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Effect Of Multiple Cracks On Gear Mesh Stiffness And Cross Correlation Of Spur Gears

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Abstract

An important indicator for spectrum analysis of gears is gear mesh stiffness as its value has been found to fluctuated with multiple cracks and increase in crack lengths. In the current study, the effect of multiple cracks on gear mesh stiffness and cross correlation has been investigated. Multiple cracks with different values of crack lengths and fixed input frequency, output frequency and lay frequency have been considered along these paths. MATLAB software along with FRANC 3D has been used to model the gears with multiple cracks induced. The values of mesh stiffness and cross correlation have been found to vary significantly with multiple cracks of varying crack lengths
Keywords: *Gear mesh stiffness, cross correlation, crack length, input frequency.*

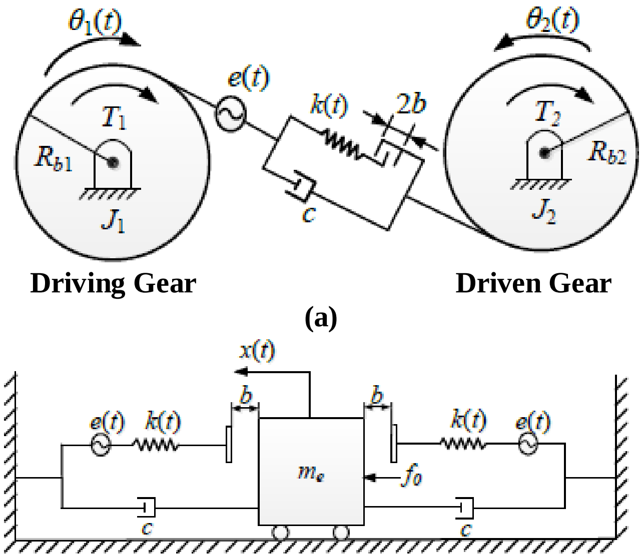
1. Introduction

Gear dynamics research focusses mainly on reducing high levels of noise and vibrations arising due to constantly changing mesh stiffness.Dynamic analysis of gears is essentially nonlinear due to changing nature of gear mesh stiffness. A linear model is developed by averaging mesh stiffness parameter and averageteeth in contact is assumed. Gear cracks often results in catastrophic damage with sufficient time and therefore its necessary to diagnose them to avoid major losses.To reduce the overall vibrational signal and increase the accuracy of vibration parameters, application of filter to gear mesh frequencies is desirable.Effects on free vibration properties are quantitatively examined due to mesh stiffness variation and manufacturing and assembling errors. Many undesired components and unwanted peaks arising due to multiple cracks in gears are analysed and investigated in this study. Direction of applied load, tooth geometry, gear material specifications, profile error and faults are the parameters on which gear mesh stiffness depends. Fatigue loading is found when teeth interaction is there which results in generation of crack.

As per the current situation, there is huge demand of silent gears and hence huge work has been carried out on analyzing the noise and vibration of transmission gears [1-6]. Gear strength analysis depends to a major extent on gear mesh misalignment which can be computed using deflection of shafts and gear mesh frequency fluctuation.During the analysis of gears, tip interference due to tooth flexibility is neglected. For healthy gears, mesh stiffness has been determined by several researchers over the last three decades [6-9]. During the gear meshing, neglecting the corner contact is valid only for double mesh zone but it influences single mesh zone to a large extent.Gear tooth has beensubjected to bending, axial and compressive loads for gear mesh stiffness calculation.Hertzian contact and fillet foundation deflection is also considered.Internal excitation is periodic in nature of transmission of gears.Chaari et al.[10] observed decrease in values of mesh stiffness.

From the above discussion, it is evident that effects of different crack angles and single crack effect on gear mesh stiffness have been the main focus of earlier researchers. Though the path of crack propagation was considered in some of these studies, multiple cracks effect and cross correlation effect on gear mesh stiffness has not been studied in detail. Therefore, the effect of multiple cracks variation in different crack propagation path on gear mesh frequency and mesh stiffness has been investigated in the current study. Multiple crack propagation paths with different crack length have been considered.Cross Correlation and Power spectrum analysis have been used to study variations instiffness of gears and its effect on cross correlation parameters concludes the paper.

2. Equivalent Models of Gear Meshing and Theoretical background



(b)
Figure 1: Gear Model : (a) Torsional Model
(b) Equivalent Single Degree of freedom Model

As per the work carried out by earlier researchers [15], Figure 1 shows a rotary model for a pair of gears. It has rotation motions θ_i of driving gear and driven gear. R_{bi} represents the base radii and T_i represents torques of gears respectively. J1 and J2 are the moment of inertia of two disks gears respectively which are joined by time varying gear mesh stiffness k(t). It consists of backlash b,displacement excitation of gear profile deviationand viscous damping c. Due to deviations of gear profile there are errors e(t) induced. The equations of motion can be represented as follows:

$$\begin{cases} J_1\ddot{\theta}_1(t) + R_{b1}\, c\, \left(R_{b1}\dot{\theta}_1(t) - R_{b2}\dot{\theta}_2(t)\right) + R_{b1}\, k(t)\, g\left(R_{b1}\theta_1(t) - R_{b2}\theta_2(t) - e(t)\right) = T_1 \\ J_2\ddot{\theta}_2(t) + R_{b2}\, c\, \left(R_{b1}\dot{\theta}_1(t) - R_{b2}\dot{\theta}_2(t)\right) + R_{b2}\, k(t)\, g\left(R_{b1}\theta_1(t) - R_{b2}\theta_2(t) - e(t)\right) = -T_2 \end{cases} \quad (1)$$

$$x(t) = R_{b1}\theta_1(t) - R_{b2}\theta_2(t) \quad (2)$$

Dynamic Transmission error $x(t)$ changes the system representation as seen in equation (3)

$$m_e\ddot{x}(t) + c\dot{x}(t) + k(t)g\left(x(t) - e(t)\right) = f_0 \quad (3)$$

Where m_e is static load and f_0 is static load which is the ratio of torque to base radius.
g(.) is a contact function and can be represented with error function as:

$$g\left(x(t) - e(t)\right) = \begin{cases} x(\tau) - e(t) - b;\ x(t) - e(t) > b \\ 0; \qquad Otherwise \\ x(t) - e(t) + b;\ x(t) - e(t) < -b \end{cases} \quad (4)$$

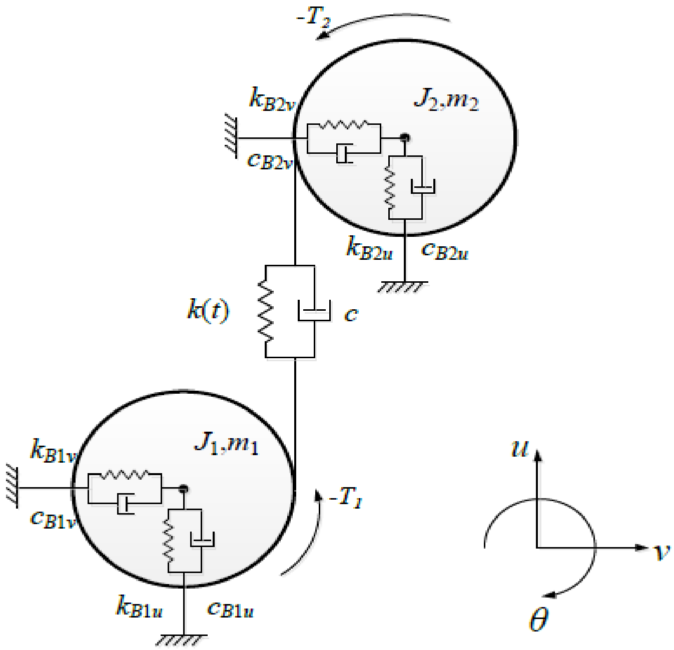


Figure 2: Equivalent 6 Degree of Freedom (DOF) Model

If we consider the 6 DOF model, then the equation of motion changes as follows:

$$\begin{cases} m_1\ddot{u}_1 + c_{B1u}\dot{u}_1 + k_{B1u}u_1 = F_t \\ m_1\ddot{v}_1 + c_{B1v}\dot{v}_1 + k_{B1v}v_1 = 0 \\ J_1\ddot{\theta}_1 + F_t R_1 = T_1 \\ m_2\ddot{u}_2 + c_{B2u}\dot{u}_2 + k_{B2u}u_2 = -F_t \\ m_2\ddot{v}_2 + c_{B2v}\dot{v}_1 + k_{B2v}v_2 = 0 \\ J_2\ddot{\theta}_2 + F_t R_2 = T_2 \end{cases} \quad (5)$$

Where F_t is net contact force and is sum of elastic force F_k and damping force F_c.

$$F_t = F_k + F_c = k_g\, \partial + c_g \dot{\partial} \quad (6)$$

Where

$\partial = R_{b1}\theta_1 + R_{b2}\theta_2 + u_2 - u_1$ is the transmission error. (7)

3. Numerical Analysis of Gear Tooth Engagement and Gear Meshing

Stiffness of gear pair in this work has been determined by considering Hertzian contact deformation between mating teeth, tooth deflection. Fillet foundation deflection has not been considered in this study. Considering the deflection due to external load, the bending stiffness, K_b, axial stiffness, K_a, and shear stiffness, K_s has been inserted in MATLAB program and total mesh stiffness has been computed. ANSYS 18 software has been used to compute gear mesh stiffness and gear meshing frequency [16]. Using Mechanical APDL, sequence of gear tooth engagement has been found as follows:

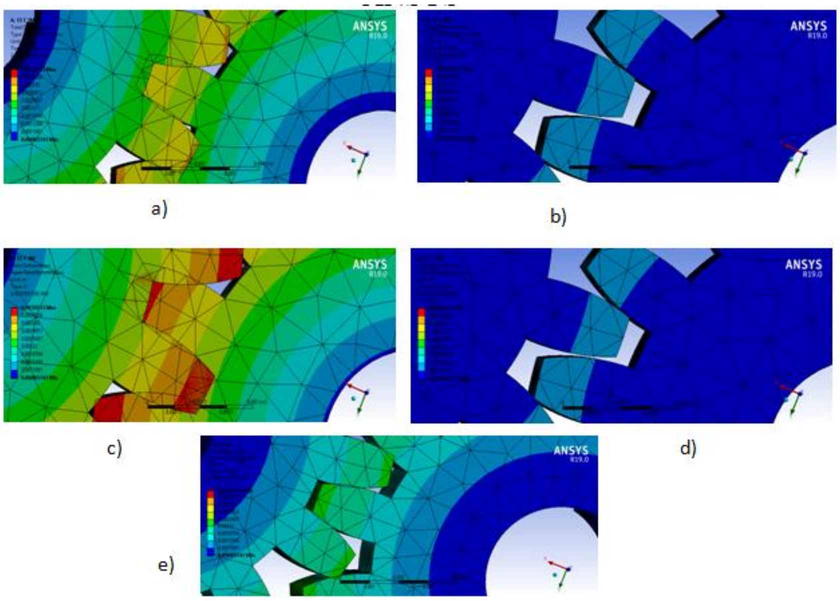


Figure 3: a) Cracked Tooth Double Contact (Initiation of Engagement)
b) Cracked Tooth Single Contact
c) Cracked Tooth Double Contact (End of Engagement)
d) Single Contact (Healthy Tooth)
e) Double Contact (Healthy Teeth)

Tooth stiffness is affected to a major extent by presence of tooth crack. When one tooth is on contact with another tooth at 2 points these 2 contacts together reduces value of the mesh stiffness as shown in Figure 3.Single tooth space occupies whole arc only when arc of action is equal to circular pitch. When one tooth is just beginning contact at one point the tooth which was in contact before ends contact at other point together with the other tooth. Therefore, there will be exactly one pair of teeth in contact during the tooth action from one to another point. But, when the arc of action is greater than circular pitch, then one pair of teeth is just entering contact at one point and another pair already in contact will not yet have reached another point. At two different separate points for very less duration of time, two teeth touch each other in different areas at same time [16].

4. Results and Discussion

Numerical analysis of gear mesh stiffness formulation as discussed in the above section has been applied to gears. The geometrical details of the gear pair are provided in Table 1. As stated in earlier section, power spectrum analysis has been performed and mesh stiffness is found out for a cracked tooth in the gear pair. The crack has been propagated towards tooth and towards rim [16].

Table 1. Geometrical details of Involute Spur Gear pair

Parameter	Driving pinion	Driven gear
Module (mm)	4	4
Number of teeth z	27	30
Pressure angle (^{0})	20	20
Addendum coefficient (ha*)	1	1
Width (mm)	20	20
Headspace coefficient (c)	0.3	0.3
Young's Modulus	210 GPa	210 GPa

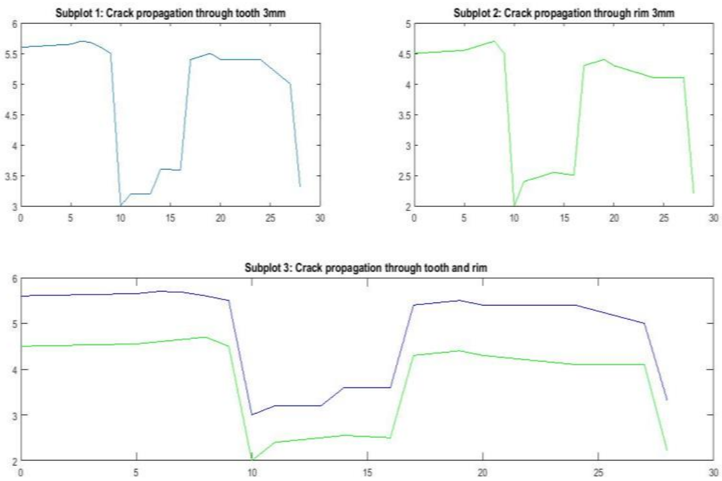


Figure 4: MATLAB output of Multiple Crack Propagation towards tooth and towards rim for crack length 2 mm.

Significant increase in mesh stiffness value is observed in Fig 2. when crack propagates towards tooth as compared to crack propagation towards rim. Multiple cracks have been inserted and the values of mesh stiffness is foundfor all cases and have been plotted against pinion tooth rotation for crack lengths 2 mm , 3 mm and 3.5 mm. The plots obtained are shown as follows in Fig 3.

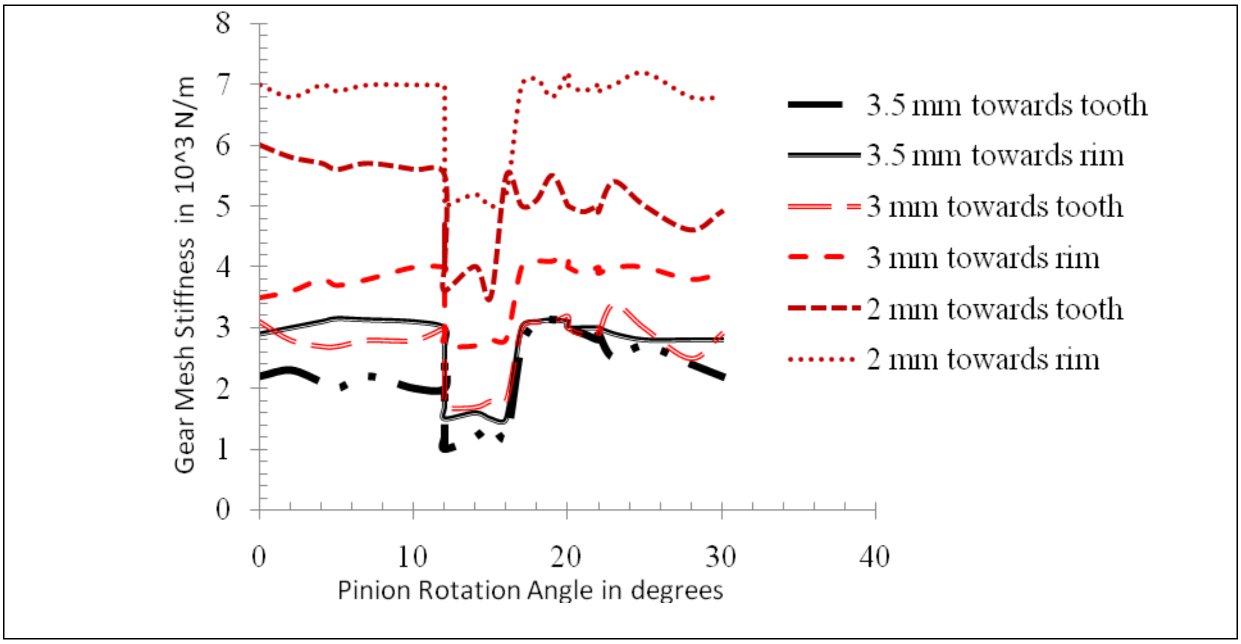


Figure 5: Gear Mesh Stiffness plot of multiple cracked gear of crack lengths 2, 3 and 3.5 mm

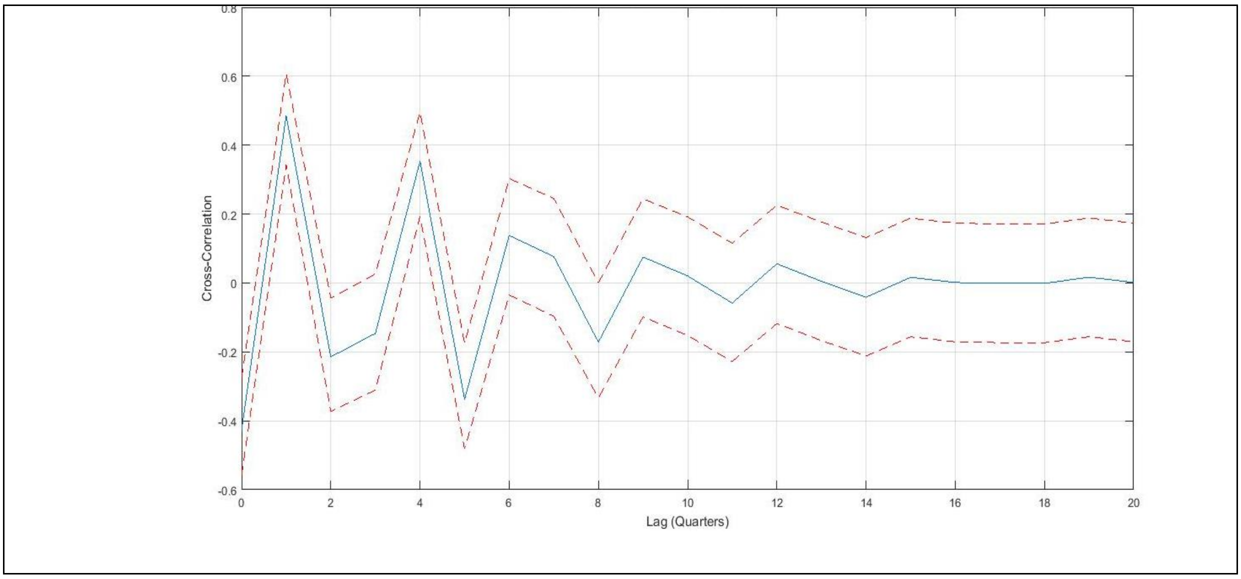


Figure 6: Cross Correlation plot of single cracked gear and multiple cracked gear of crack length 2 mm

Cross correlation plot clearly shows the secondary effects of multiple cracks. However, when the crack length is changing, multiple cracks in the same path doesn't show any significant changes for different crack lengths. Crack lengths of 2 mm, 3 mm and 3.5 mm are investigated in this study and since significant variations are observed in 2 mm crack length results, comparison of single crack and multiple cracked gear has been investigated.There is a negative correlation as seen in figure 4 when it comes to comparison between single and multiple cracked gear. Variations in cross correlation for single cracked gears are in negative scale while those for multiple cracked gear in positive scale which clearly states that distortions are more in multiple cracks as compared to single cracked gear.

Ansys software has been used to model the gears. The effect of path variation isdifferent for the three cases. Mesh stiffness values are found to be quite close to each other when the same crackmoves towards tooth and rim except for the case of 2 mm. Changes in mesh stiffness values for multiple cracks towards tooth and towards rim are most significant in case of 2mm crack length as shown in Fig.3. For 2 mm defect, highest values of mesh stiffness appear for crack towards tooth followed by crack towards rim. For 3 mm and 3.5 mm crack lengths, mesh stiffness value changes in decreasing order of crack towards tooth followed by crack towards rim.

5. Conclusions

The effect of multiple cracks on the cross correlation and mesh stiffness of a spur gear pair has been investigated in the current study. The cracks are propagated towards tooth and towards rim. Crack lengths of 2 mm, 3 mm and 3.5 mm have been considered on all the paths for a comparative analysis. FRANC 3D software is used to insert crack in the model developed in ANSYS. Static structural analysis was also performed using Mechanical APDL. The numerical results obtained from this study show that there is significant variation in values of gear mesh stiffness for different paths of progression, the variation being maximum for 2 mm crack length. Negative cross correlation is found between single crack propagation and multiple crack propagation in gears. Significant variation has been observed between mesh stiffness values for crack propagation towards tooth and rim for 2 mm crack length.

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