$P(X) \setminus P(X \setminus A) = S_P(A)$ . In data, case  $P(X \setminus A)$  is a boundary subset in  $P(X)$  [8]. Then its addition  $S_P(A) \sqcup \mathbb{R}^2$ . The theorem it is proved.

**The theorem 13.** Let  $X$  infinite compact consisting of final number a connectivity component then space  $P(X) \setminus P_n^C(X)$  homeomorphous is dense  $P(X) \setminus P_n^C(X)$ .

**The proof.** Let  $x_1, x_2, \dots, x_k$  connectivity components of compact  $X$ . Owing to infinity compact  $X$ , exists at least one component, which non-single pointed. Let  $x_1, x_2, \dots, x_r$ -one-dot components of connectivity and  $x_{r+1}, x_{r+2}, \dots, x_k$  - not one-dot components of connectivity. Means, compact  $X$  is

association of these  $X_i$  i.e.  $X = \bigcup_{i=1}^k X_i$ ,  $X_i \cap X_j \neq \emptyset$ ,  $i \neq j$ . Owing to definition of space  $P_n^C(X)$

it is had  $P_n^C(X) = P_n^C(\bigcup_{i=1}^k X_i) = \bigcup_{i=1}^k P_n^C(X_i)$ . Means, for  $i = \overline{1, r}$  space  $P_n^C(X_i)$  consists from one points-measures of Dirak. For  $i \in \overline{r+1, k}$  by definition of spaces takes place  $P_n^C(X_i) = P_n(X_i)$  i.e.

$$P_n^C(X) = \{\sigma_{x_1}, \sigma_{x_2}, \dots, \sigma_{x_r}\} \cup \bigcup_{i=r+1}^k P_n(X_i)$$

In work [8] we have shown, that for any  $n \in N$  spaces  $P_n(X_i)$  are  $Z$ -multitude in  $P(X)$ . It is obvious, that  $P_n^C(X)$  it is closed in  $P(X)$  and  $P_n^C(X) \neq P(X)$ . We fix one point  $\mu_0 \in P(X) \setminus P_n^C(X)$ . For example

$$\mu_0 = m_1 \sigma_{x_1} + m_2 \sigma_{x_2} + \dots + m_k \sigma_{x_k}, \text{ where } x_i \in X_i, i > r, m_i > 0, \sum_{i=1}^k m_i = 1$$

For any  $\varepsilon > 0$  we will construct to display  $f_\varepsilon(\mu) : P(X) \rightarrow P(X)$  flat  $f_\varepsilon(\mu) = (1 - \varepsilon)\mu + \varepsilon\mu_0$ . We will consider a difference

$$|f_\varepsilon(\mu) - \mu| = |(1 - \varepsilon)\mu + \varepsilon\mu_0 - \mu| = |\mu - \varepsilon\mu + \varepsilon\mu_0 - \mu| = |\varepsilon(\mu - \mu_0)| = \varepsilon \cdot |\mu - \mu_0|$$

If  $\mu \in P_n^C(X)$ ,  $f_\varepsilon(\mu) \in P_n^C(X)$ . Now we will show, that space  $P(X) \setminus P_n^C(X)$  homeomorphous is dense in  $P(X)$ . Required homeomorphous  $h(\mu, t): P(X) \times [0, 1] \rightarrow P(X)$  we build flat  $h(\mu, t) = (1-t)\mu + t\mu_0$ . It is obvious, that

$$h(\mu, 0) = (1-0)\mu + 0 \cdot \mu_0 = \mu = h(\mu, 0) = id_{P(X)}.$$

If  $t > 0$ ,  $h(\mu, t) \in P_n^C(X)$ . I.e.  $P(X) \setminus P_n^C(X)$  homeomorphous is dense in  $P(X)$ . The theorem is proved.

From this theorem follows

**Consequence 17.** For any infinite compact  $X$  consisting of final number a connectivity component subspace  $P_n^C(X)$  homeomorphous indignity in  $P(X)$ .

We say, that topological spaces  $X$  satisfies strong discrete approximated to property (we write SDAP), if for each display  $f: Q \times N \rightarrow X$  and each covering  $u \in cov(X)$  there is such display  $\bar{f}: Q \times N \rightarrow X$ , that  $(f, \bar{f}) \prec U$  and  $\{\bar{f}(Q \times N): n \in N\}$  discrete family in  $X$  [10].

Speak, set  $A \subset X$  is called (strong)  $Z_\sigma$ -multitude in  $X$  if  $A = \bigcup_{n=1}^{\infty} A_n$  and  $A_n$  is (strong)  $Z$ -multitude in  $X$ .

Space  $X$  is called (strong)  $Z_\sigma$ -space if itself  $X$  is (strong)  $Z_\sigma$ -multitude in  $X$ .

There is a following

**The theorem 1.4.10 [10].** Strong  $Z_\sigma$ -is space being ANR space satisfies everyone SDAP.

Means, owing to the theorem 1.4.10 takes place

For everyone everywhere dense opened subspace  $U$  infinite compact  $X$  distinct from  $X$ , space  $P(U) \in SDAP$ ;

If  $A_1 \subset A_2 \subset \dots \subset A_n \subset \dots$ ,  $A_i$  it is closed in  $X$  and  $\bigcup_{i=1}^{\infty} A_i \neq X$ ,  $X$ -unlimited,  $\bigcup_{i=1}^{\infty} P(A_i) \in SDAP$ ;

For any counted everywhere dense subset  $A$  infinite compact  $X$  spaces  $P_\omega(A) \in SDAP$  and  $P(A) \in SDAP$ ;

In particular,  $P_\omega(X) \in SDAP$ .

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